Entanglement and Complementarity

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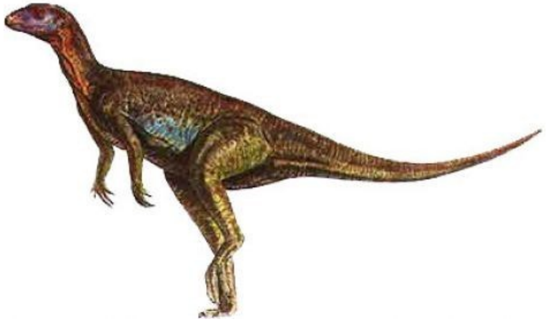
What I'm going to talk about

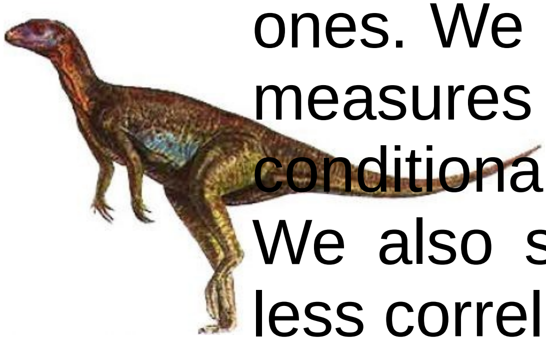
We always say that entangled states are more correlated... WHAT DOES IT MEAN exactly?



We always say that entangled states are more correlated... WHAT DOES IT MEAN exactly?

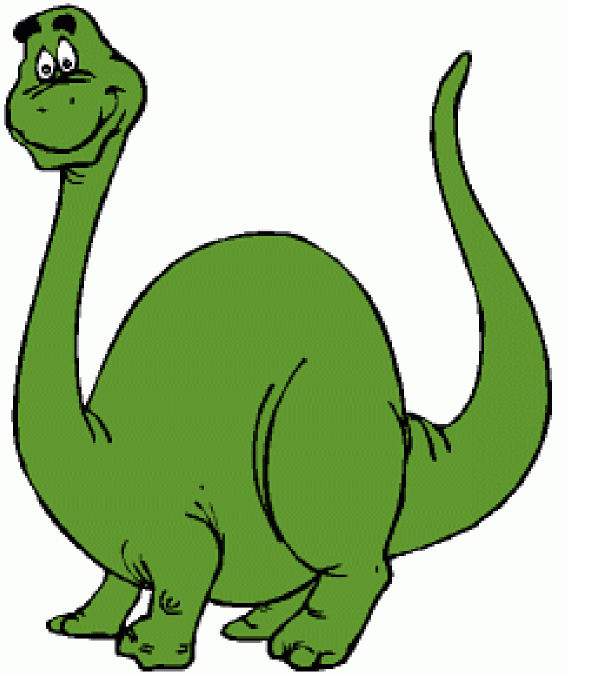
they have more correlations among complementary observables than separable ones

Abstract: We show that states that have more correlations among complementary observables must be entangled. The reverse is false: general entangled states do not have more correlations on complementary observables than separable ones. We either prove or conjecture that this is true for different measures of correlation: the mutual information, the sum of conditional probabilities and the Pearson correlation coefficient. We also show that states with nonzero discord typically have less correlation than classically correlated states.



Usual approaches to study entanglement

- Non locality
- Negative partial transpose
- Bell inequality violations
- Enhanced precision in measurements
- etc.



Here: we use correlations among two (or more) COMPLEMENTARY PROPERTIES

Remember: Complementary properties.

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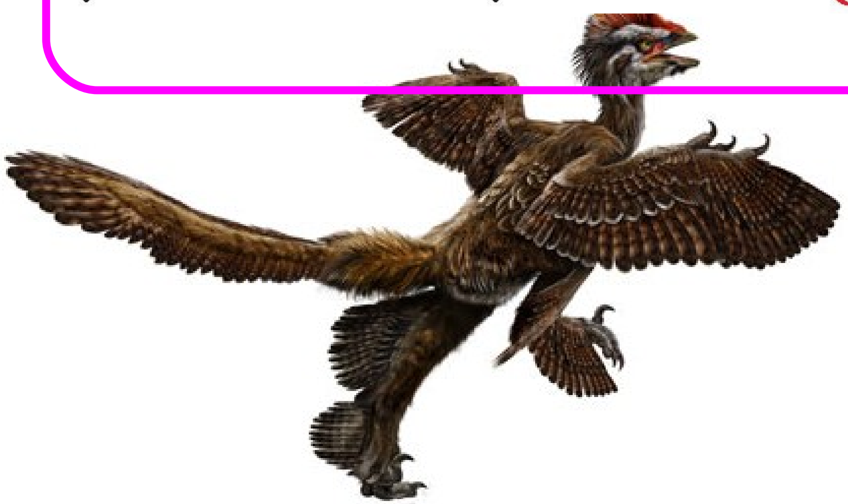
Two observables: the knowledge of one gives no knowledge of the other

$$A = \sum_a f(a)|a\rangle\langle a|$$

$$C = \sum_c g(c)|c\rangle\langle c|$$

$$\left|\langle a|c\rangle\right|^2 = \frac{1}{d}$$



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$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{|++\rangle + |--\rangle}{\sqrt{2}}$$

Maximally entangled state: perfect correlation BOTH on 0/1 and on +/-

$$|\pm\rangle \equiv \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

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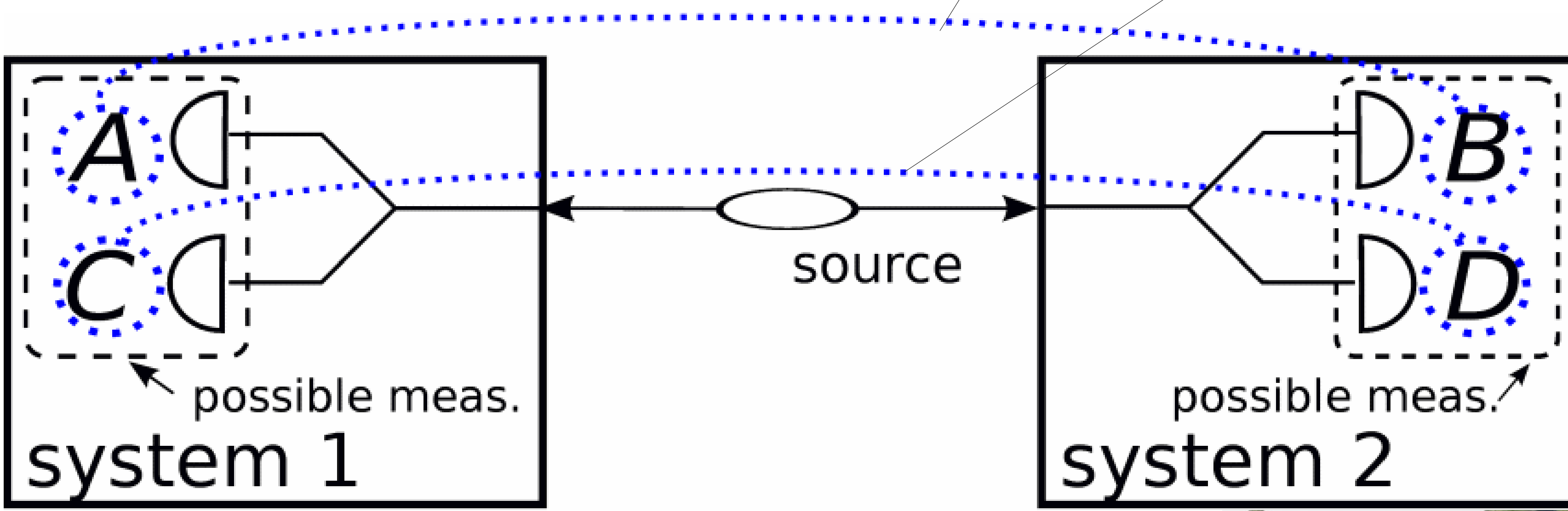
Maximally entangled state: perfect correlation BOTH on 0/1 and on +/-

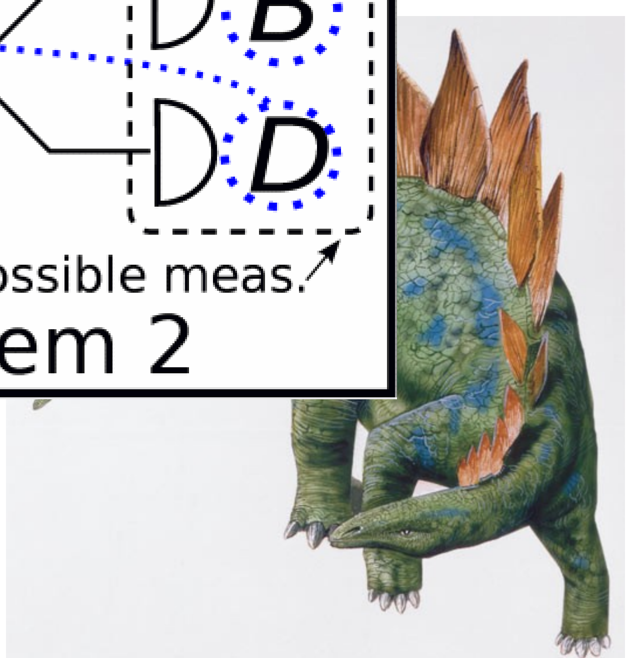
$$(|00\rangle\langle 00| + |11\rangle\langle 11|)/2 =$$
$$(|+\rangle\langle +| + |-\rangle\langle -|)/2 \otimes (|+\rangle\langle +| + |-\rangle\langle -|)/2$$

separable state: perfect correlation for 0/1, no correlation for +/-

Simple experiment

- On system 1 measure either *A* or *C*
- On system 2 measure either *B* or *D*
- Calculate correlations *A-B* and *C-D*





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$$I_{AB} = H(A) + H(B) - H(A,B)$$

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- Sum of conditional probabilities

$$\mathcal{S}_{AB} = \sum_i p(a_i|b_i)$$

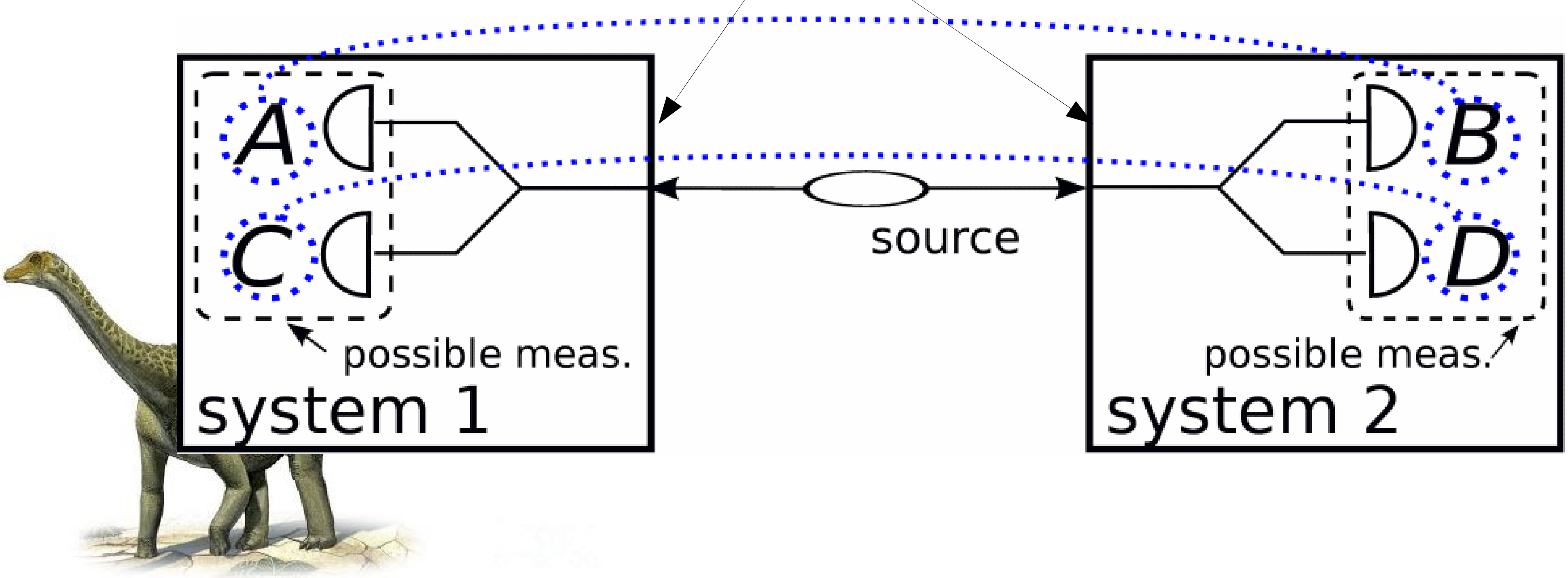
$\mathcal{S}_{AB} = 0, d \Rightarrow$ perfect correlation or anticorrelation

Use these to measure correlations among

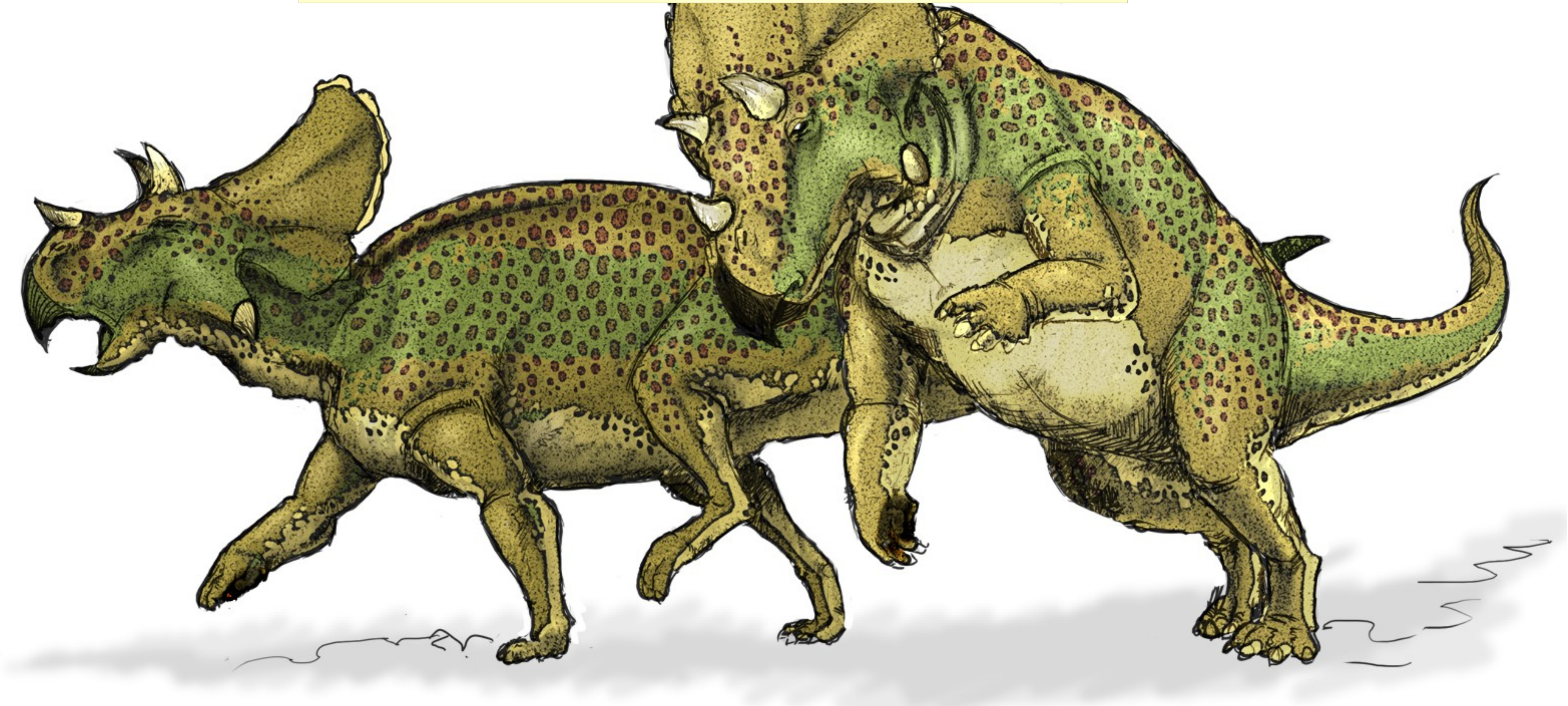
2 complementary properties

$A \otimes B$ complem to $C \otimes D$

of 2 systems



Some results...

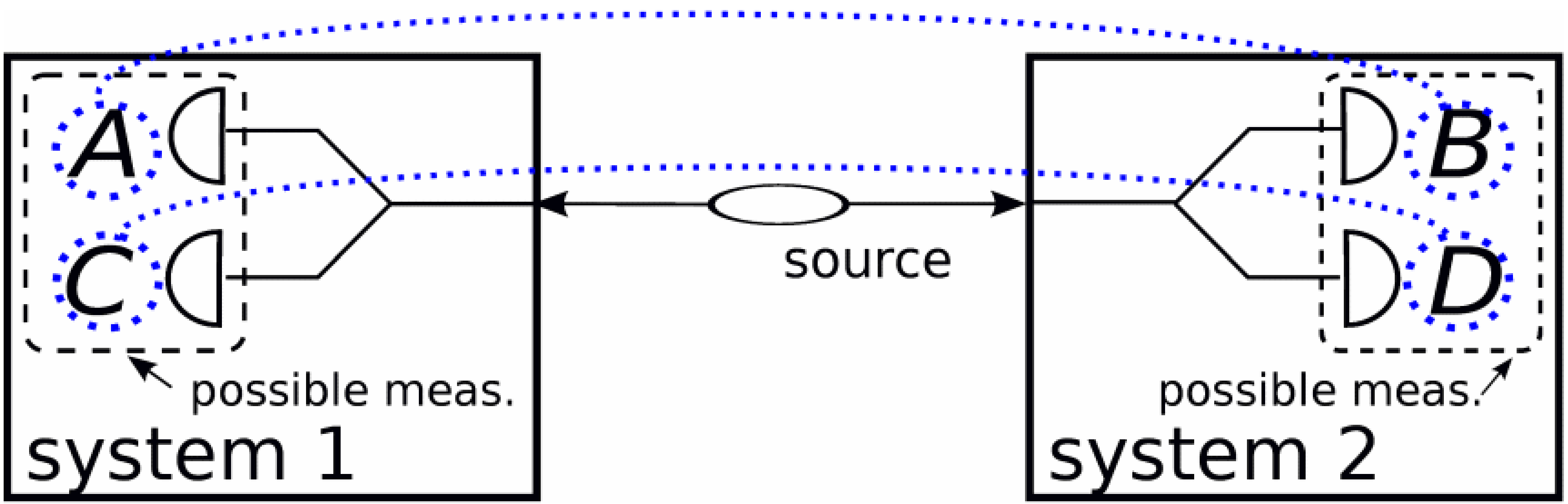


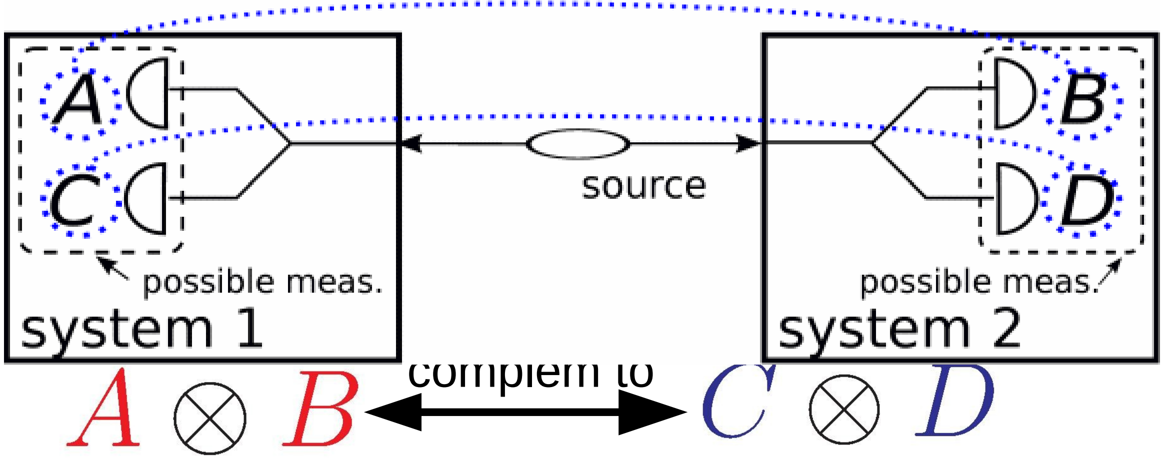
Start with mutual information

$$I_{AB} = H(A) + H(B) - H(A,B)$$

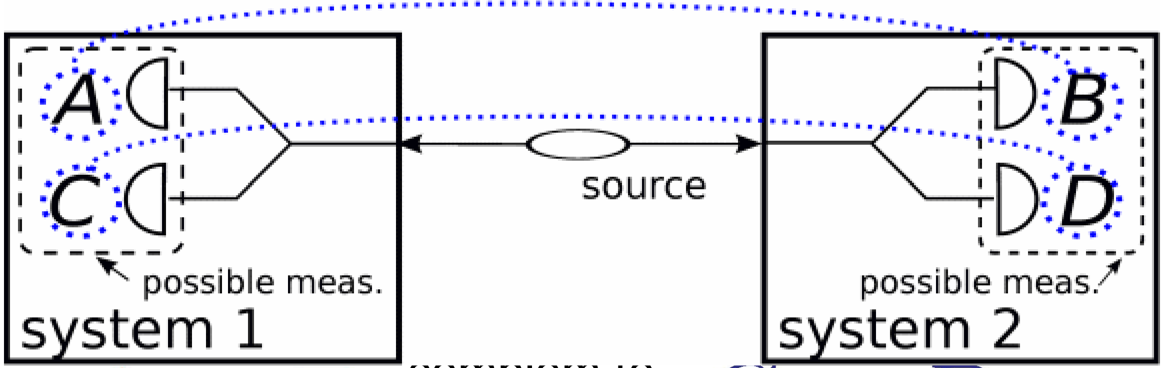
"total" correlation given by the sum

$$I_{AB} + I_{CD}$$





The system state is **maximally entangled** iff perfect correlation on **both** *A*-*B* and *C*-*D*



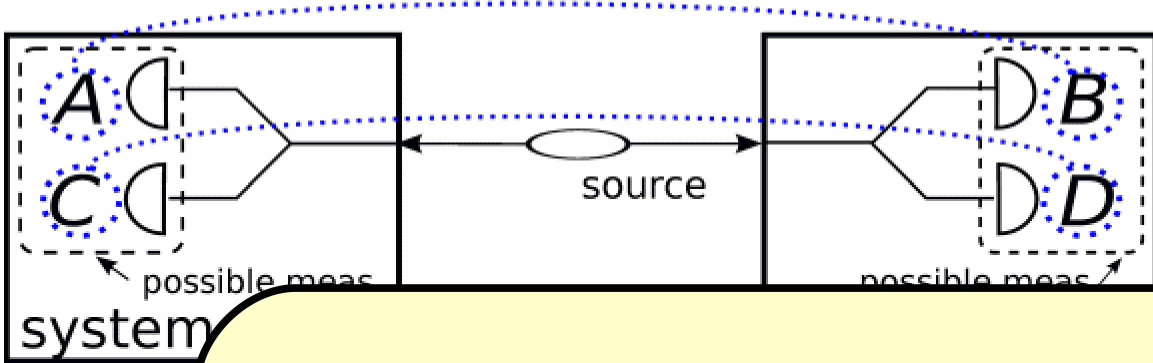
$A \otimes B$ ⟵ complem to ⟶ $C \otimes D$

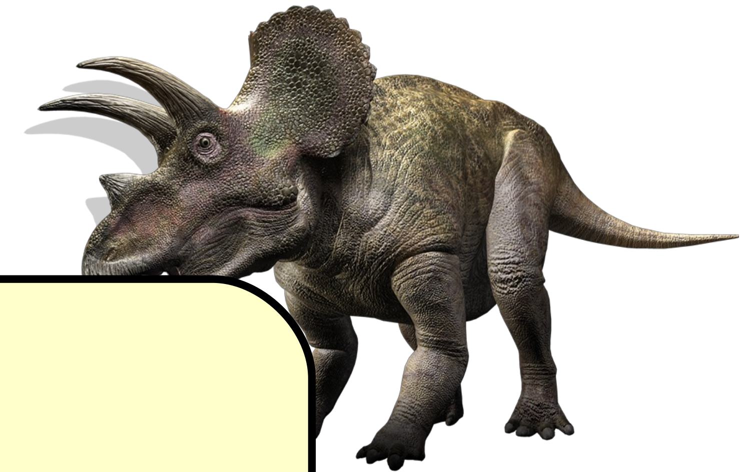
The system state is **maximally entangled** iff perfect correlation on **both** *A*-*B* and *C*-*D*

$$I_{AB} + I_{CD} = 2 \log d$$

(for some observ *ABCD*)

$\Leftrightarrow \quad |\Psi_{12}\rangle$ maximally entangled





A

$$I_{AB} \leqslant \log_2 d$$

$$I_{CD} \leqslant \log_2 d$$

T… **…mally**
e… …lation
on **both** *A-B* and *C-D*

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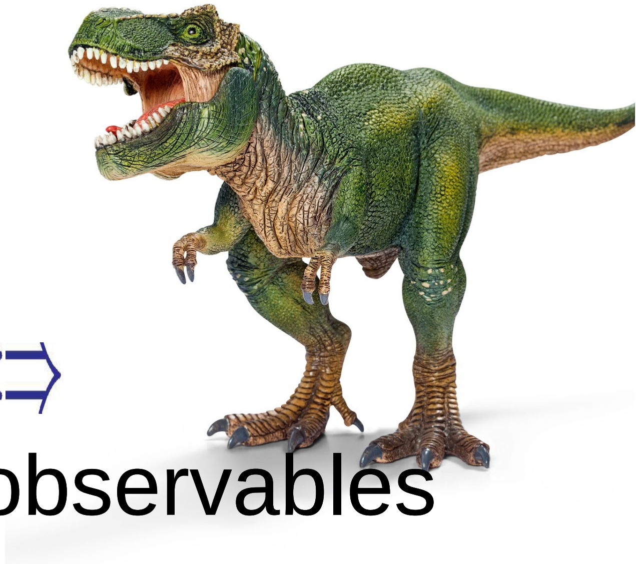
Easy to prove:

the state is maximally entangled $\Longleftrightarrow$

perfect correlations on all compl observables



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Just use simple properties of conditional probabilities, e.g.

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Just use simple properties of conditional probabilities, e.g.

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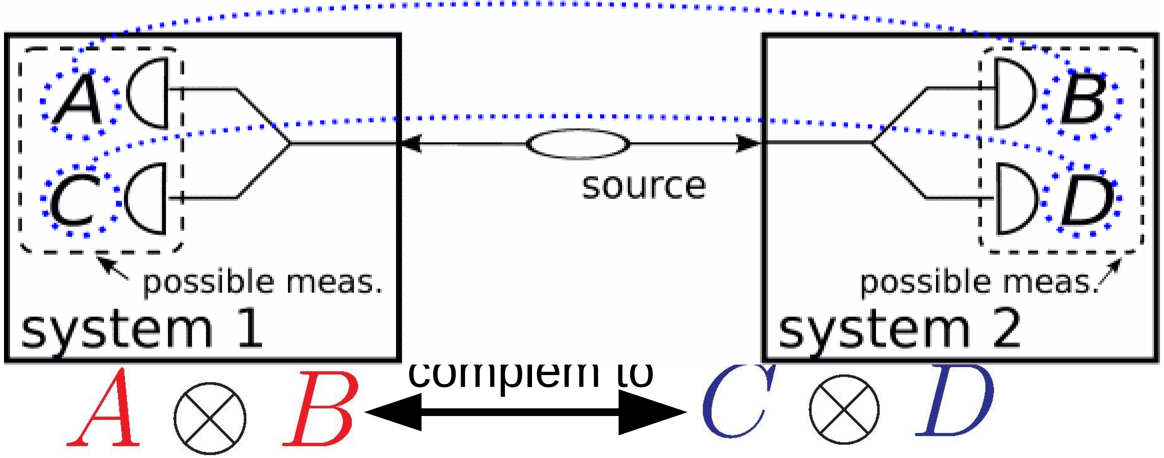
...and write the mutual info as a function of conditional probs.



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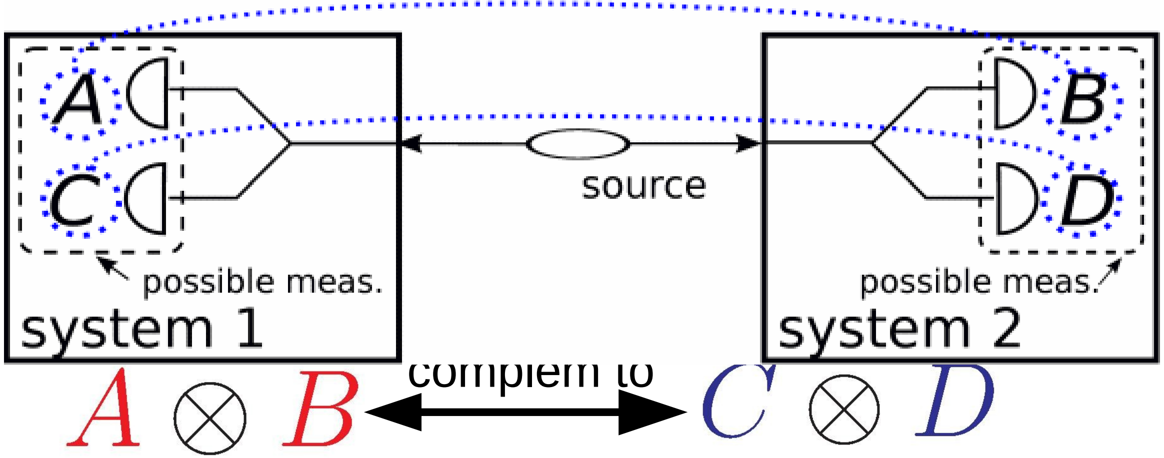

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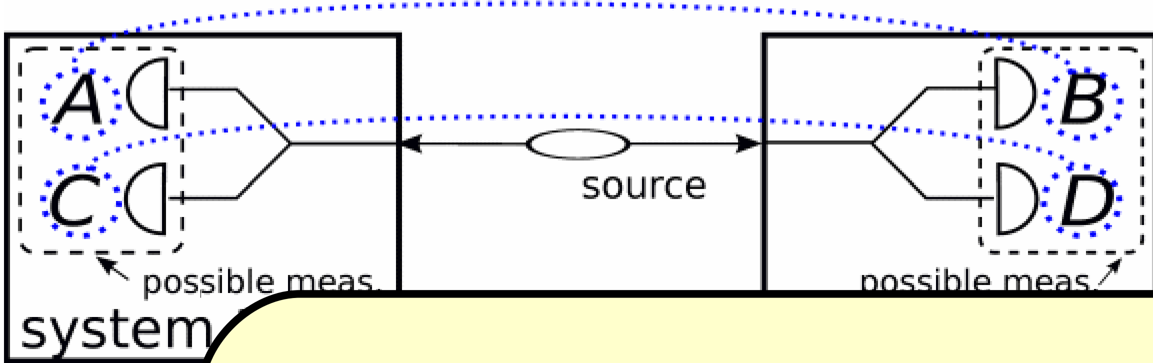
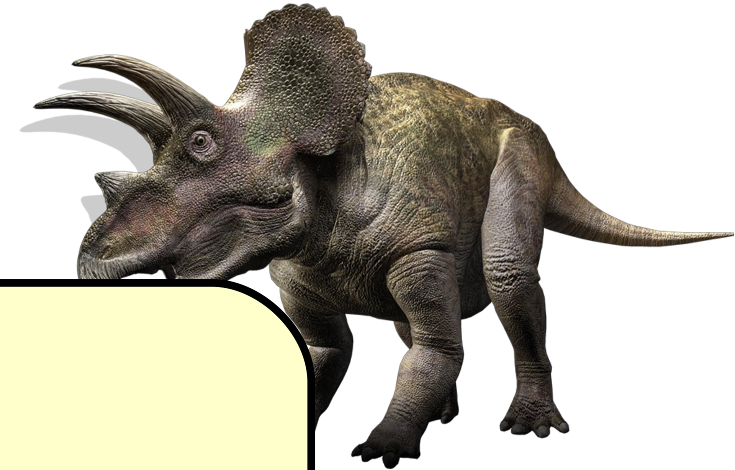
$$I_{AB} + I_{CD} > \log d \quad \Rightarrow$$

ρ_{12} ent





$A \otimes$

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Can the bound be made **tighter**?

The system state is **entangled** if correlations on **both** *A*-*B* and *C*-*D* are large enough

$$I_{AB} + I_{CD} > \log d \Rightarrow \rho_{12} \text{ ent}$$

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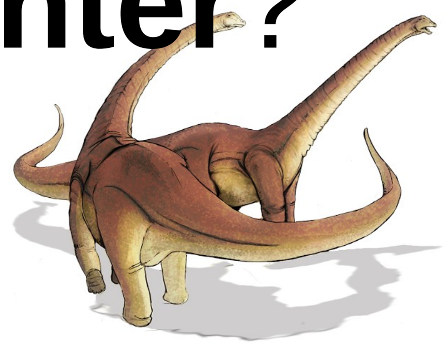
NO!!



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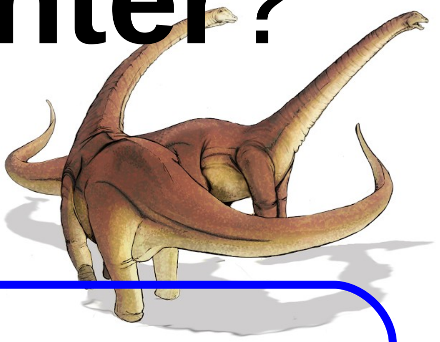


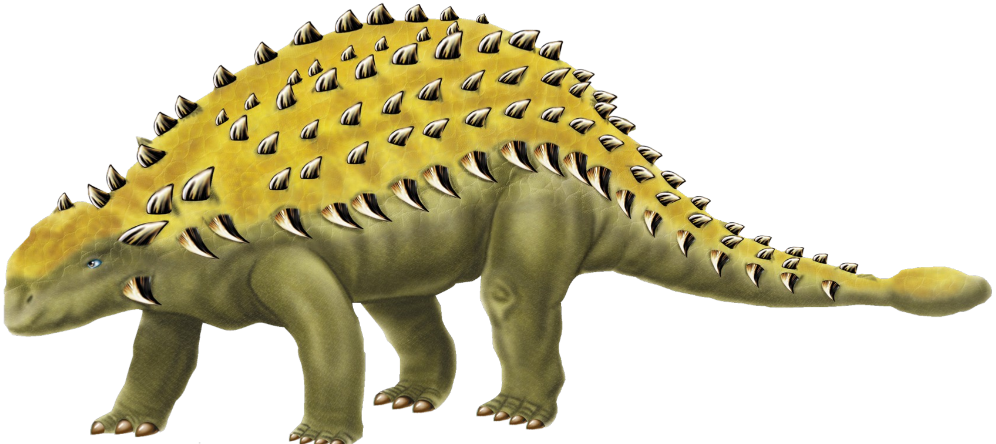
the **separable** state

$$\frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$$

saturates it: $I_{AB} + I_{CD} = \log d$

The systen state is **entangled** if correlations on **both** *A*-*B* and *C*-*D* are large enough

is the converse true?



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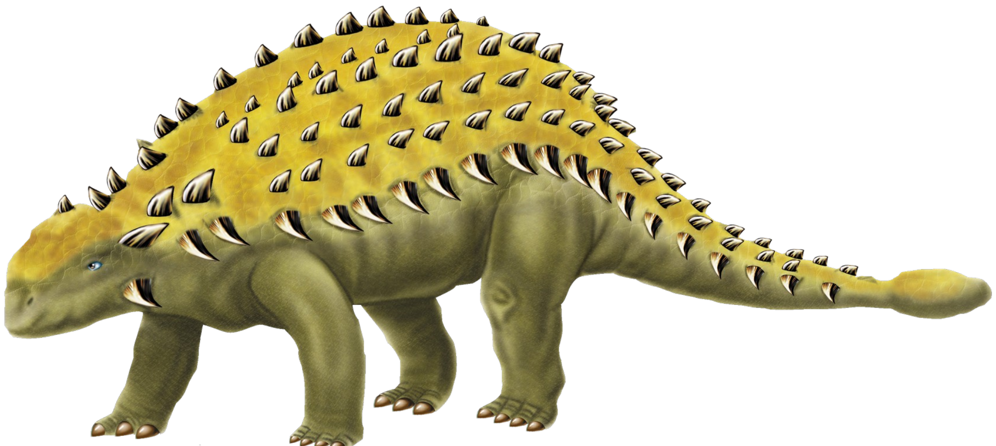

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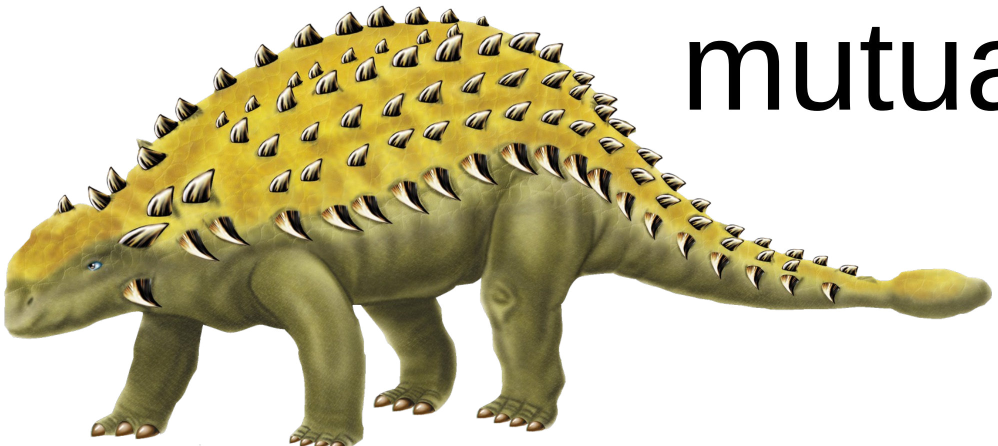
is the converse true?

➡ NO!!

$$|\psi_\epsilon\rangle = \epsilon|00\rangle + \sqrt{1-\epsilon^2}|11\rangle$$

is entangled but has negligible mutual info for $\epsilon \to 0$



Proof:

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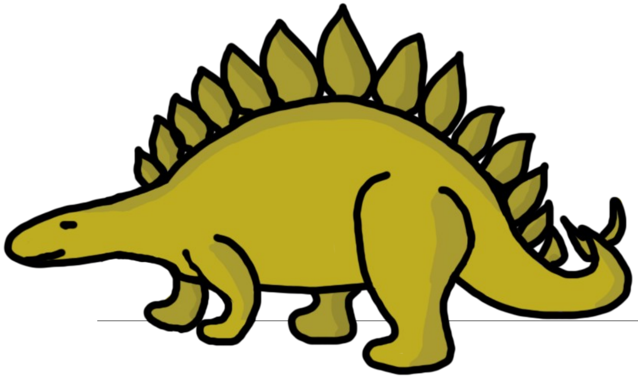

$$I_{AB} + I_{CD} > \log d \Rightarrow \rho_{12} \text{ ent}$$

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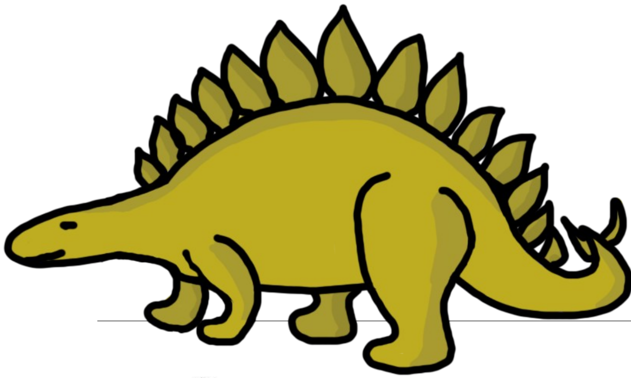

$I_{AB} + I_{CD} > \log d \Rightarrow$ ρ_{12} ent

ρ_{12} separable $\Rightarrow I_{AB} + I_{CD} \leqslant \log d$

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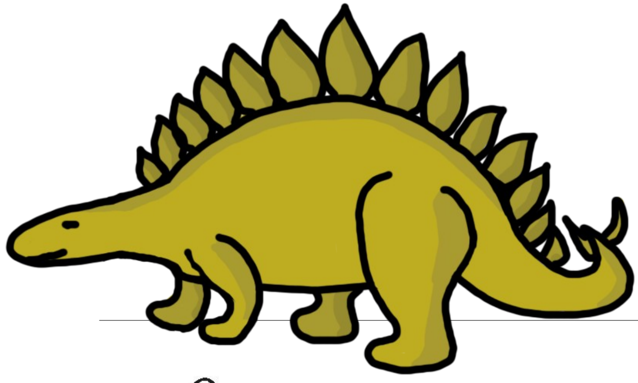

$$\rho_{12} \text{ separable} \Rightarrow I_{AB} + I_{CD} \leqslant \log d$$

- Use the **concavity** of the entropy:

$$H(A)_{\sum_i p_i \rho_i} \geqslant \sum_i p_i \, H(A)_{\rho_i}$$

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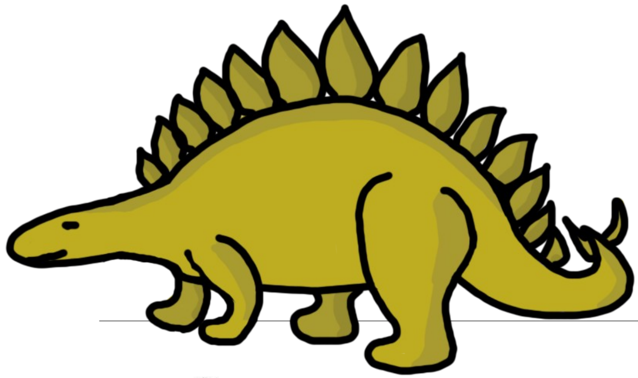

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- Use the **concavity** of the entropy:

$$H(A)_{\sum_i p_i \rho_i} \geqslant \sum_i p_i\, H(A)_{\rho_i}$$

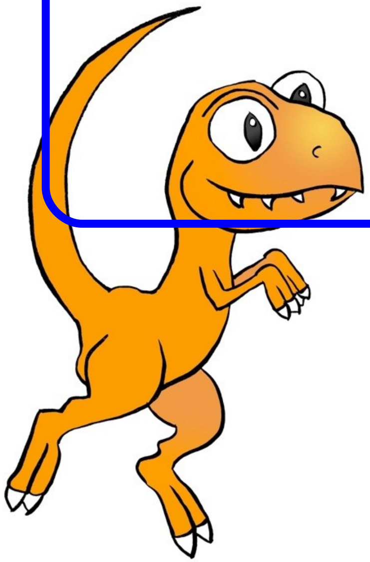
- Use **Maassen-Uffink**'s entropic uncertainty relation:

$$H(A)_\rho + H(C)_\rho \geqslant \log d$$

(for complem observ)

$$I_{AB} + I_{CD} > \log d \implies \rho_{12} \text{ ent}$$

What happens at the border with the entangled region?



$$I_{AB} + I_{CD} = \log d$$

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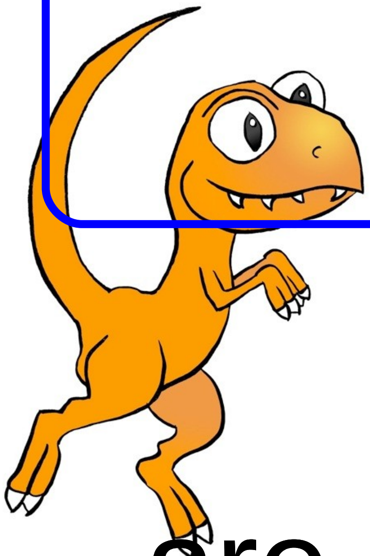

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are states more correlated than classically-correlated states?

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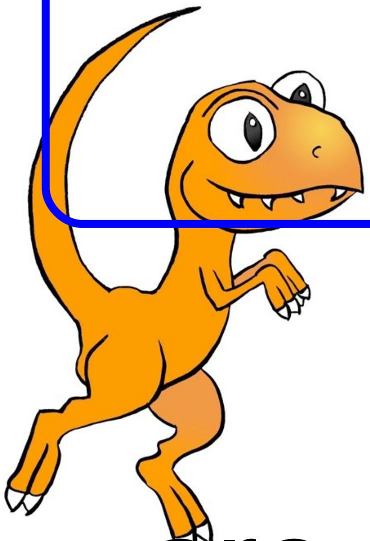

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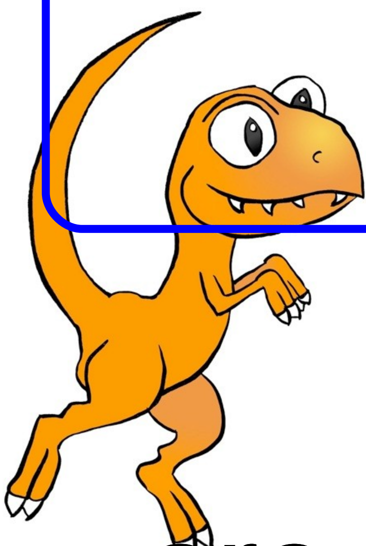

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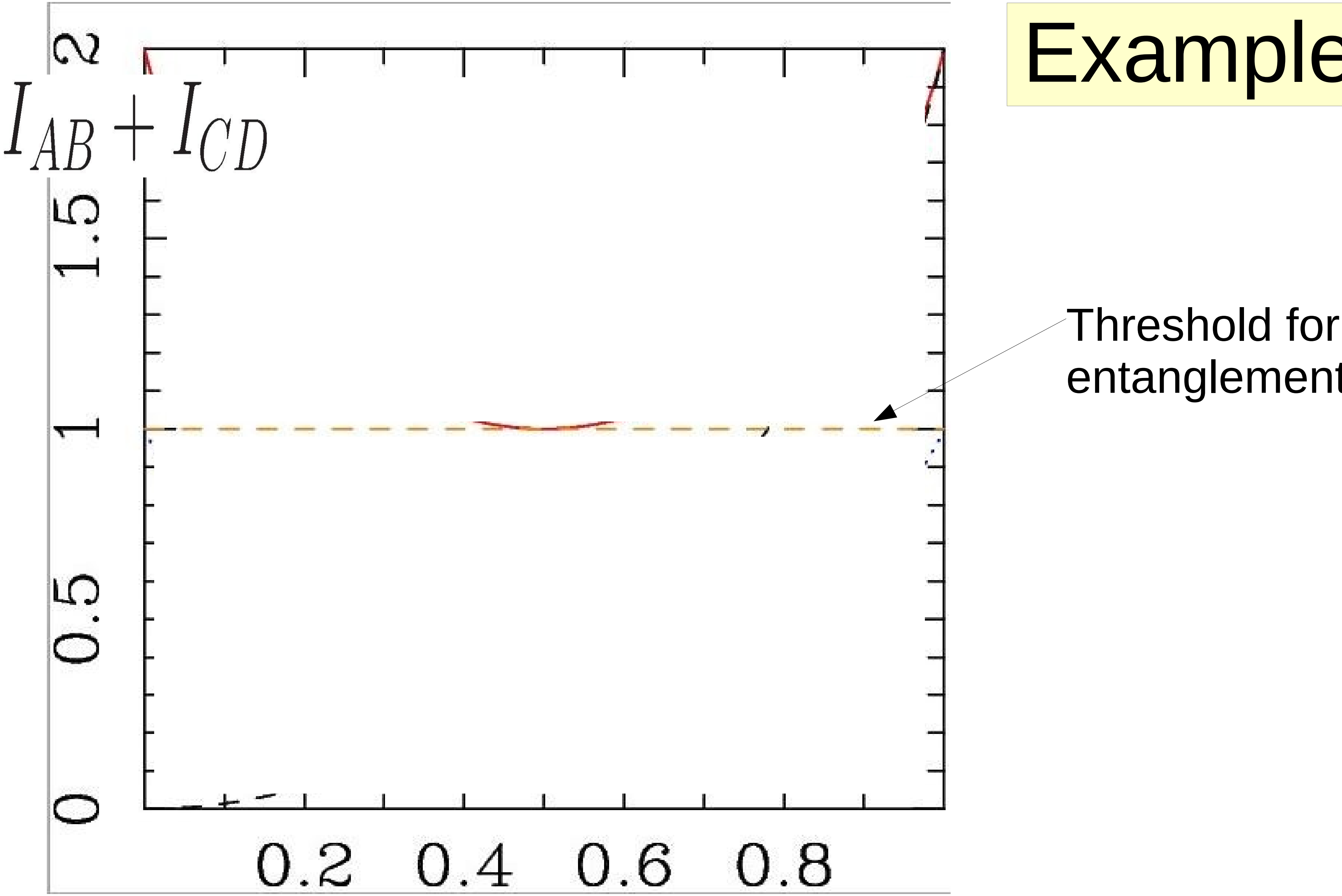
$\sum_i p_{ij} |i\rangle\langle i| \otimes |j\rangle\langle j|$ ← CC

ZERO DISCORD!

Examples



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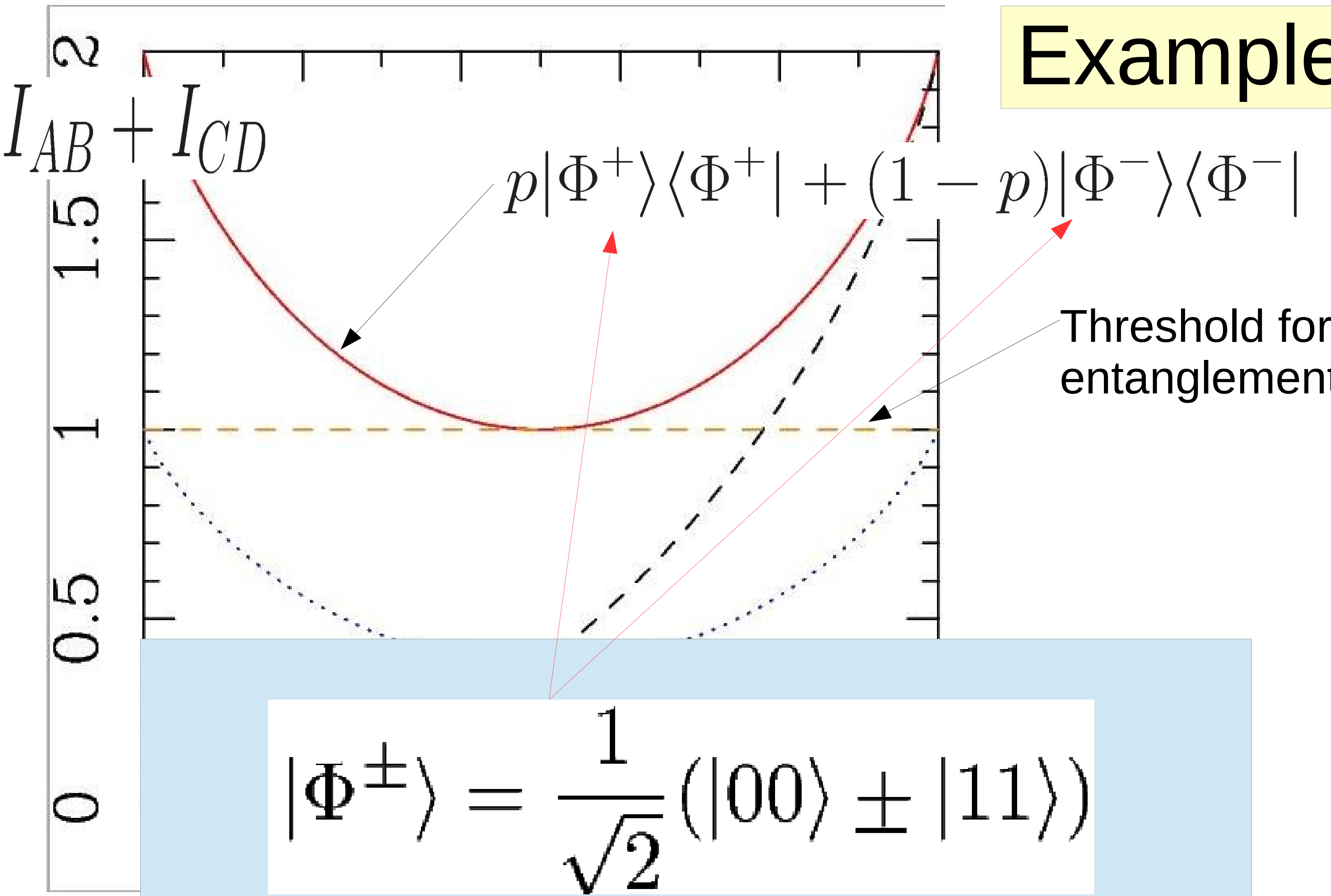
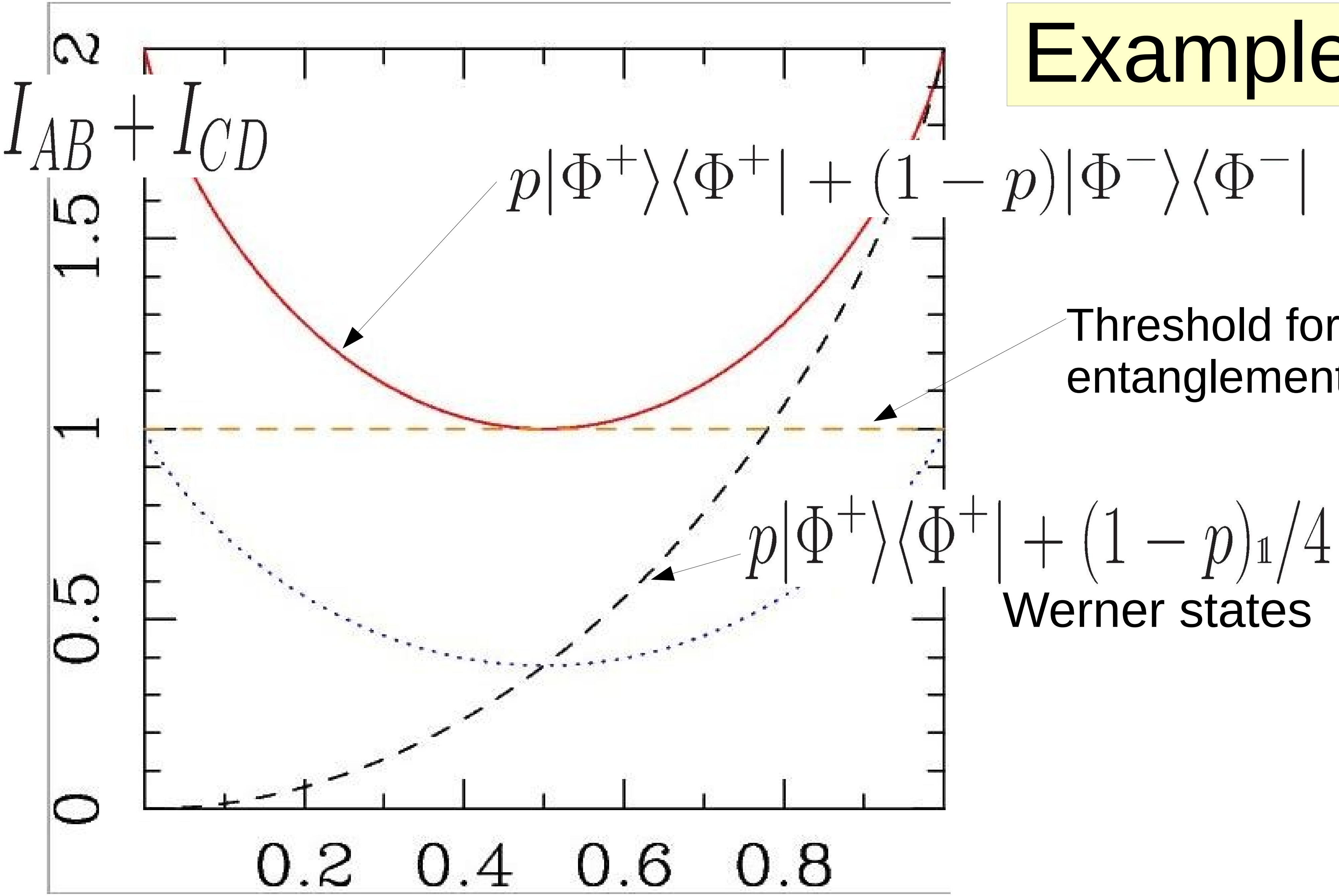


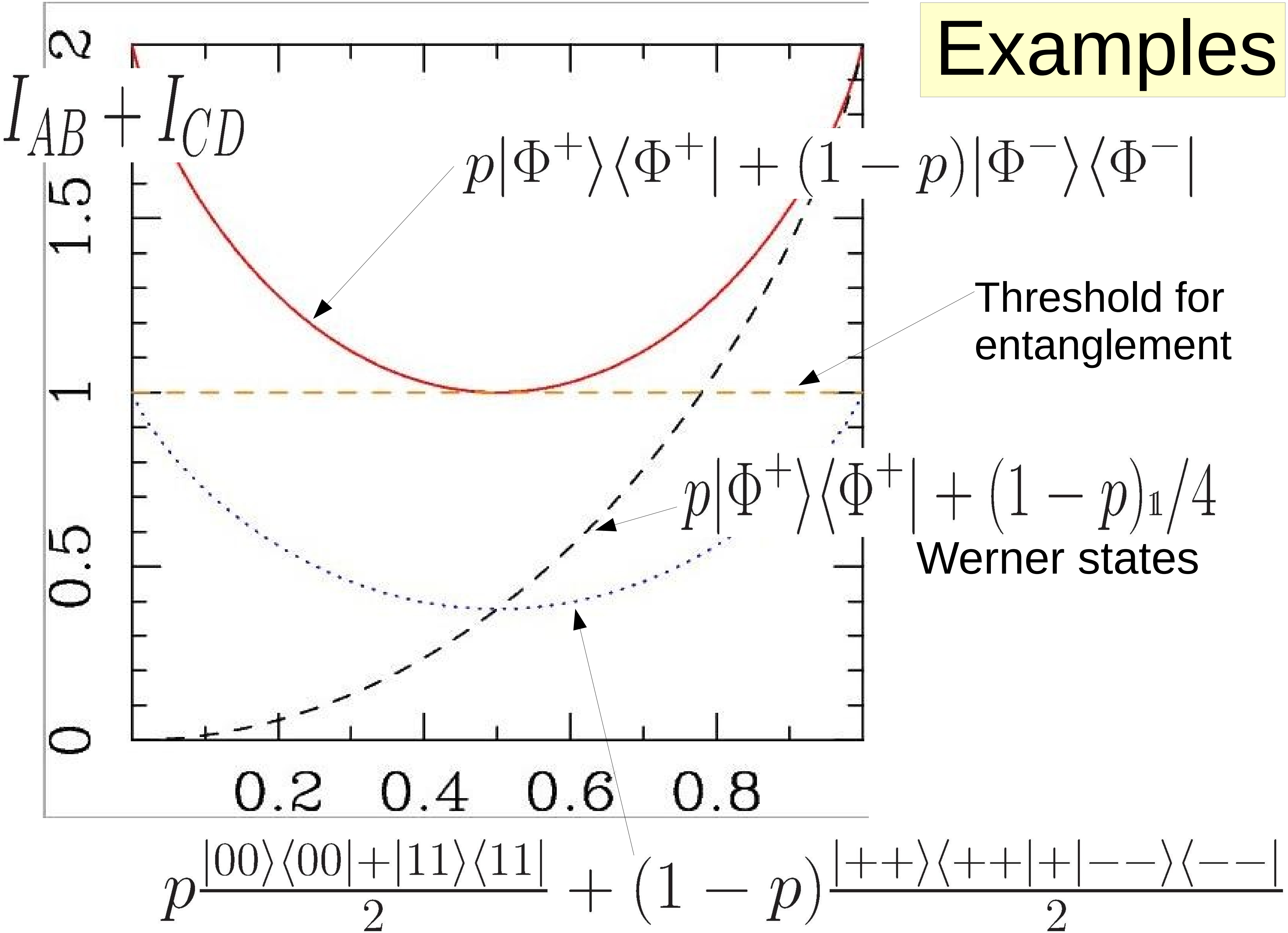
$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

Examples



$I_{AB}+I_{CD}$
$p|\Phi^+\rangle\langle\Phi^+| + (1-p)|\Phi^-\rangle\langle\Phi^-|$
Threshold for entanglement
$p|\Phi^+\rangle\langle\Phi^+| + (1-p)\mathbb{1}/4$
Werner states
0
0.5
1
1.5
2
0.2
0.4
0.6
0.8

Examples



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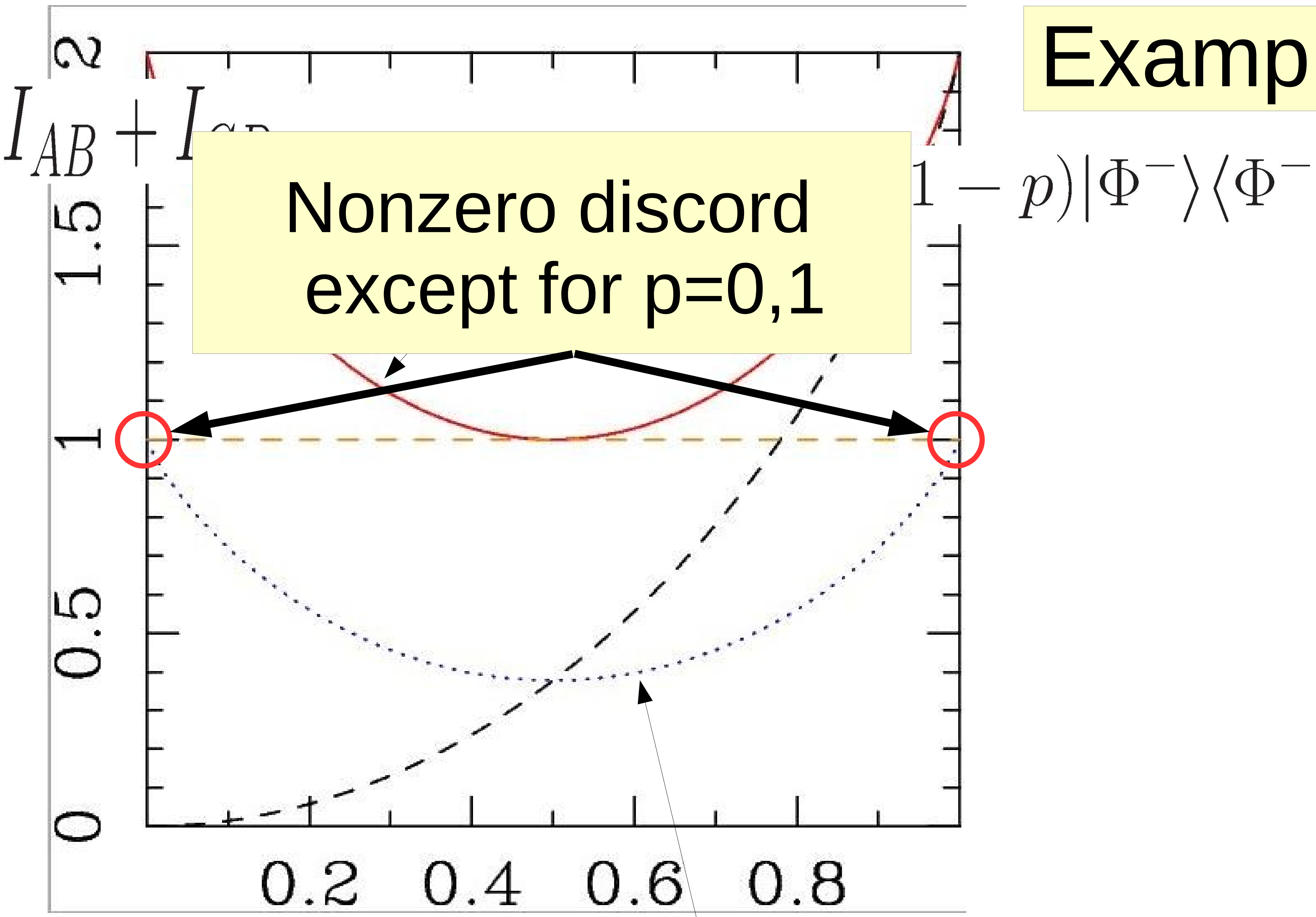
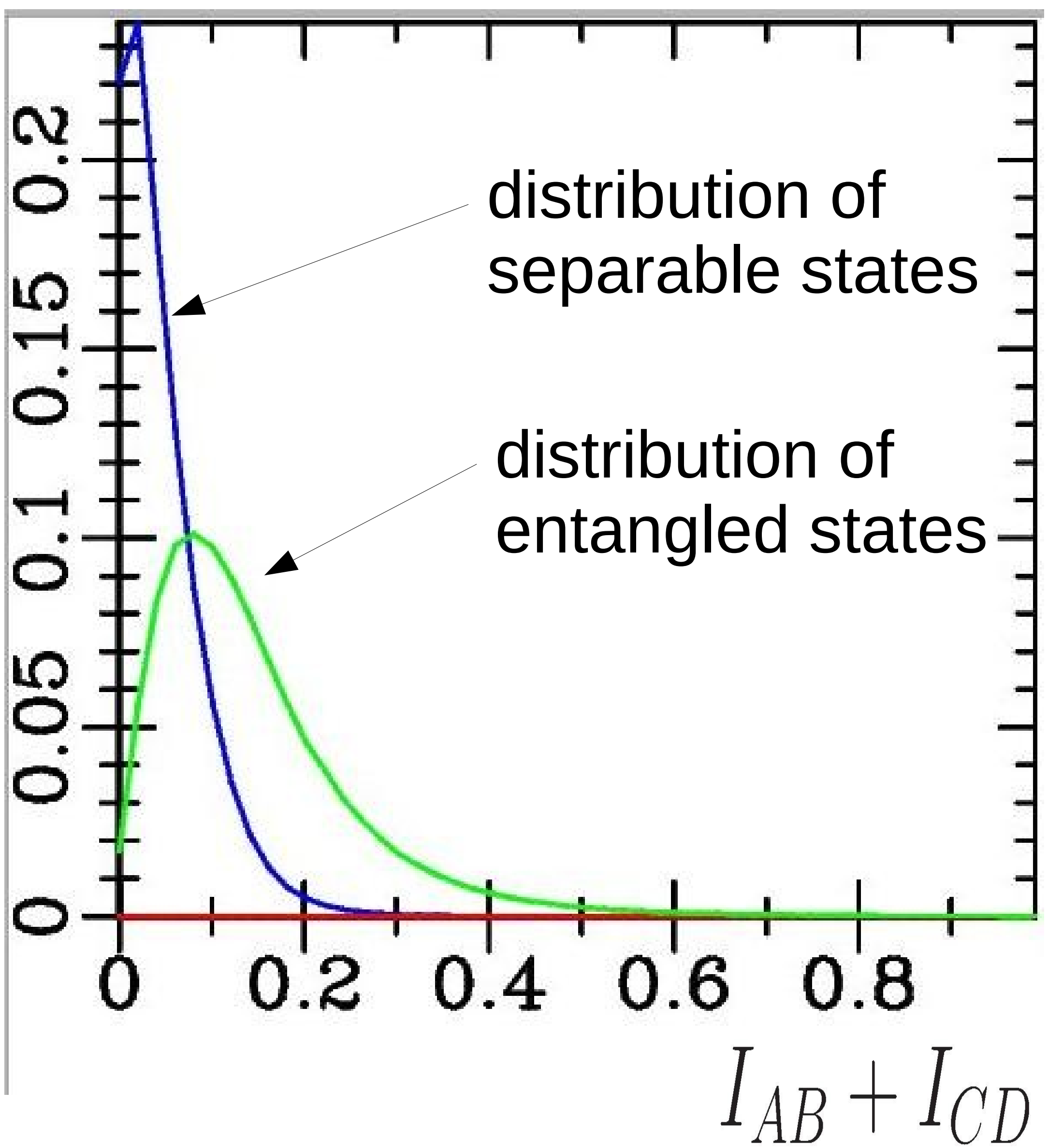
Examples



$$p\frac{|00\rangle\langle 00|+|11\rangle\langle 11|}{2}+(1-p)\frac{|++\rangle\langle ++|+|--\rangle\langle --|}{2}$$

Example

Randomly generated 2 qubit states (uniform in Haar measure)



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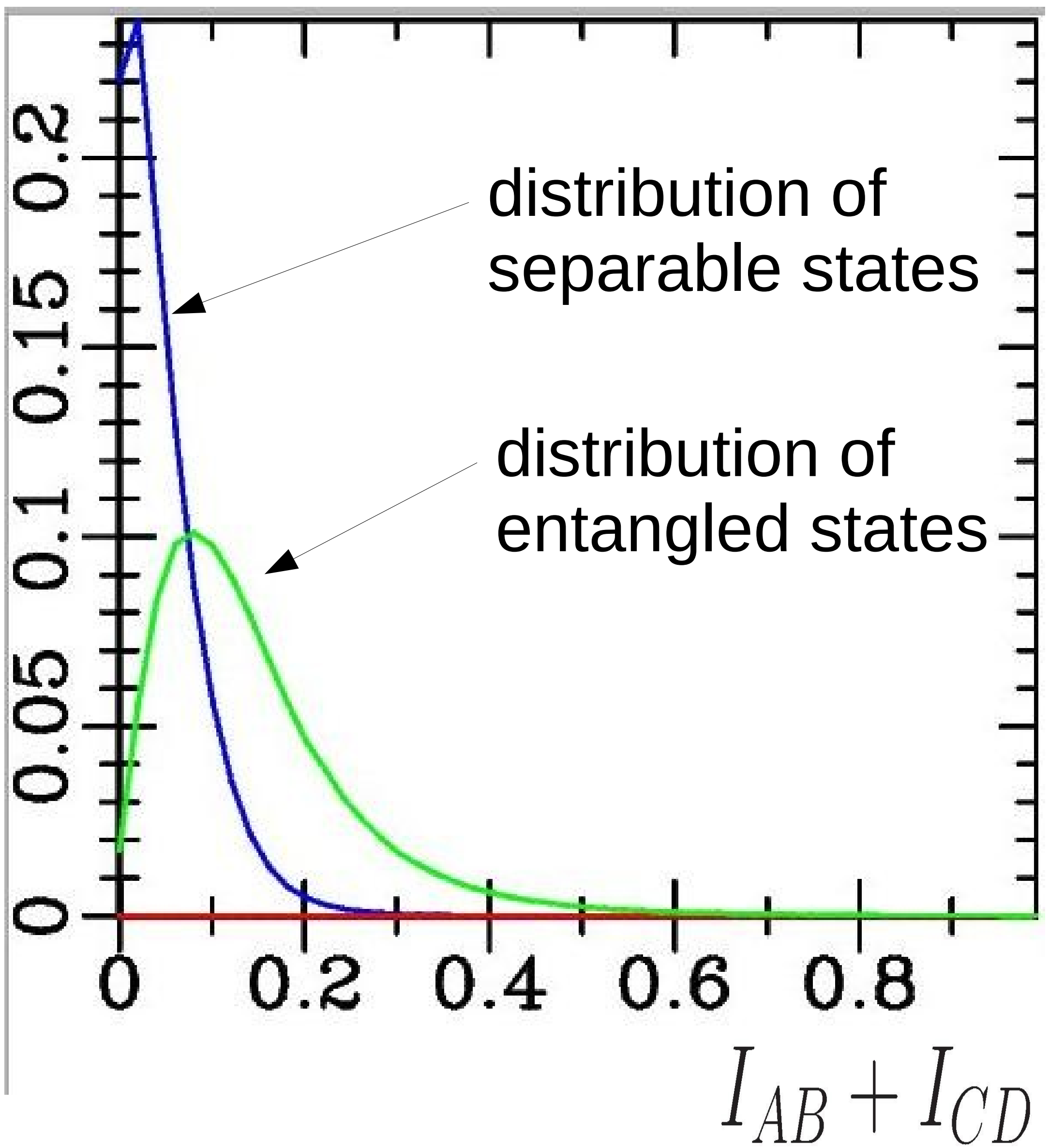

A large overlap between the two curves (but still distinguishable).

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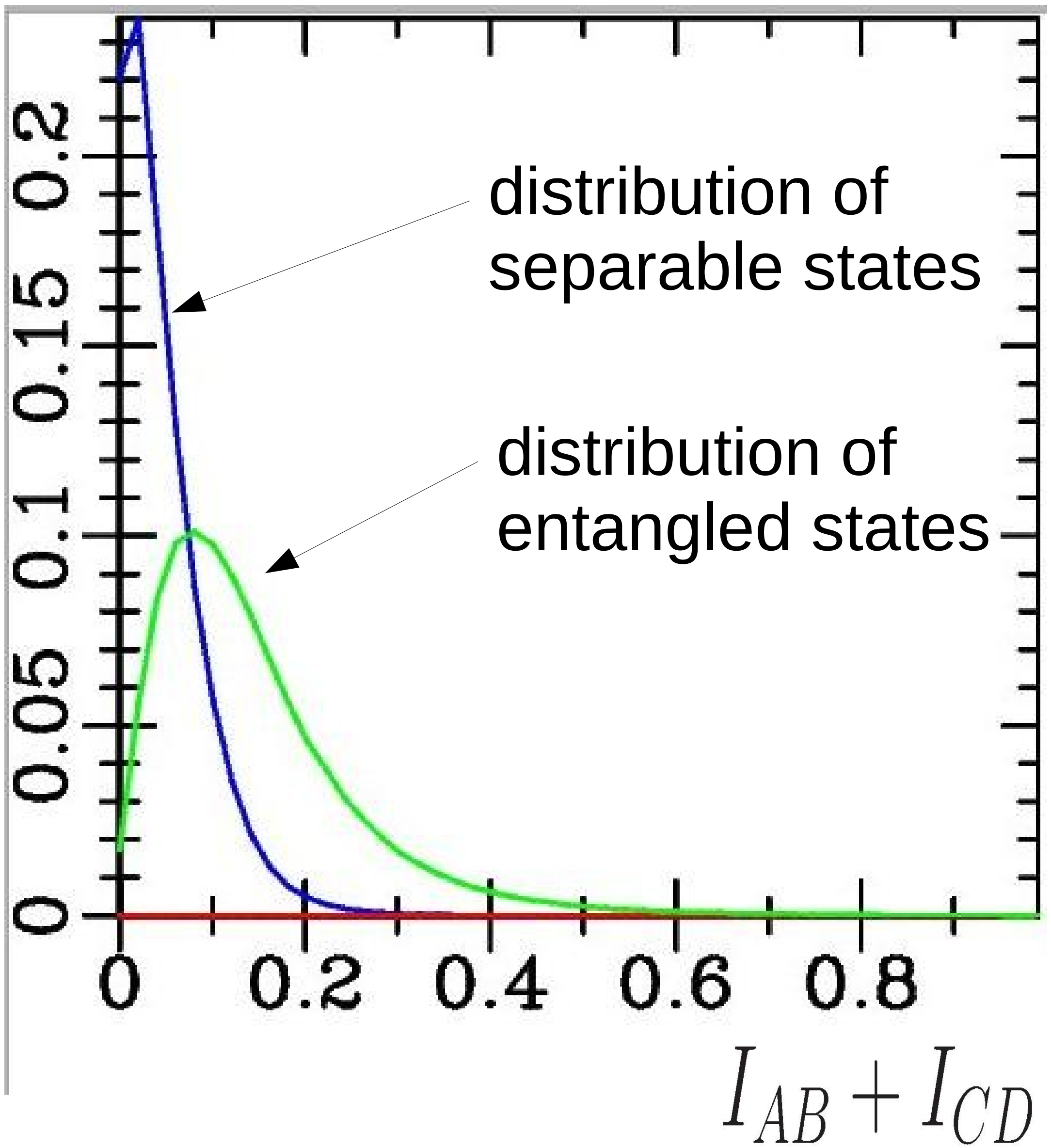

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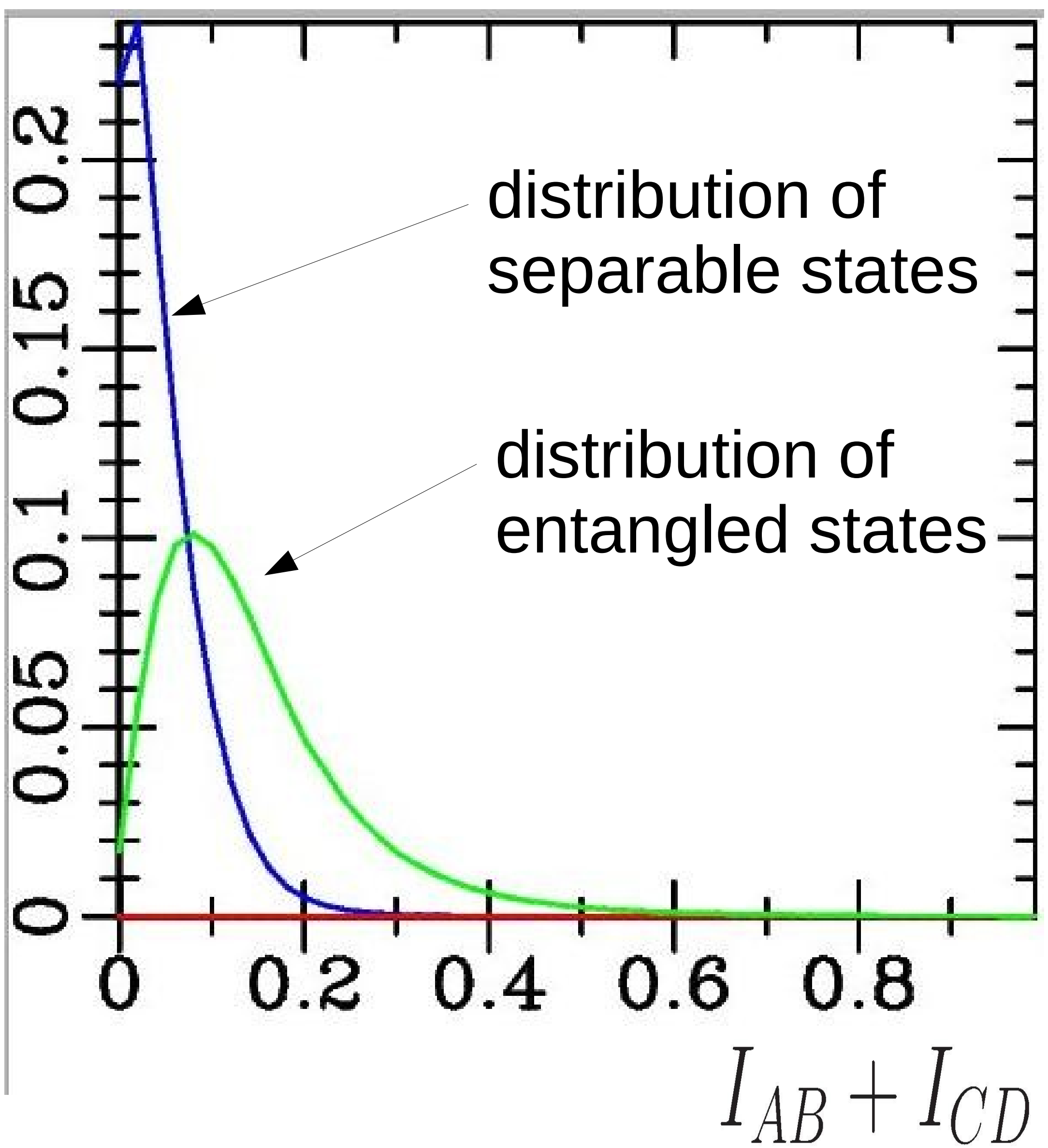

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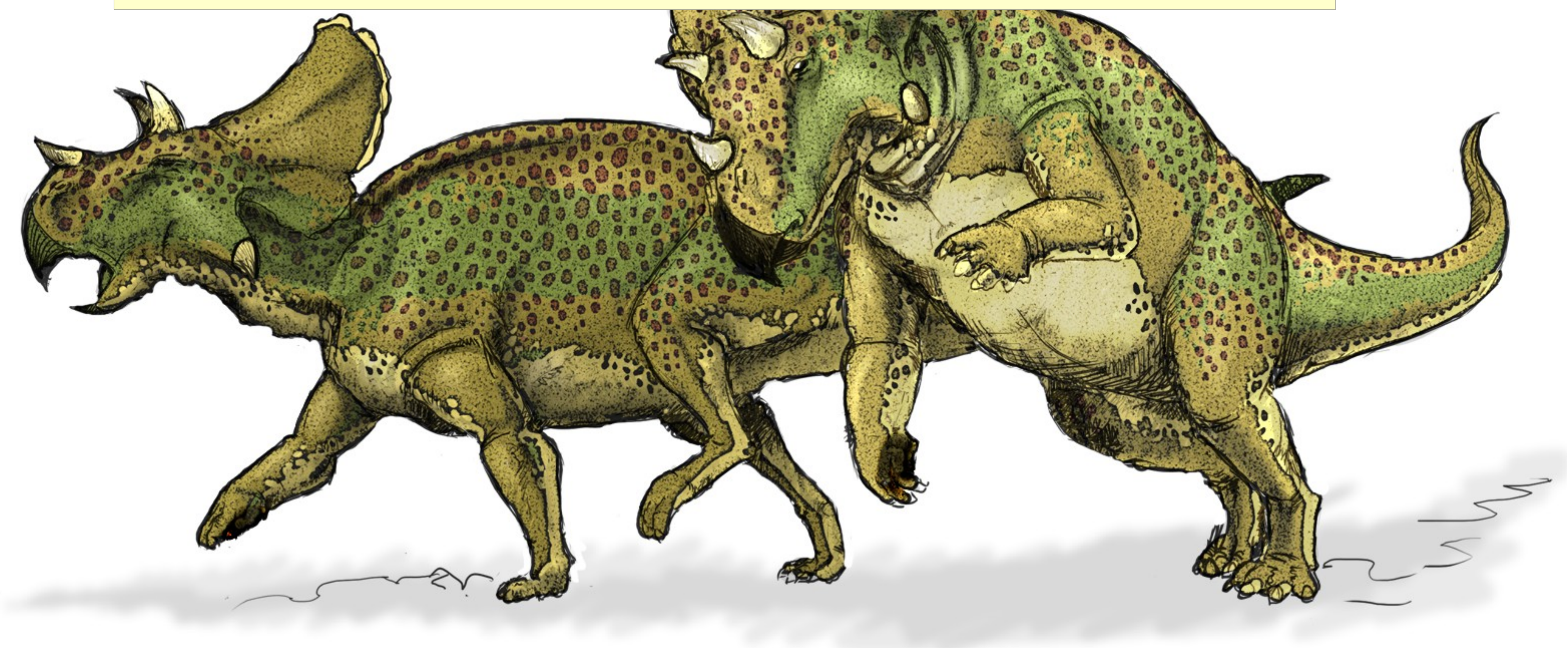
YES!

Another measure of correlation...



Pearson correlation coefficient

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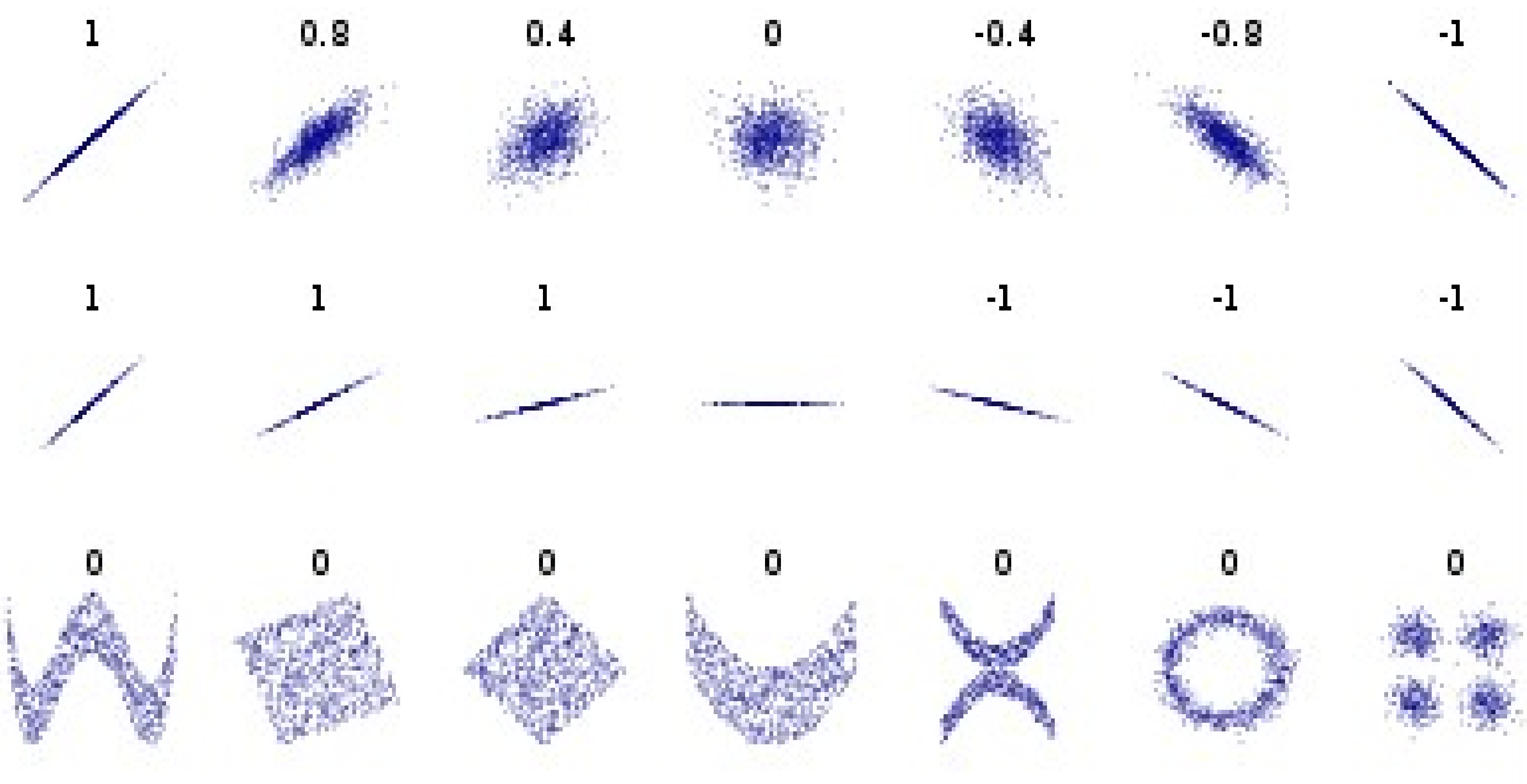

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... but its modulus is still $\leqslant |1|$:

Pearson correlation coefficient

$$\mathcal{C}_{AB} \equiv \frac{\langle AB \rangle - \langle A \rangle \langle B \rangle}{\sigma_A \sigma_B}$$

$|\mathcal{C}_{AB}| = 1 \Rightarrow$ perfect correlation or anticorrelation

it can be **complex** for quantum expectation values

... but its modulus is still $\leqslant |1|$:

$$|\langle AB \rangle - \langle A \rangle \langle B \rangle|^2 = |\tfrac{\langle [A,B] \rangle + \langle \{A,B\} \rangle}{2} - \langle A \rangle \langle B \rangle|^2 =$$

$$|\tfrac{1}{2}\langle [A, B] \rangle|^2 + |\tfrac{1}{2}\langle \{A, B\} \rangle - \langle A \rangle \langle B \rangle|^2 \leqslant \sigma_A^2 \sigma_B^2$$

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Using Schroedinger's uncertainty relation:

$$\sigma_A^2 \sigma_B^2 \geqslant |\tfrac{1}{2}\langle [A,B] \rangle|^2 + |\tfrac{1}{2}\langle \{A,B\} \rangle - \langle A \rangle \langle B \rangle|^2$$

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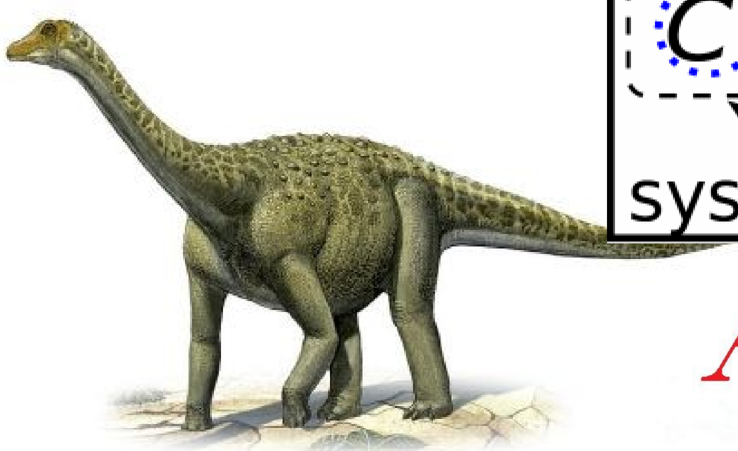
it can be **complex** for quantum expectation values

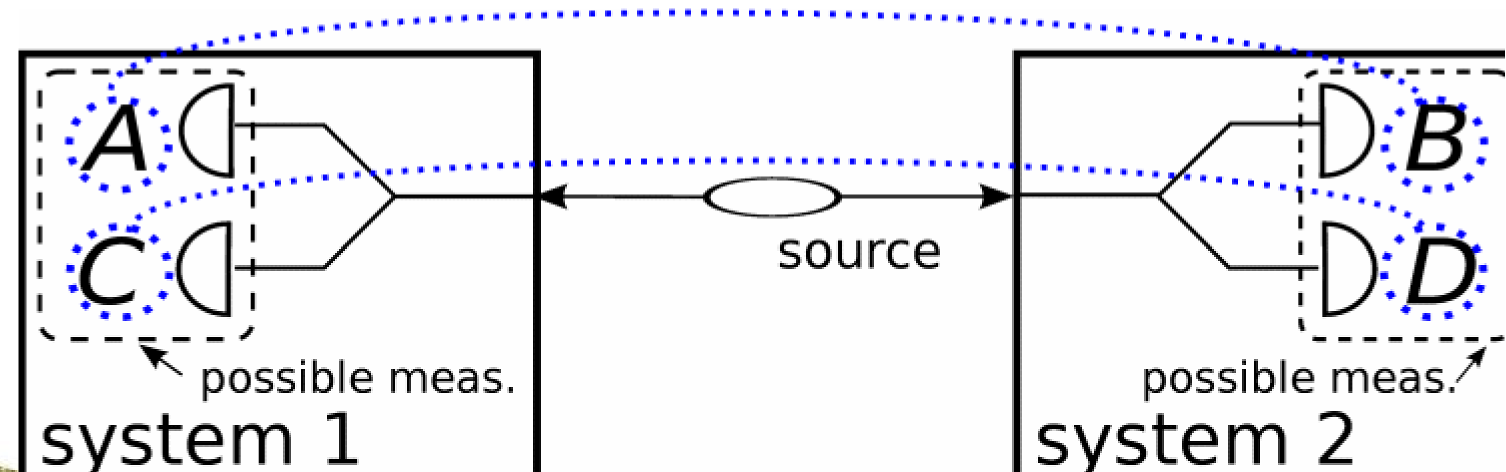
not a problem for us: *A* and *B* commute, so it's **REAL**

$$A \otimes B = A \otimes \mathbb{1} + \mathbb{1} \otimes B$$

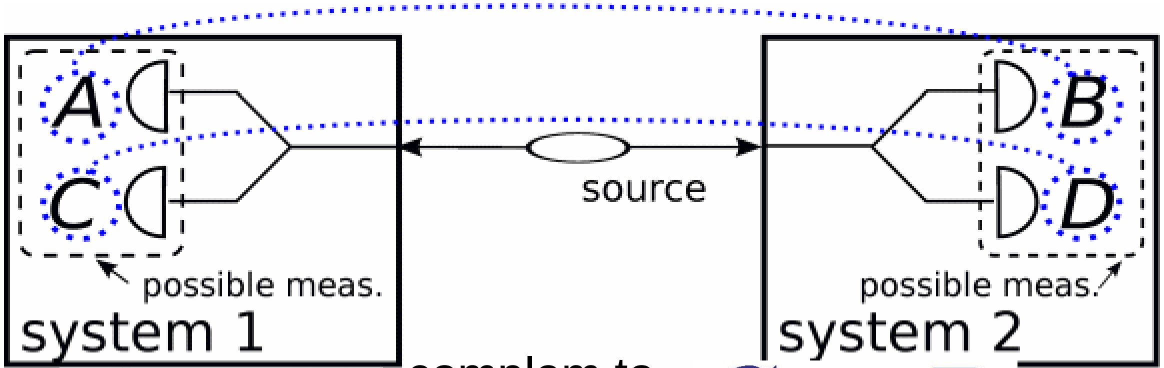
Total correlation: again use the sum

$$|C_{AB}| + |C_{CD}|$$



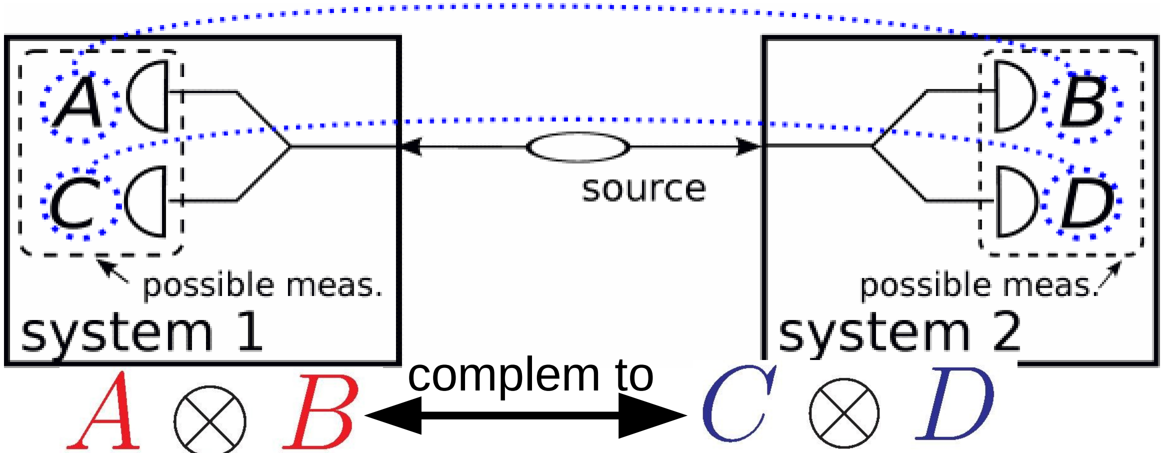


$A \otimes B$ ⟷ $C \otimes D$ (complem to)



$A \otimes B$ ⟵ complem to ⟶ $C \otimes D$

The system state is **maximally entangled** iff perfect correlation on **both** *A*-*B* and *C*-*D*



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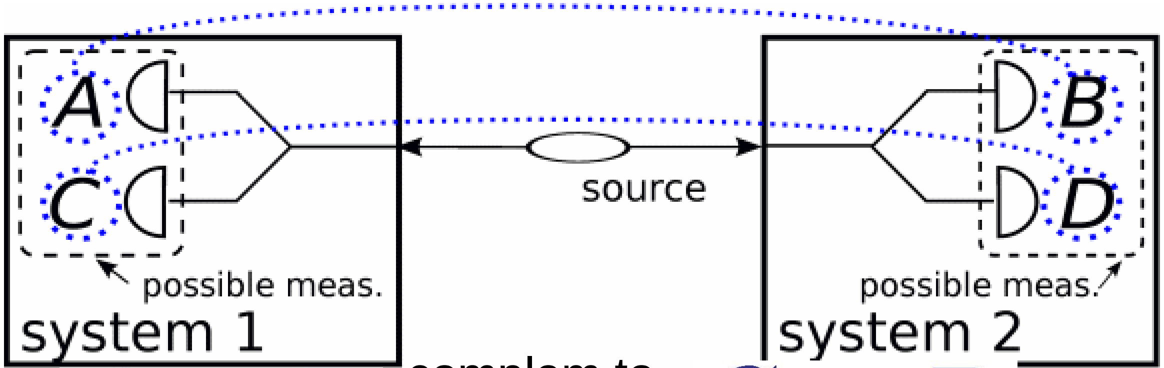
True also using Pearson! (for **linear** observables: Pearson measures only linear correl)



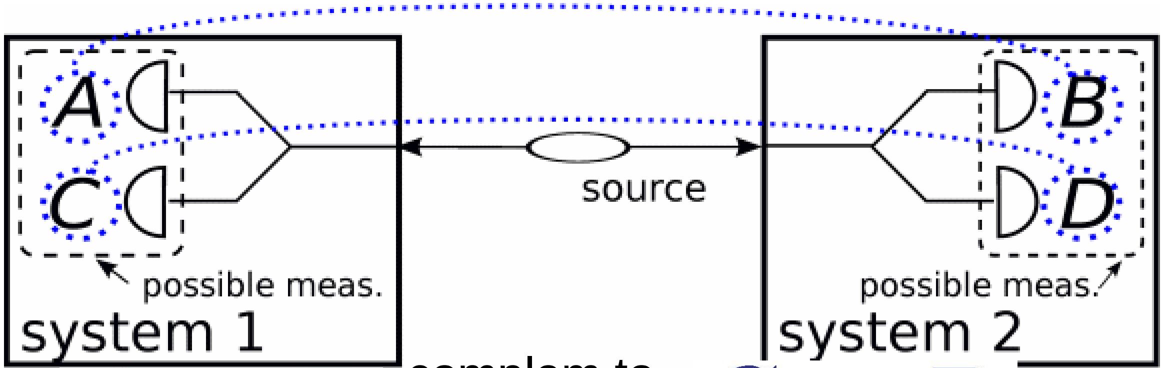
$A \otimes B$ ⟷ complem to $C \otimes D$

The system state is maximally entangled iff perfect correlation on both *A-B* and *C-D*

True also using Pearson! (for **linear** observables: Pearson measures only linear correl)

"linear" = linear in the eigenvalues

e.g. $A = \sum_i a_i |a_i\rangle\langle a_i|$ and **not** $A = \sum_i a_i^2 |a_i\rangle\langle a_i|$



$A \otimes B \longleftrightarrow C \otimes D$ (complem to)

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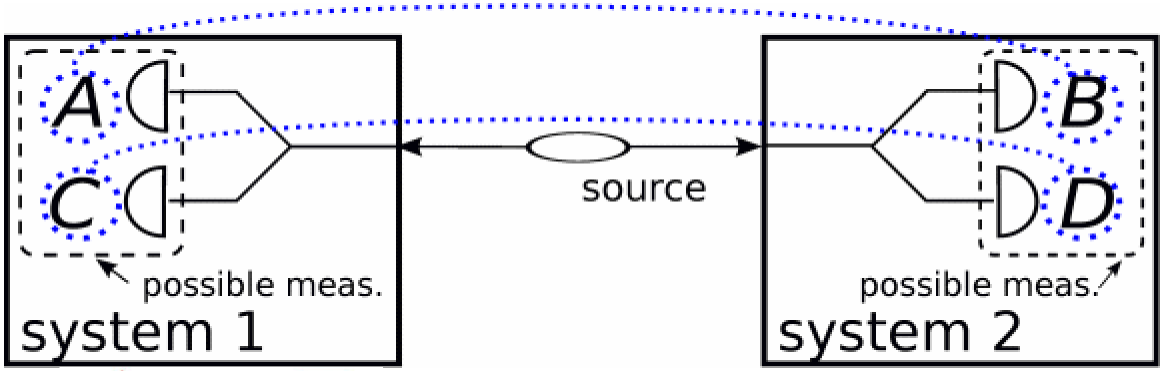
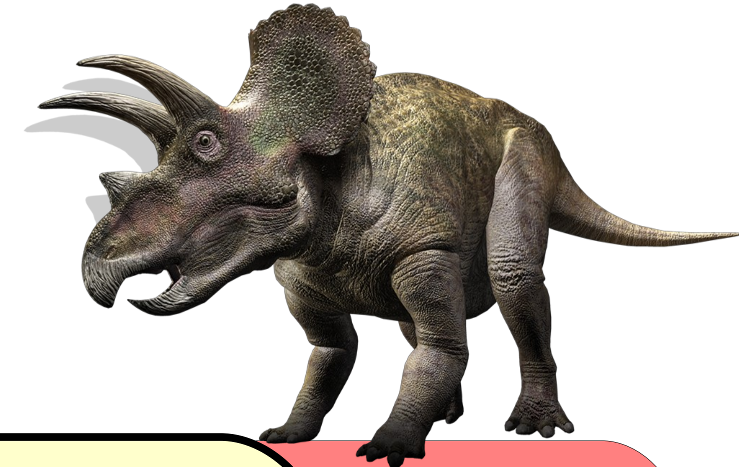
$$|\mathcal{C}_{AB}| + |\mathcal{C}_{CD}| = 2$$ (for some observ *ABCD*)

$$\Leftrightarrow \quad |\Psi_{12}\rangle$$ maximally entangled





$A \otimes B$

T **mally**

e ation

$$|\mathcal{C}_{AB}| \leqslant 1$$

$$|\mathcal{C}_{CD}| \leqslant 1$$

Tru **linear**

observables: Pearson measures only linear correl)

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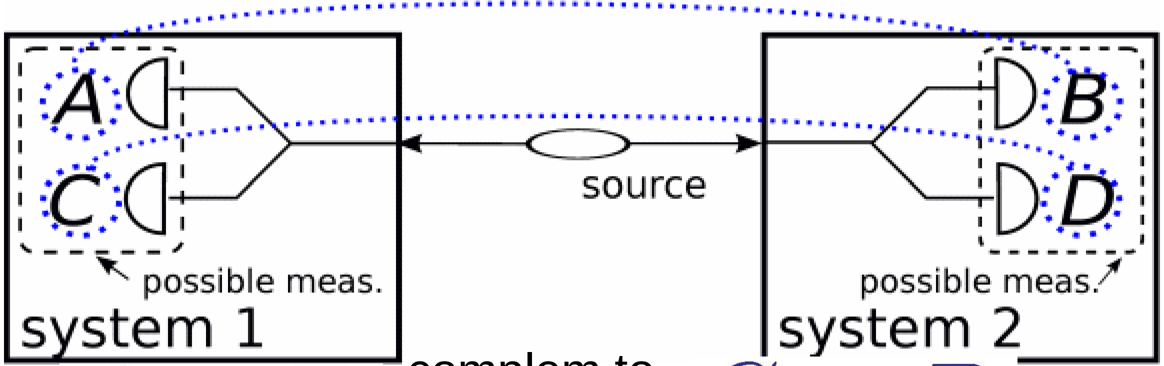

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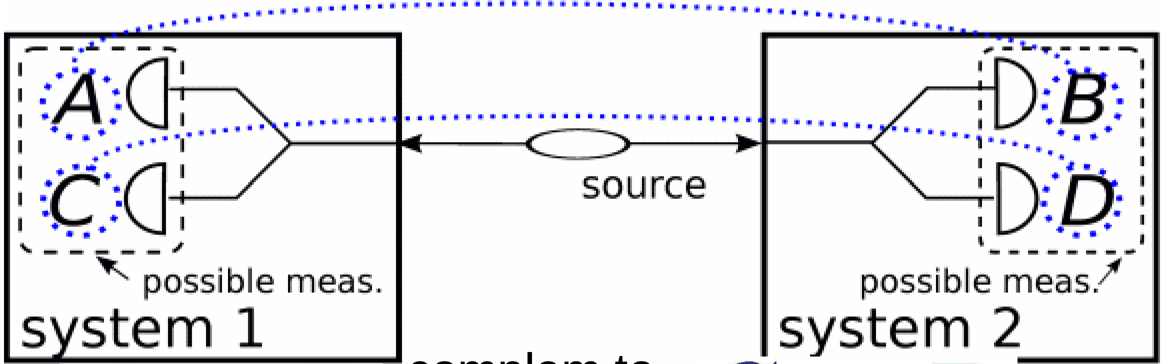

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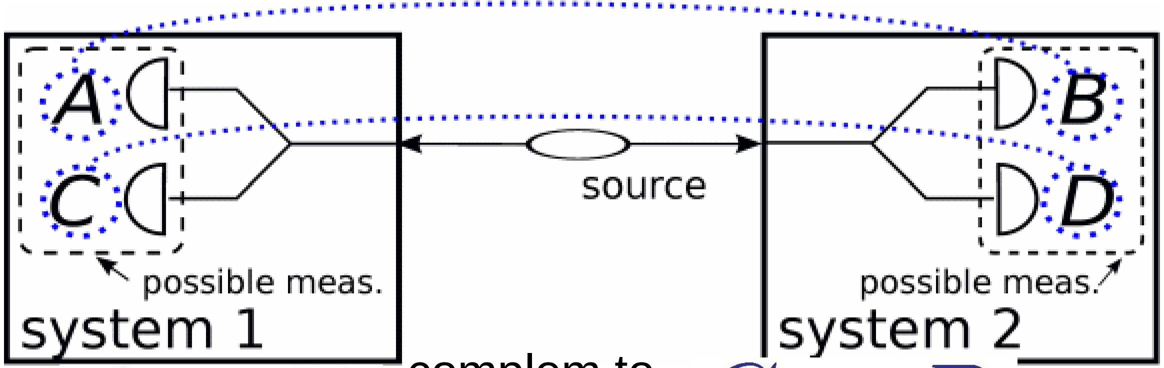

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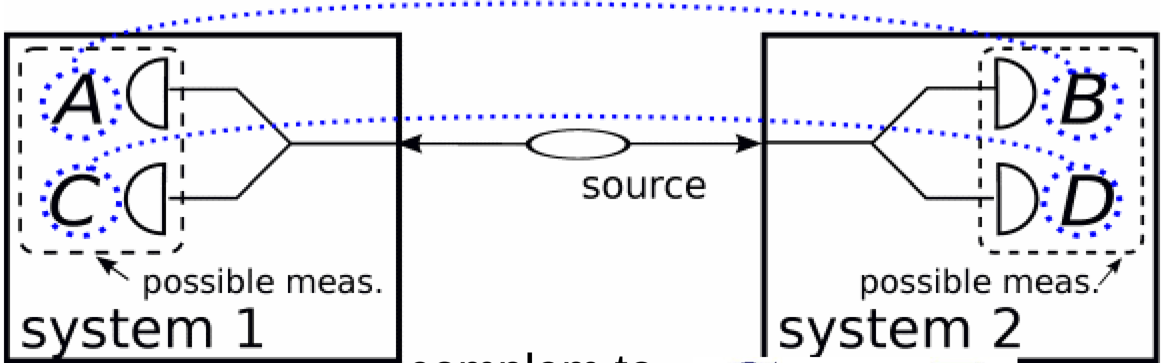
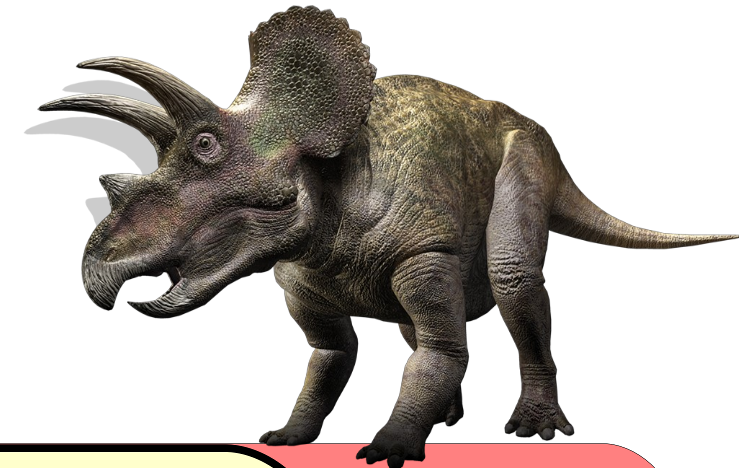
(for some observ *ABCD*)





$A \otimes B$ ←complem to→ $C \otimes D$

Th ... **gled** if

co ... d *C*-*D*

$$|\mathcal{C}_{AB}| \leqslant 1$$

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co ... ow if

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Conjecture: $|\mathcal{C}_{AB}| + |\mathcal{C}_{CD}| > 1 \;\Rightarrow$ state is ent.

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separable state $\frac{|00\rangle\langle 00| + |11\rangle\langle 11|}{2}$

$$|\mathcal{C}_{AB}| + |\mathcal{C}_{CD}| = 1$$

(perfect correl on one basis,
no correl on the complem)

States on the border are zero-discord?

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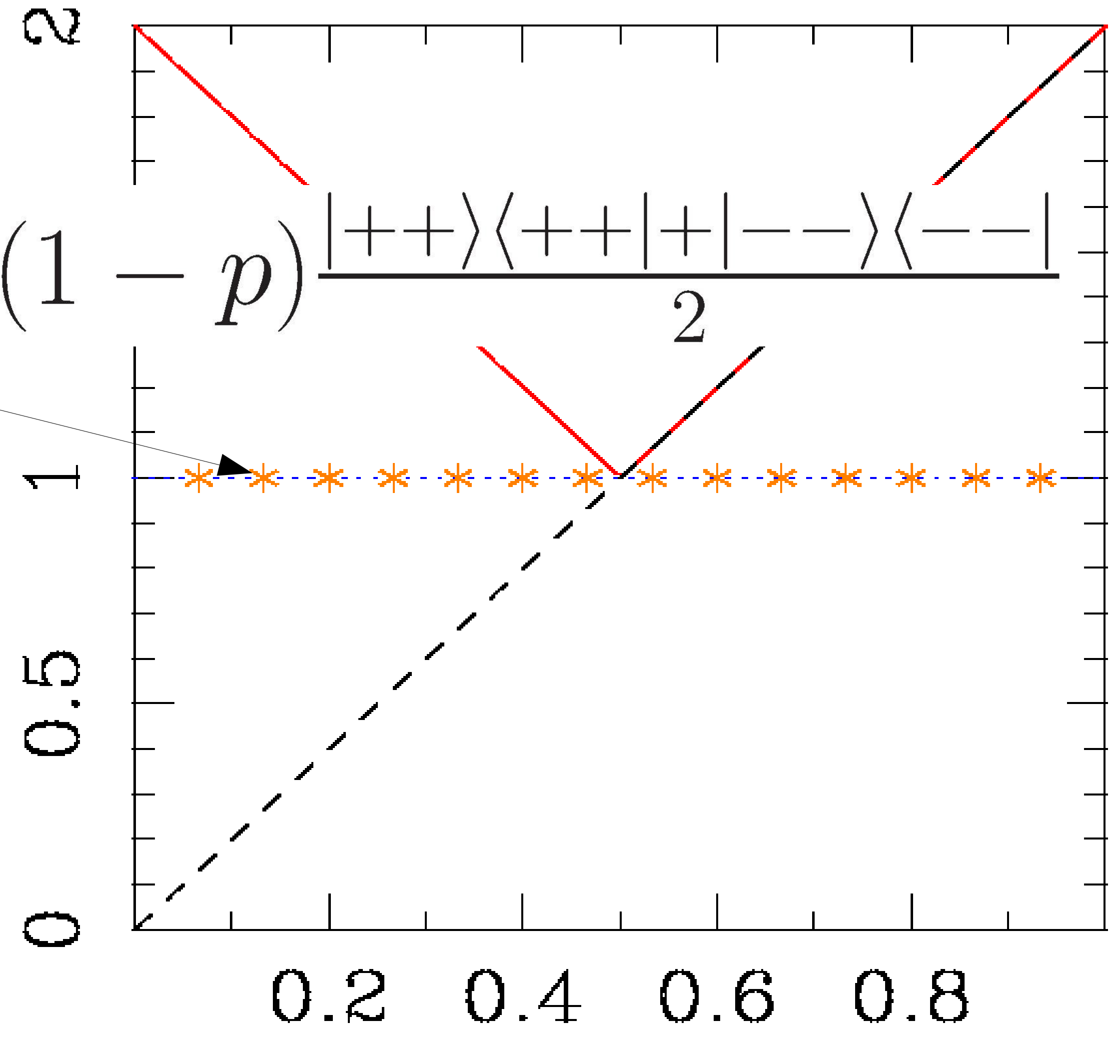
$$p\frac{|00\rangle\langle 00| + |11\rangle\langle 11|}{2} + (1-p)\frac{|++\rangle\langle ++| + |--\rangle\langle --|}{2}$$



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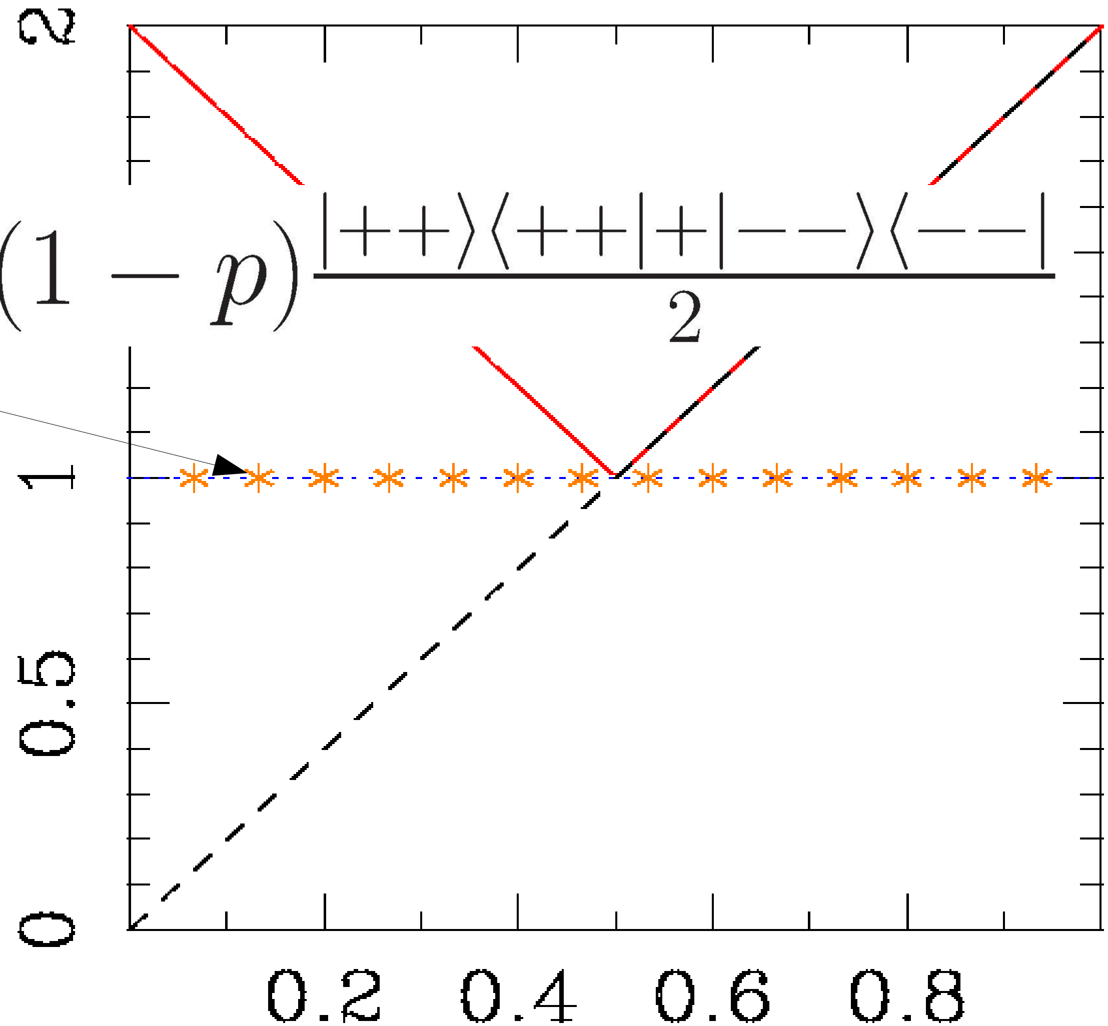
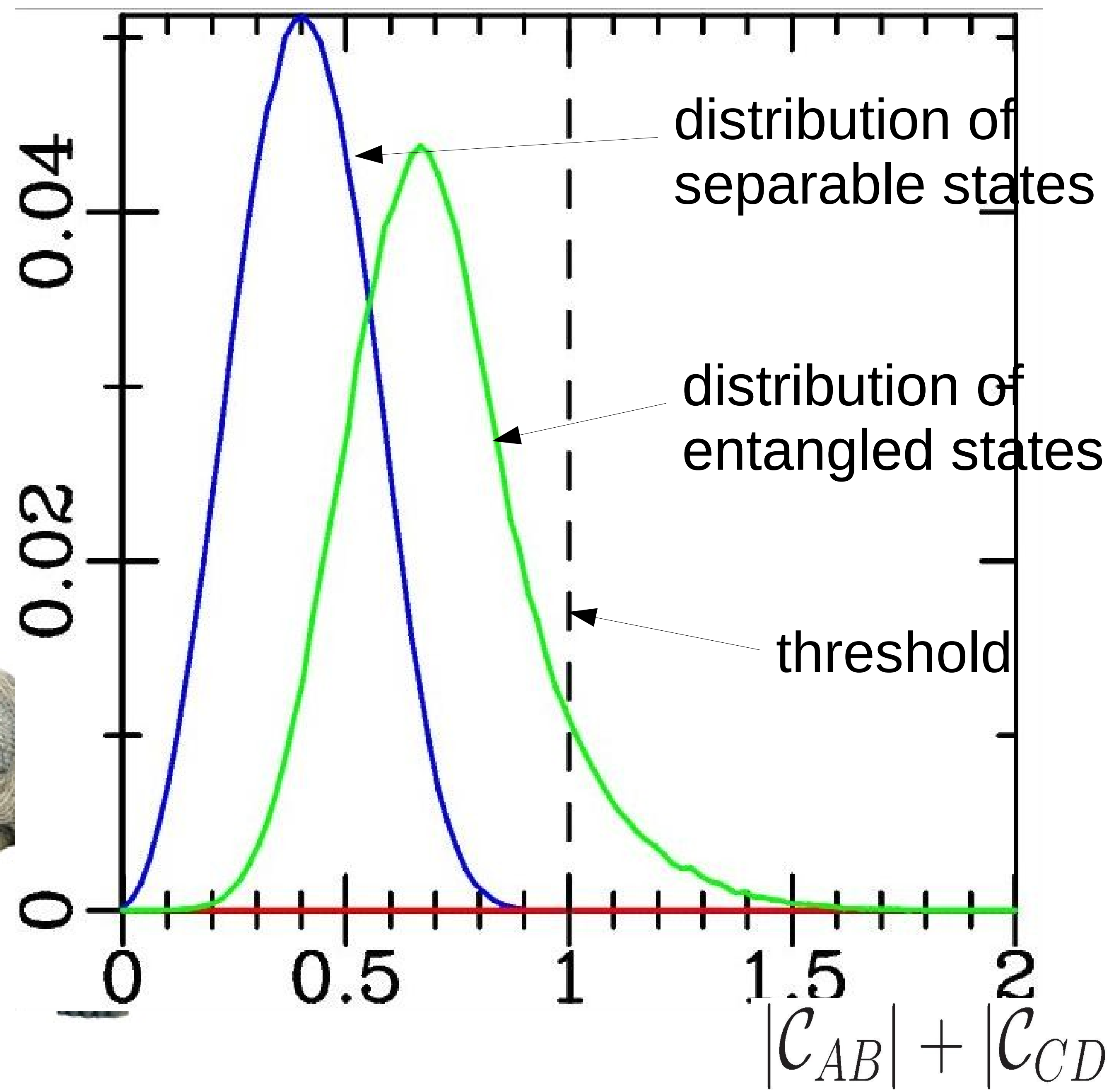


It's always at the boundary

(and has nonzero discord for p≠0,1)

Conjecture: $|\mathcal{C}_{AB}| + |\mathcal{C}_{CD}| > 1 \;\Rightarrow$ state is ent.

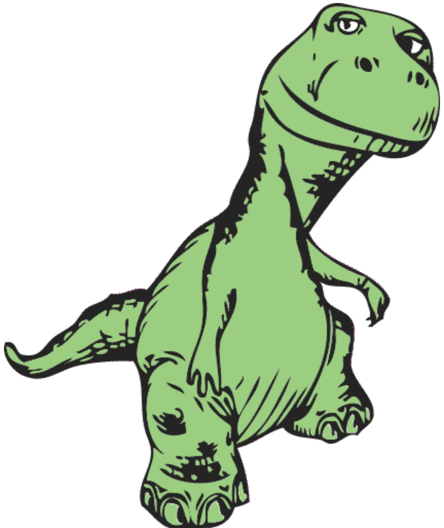
numerical evidence:



Is the Pearson correlation weaker than the mutual info?

only linear correlations

all correlations



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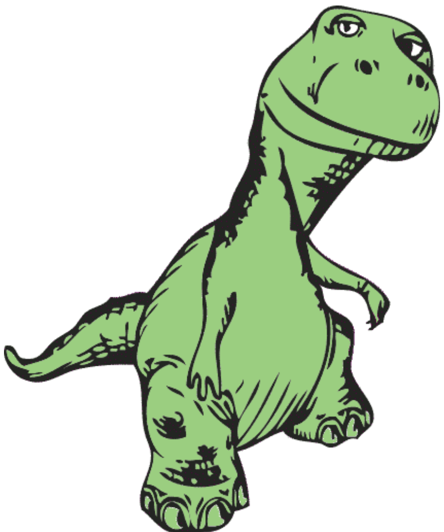
- NO!!



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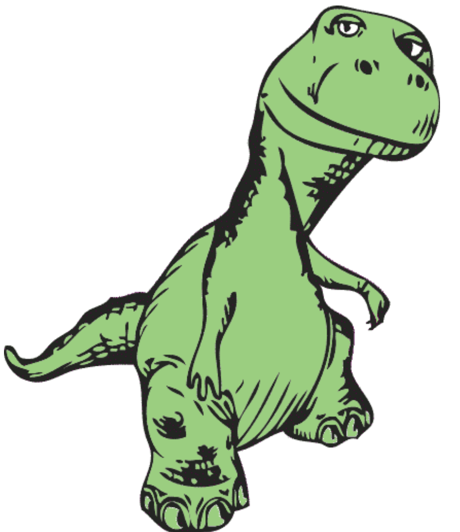
➡ NO!

$$|\psi_\epsilon\rangle = \epsilon|00\rangle + \sqrt{1-\epsilon^2}|11\rangle$$

Has negligible mutual info for $\epsilon \to 0$



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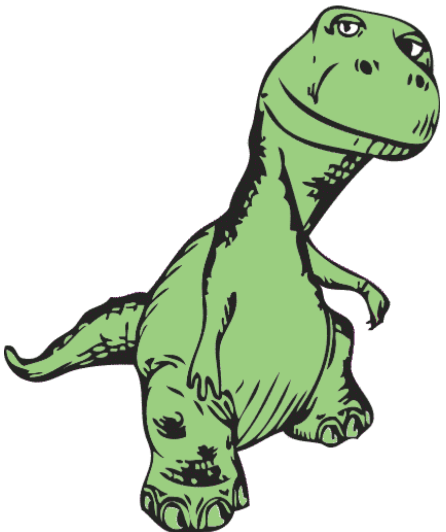
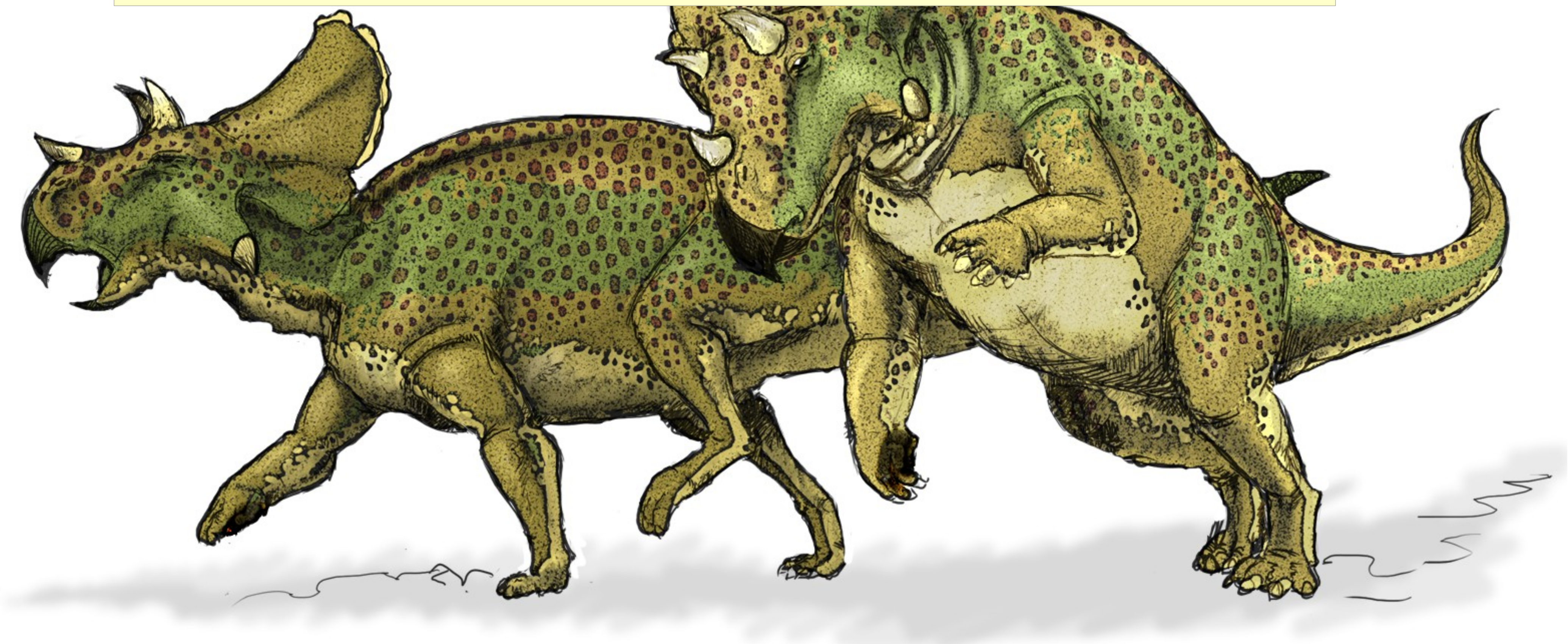
but Pearson correlation **always >1!**



Still another measure of correlation...



- Sum of conditional probabilities

$$\mathcal{S}_{AB} = \sum_i p(a_i|b_i)$$

← [similar approach (but **joint** probabilities):PRA 86,022311]

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conditional probability of finding result *i* for *A* when I found result *i* for *B:*

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[similar approach (but **joint** probabilities):PRA 86,022311]

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$$p(a_i|b_i) = 1$$

results are always the **same**

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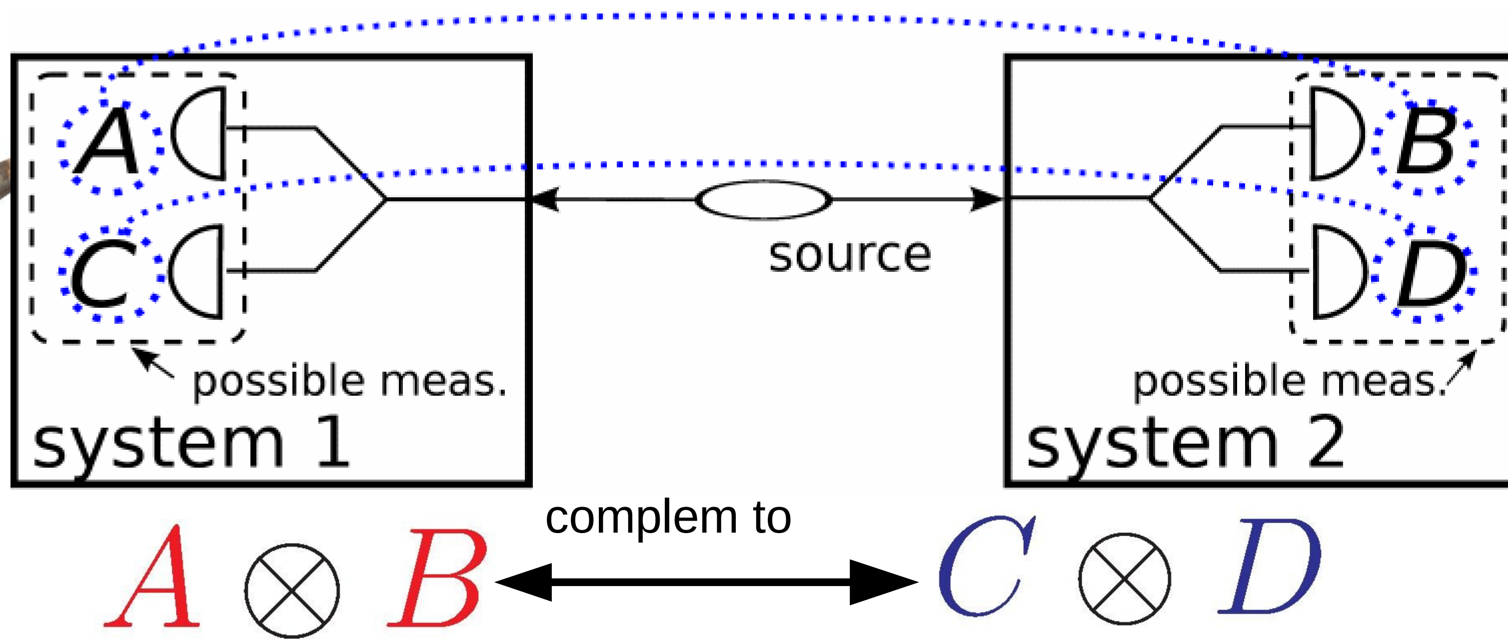
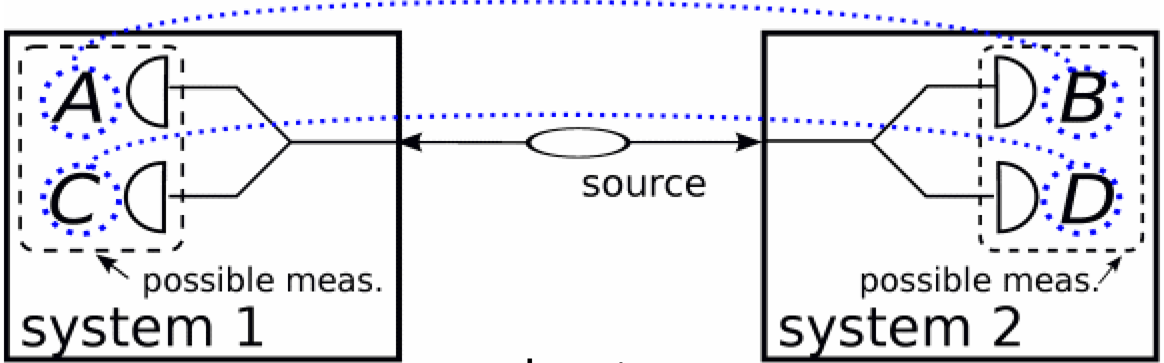
$S_{AB} = d \quad \Rightarrow$ perfect correlation

Total correlation: again use the sum

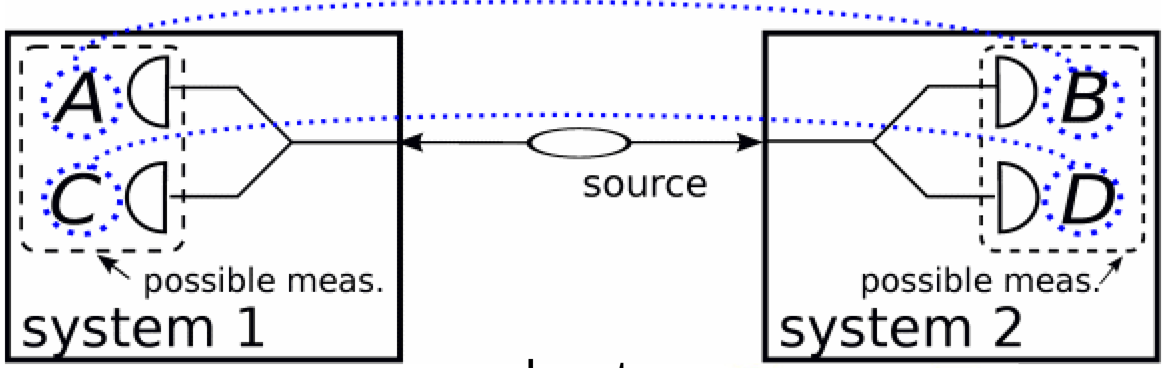
$$S_{AB} + S_{CD}$$





$A \otimes B$ ⟷ complem to ⟷ $C \otimes D$

The system state is **maximally entangled** iff perfect correlation on **both** *A*-*B* and *C*-*D*



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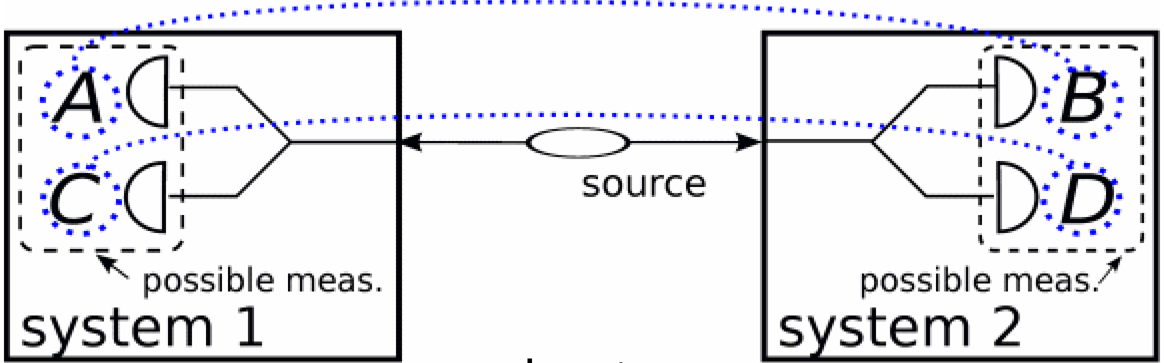

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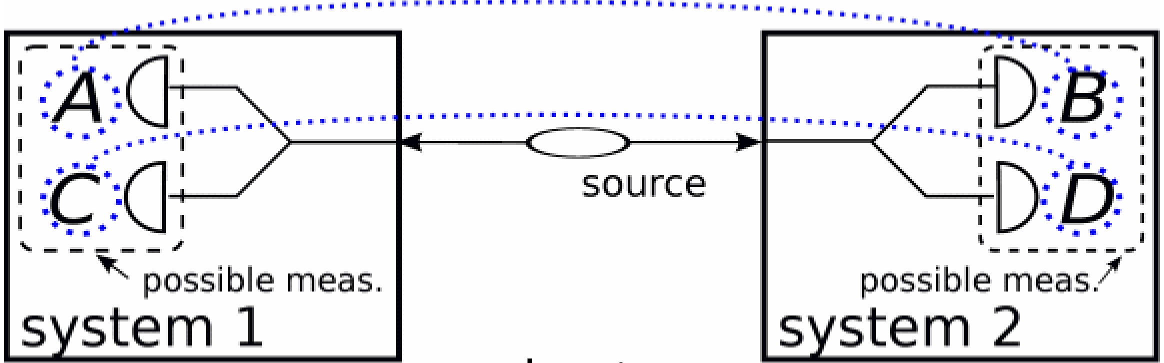
$$S_{AB} + S_{CD} = 2d$$ (for some observ *ABCD*)

$$\Leftrightarrow \quad |\Psi_{12}\rangle$$ maximally entangled



$A \otimes B$ ⟷ complem to ⟷ $C \otimes D$

T… mally

e… ation

$$|S_{AB}| \leqslant d$$

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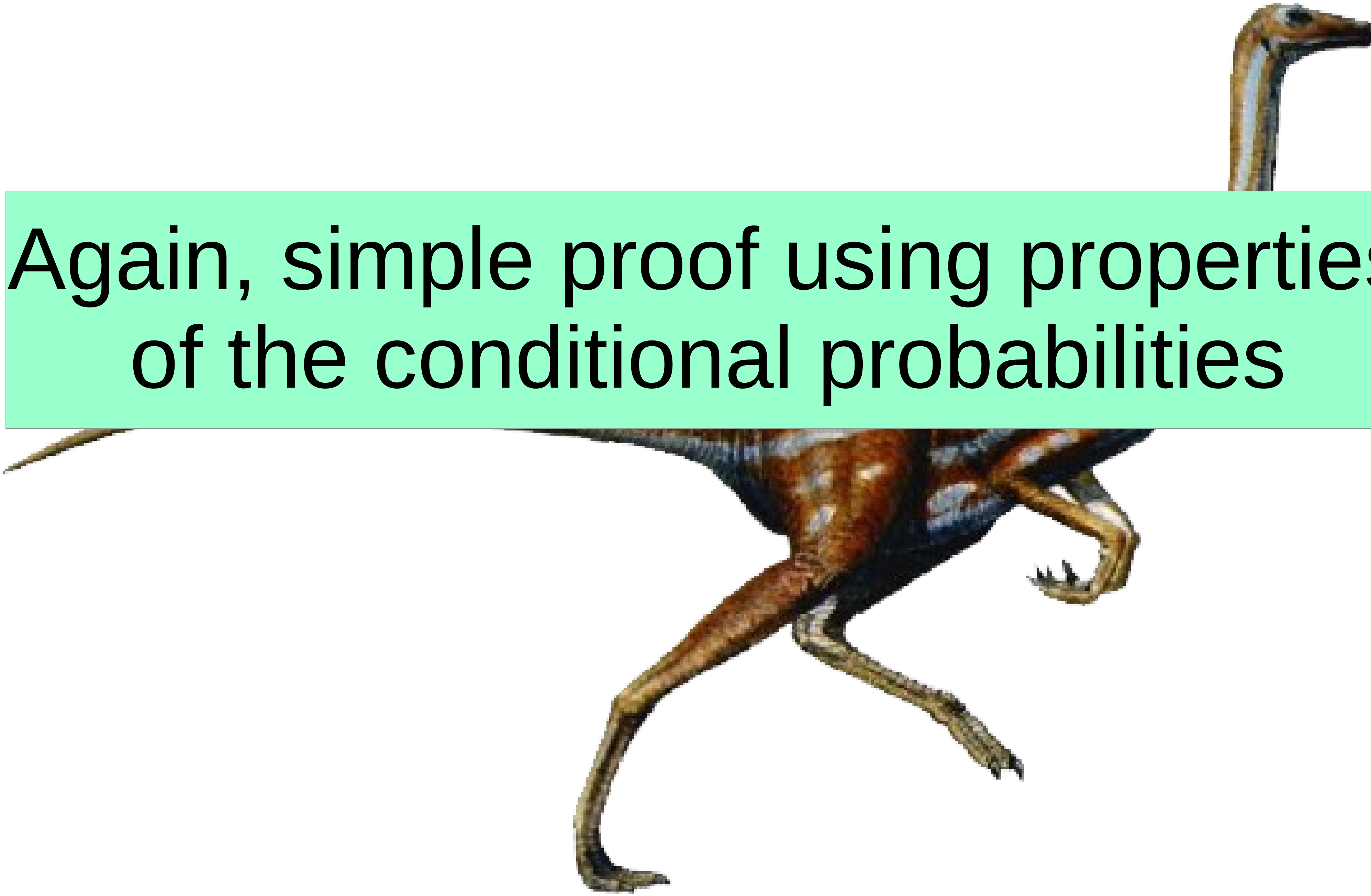
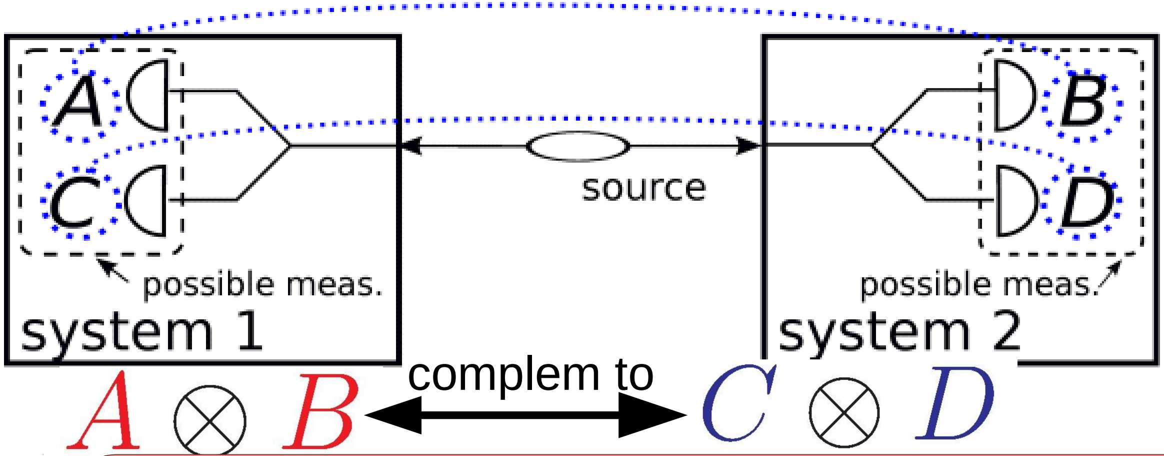
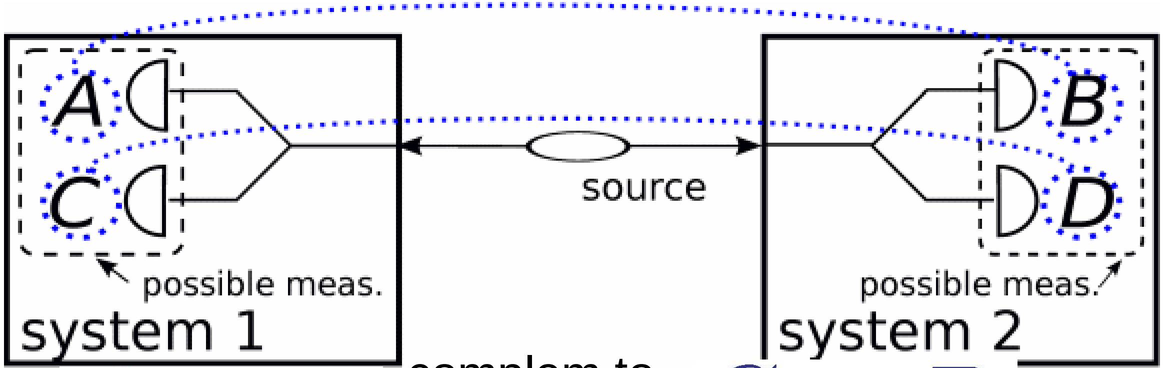


Again, simple proof using properties of the conditional probabilities



The system state is **entangled** if correlations on **both** *A*-*B* and *C*-*D* are large enough?



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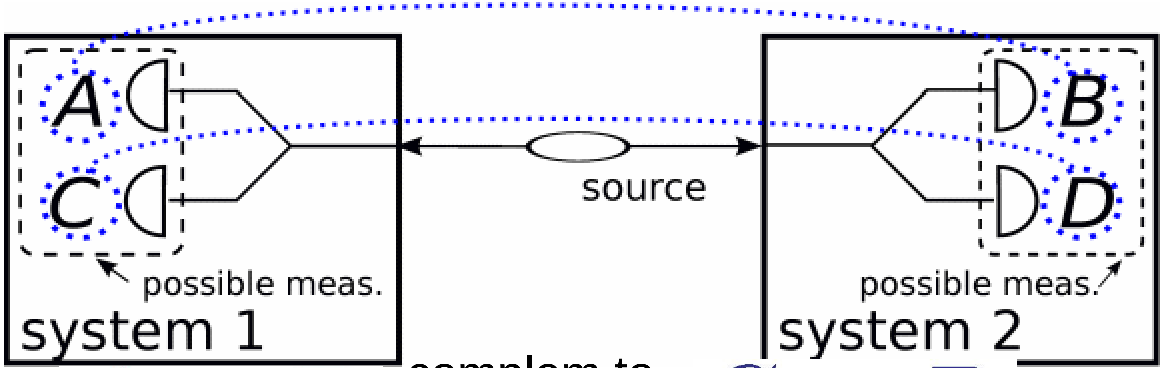
CONJECTURE: we don't know if it's true also using S_{AB}



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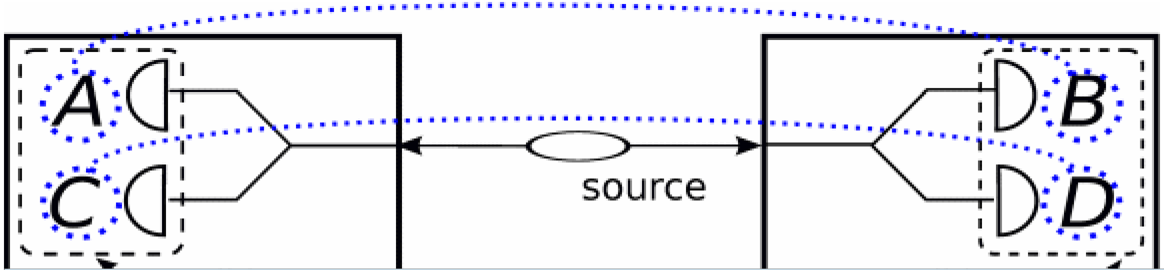

$$p(a_i|b_i) = 1$$ ← results are always the **same**

$$p(a_i|b_i) = 1/d$$ ← results are **uncorrelated**

it's true also using S_{AB}

$$S_{AB} + S_{CD} \notin [1, d + 1] \Rightarrow \rho_{12} \text{ ent}$$

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Conjecture:

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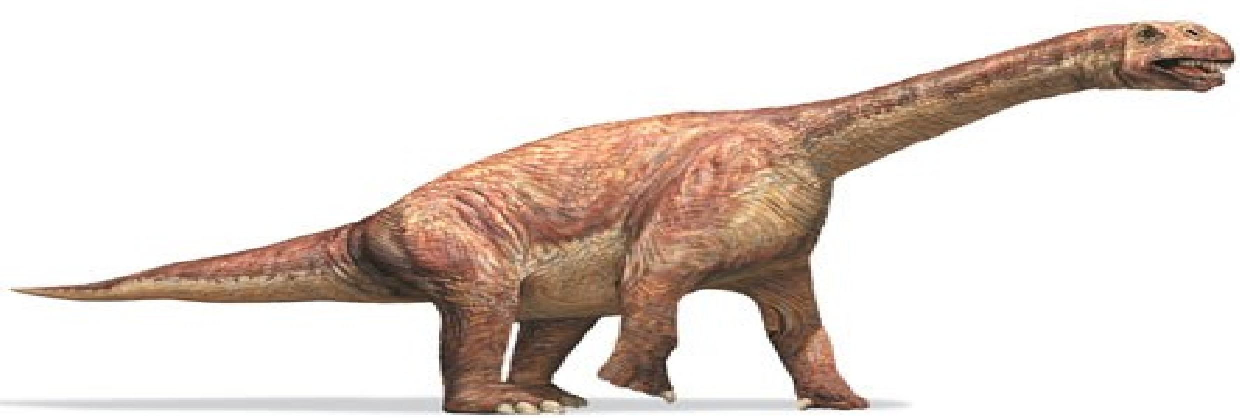

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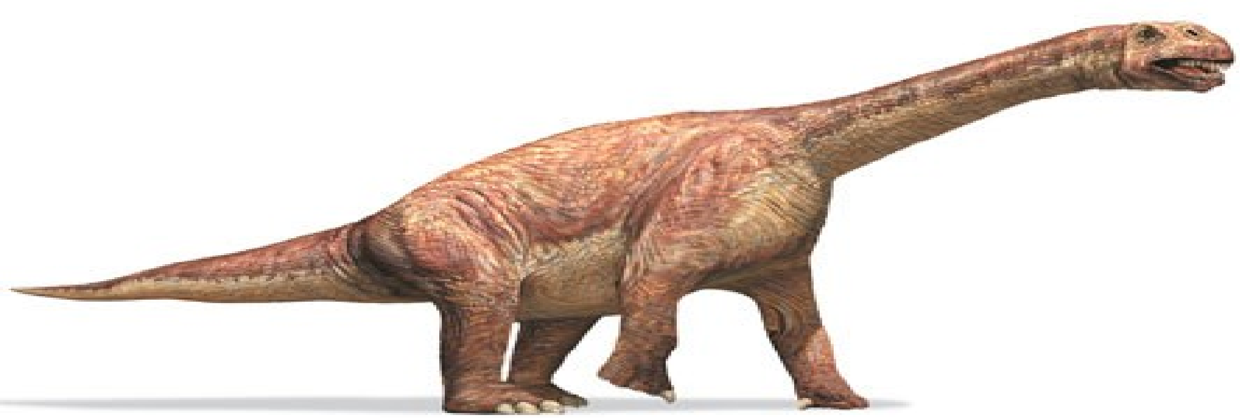

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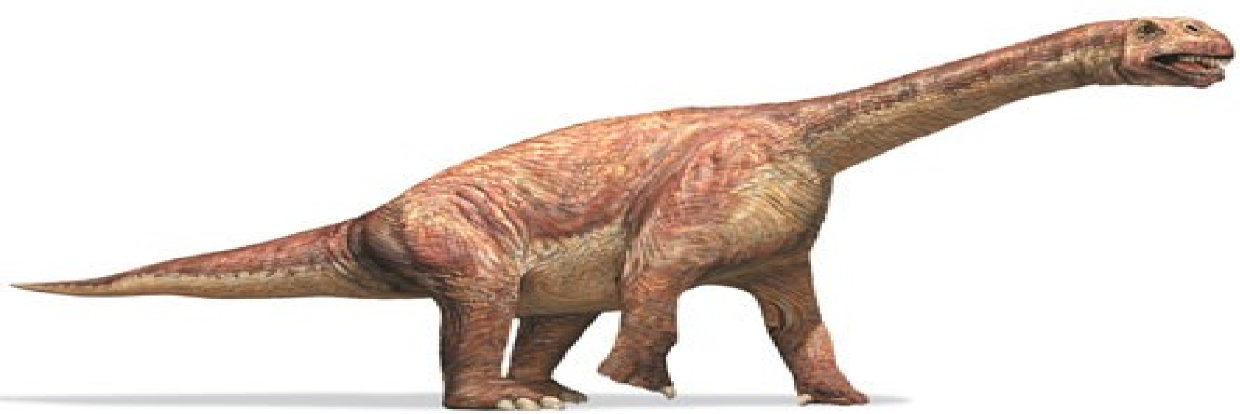
$$S_{AB} + S_{CD} = d+1$$

Perfect correlation on the |0>,|1> basis (→ d)

No correlation on the |+>,|- > basis (→ 1)



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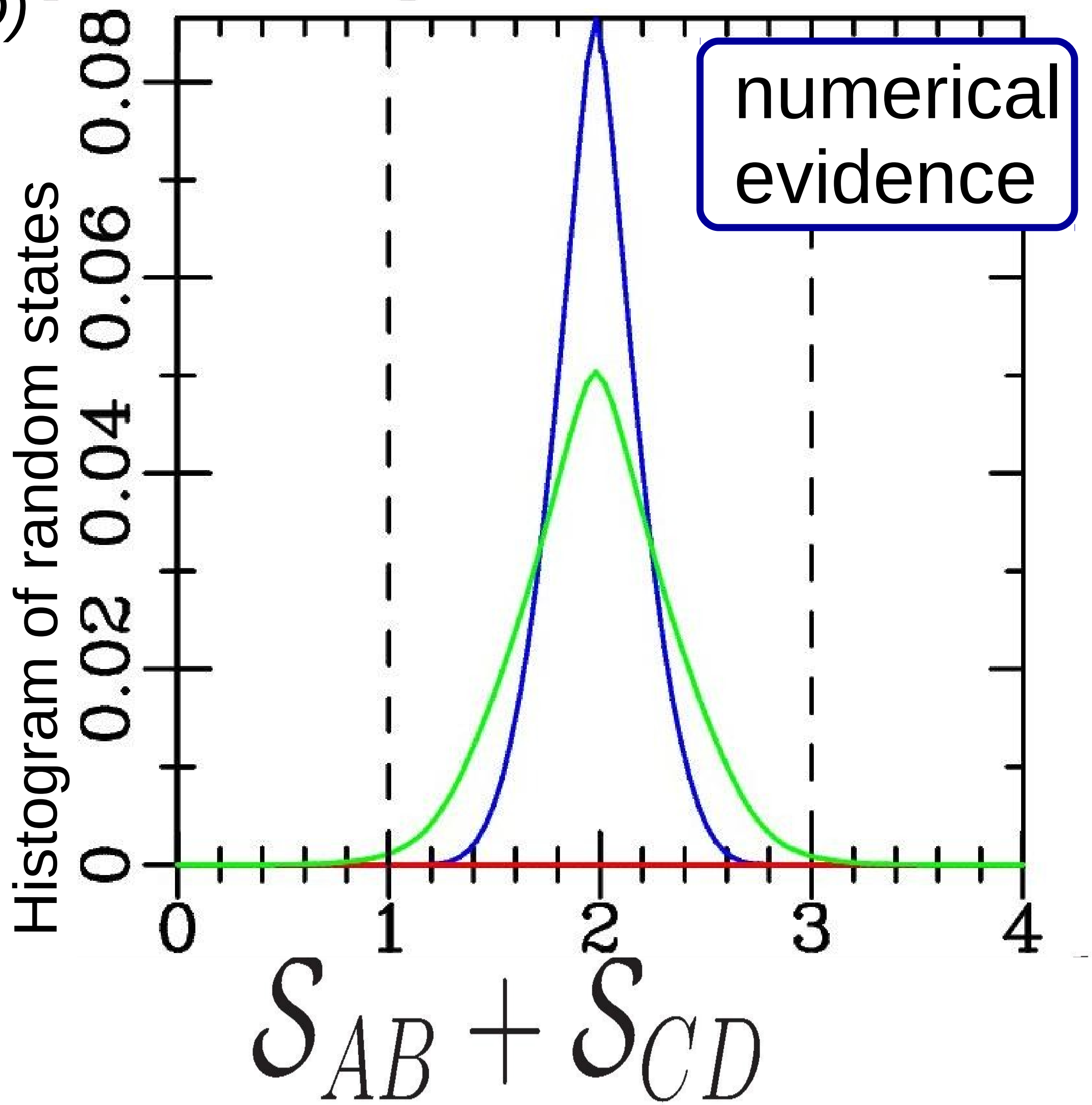
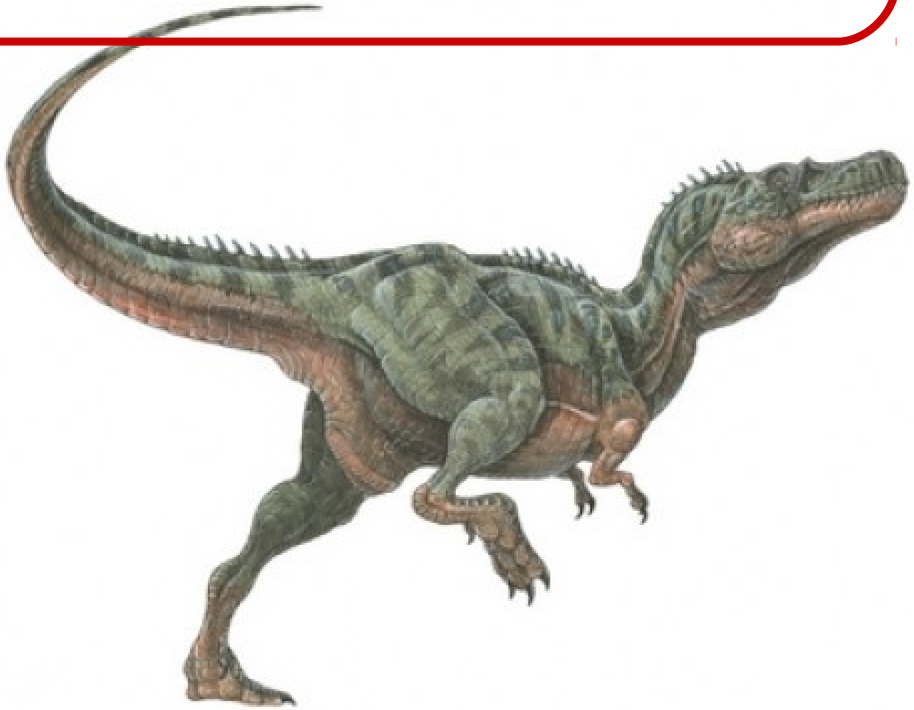


Role of discord?

Role of discord?

zero discord states can be correlated only on **one** of the complem properties. S_{AB}

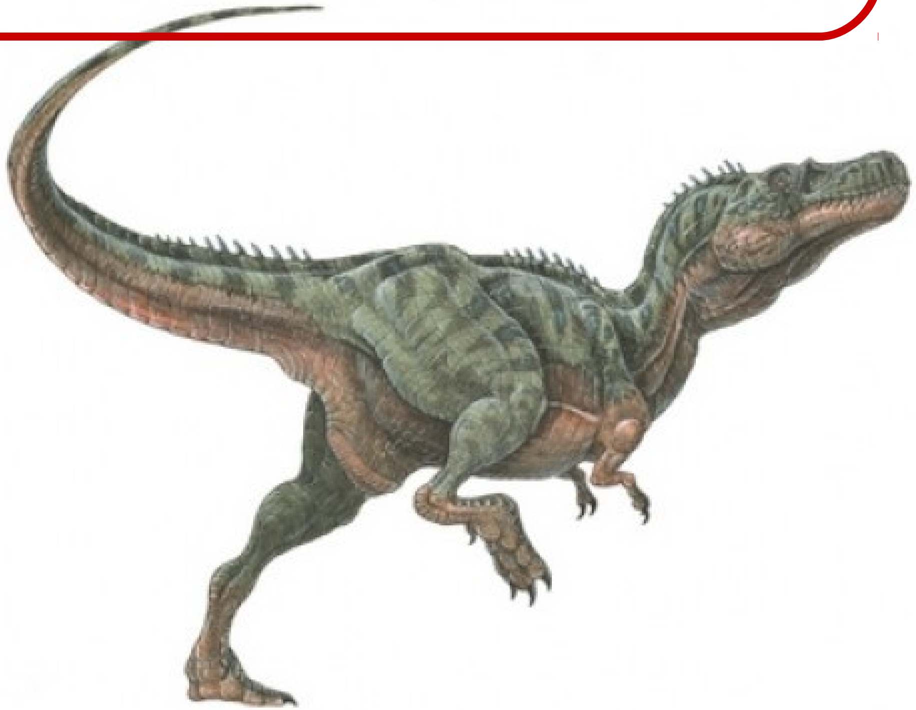


Role of discord?

zero discord states can be correlated only on **one** of the complem properties. S_{AB}

$$\frac{|00\rangle\langle 00|+|11\rangle\langle 11|}{2}$$

perfect correlation only on 0/1



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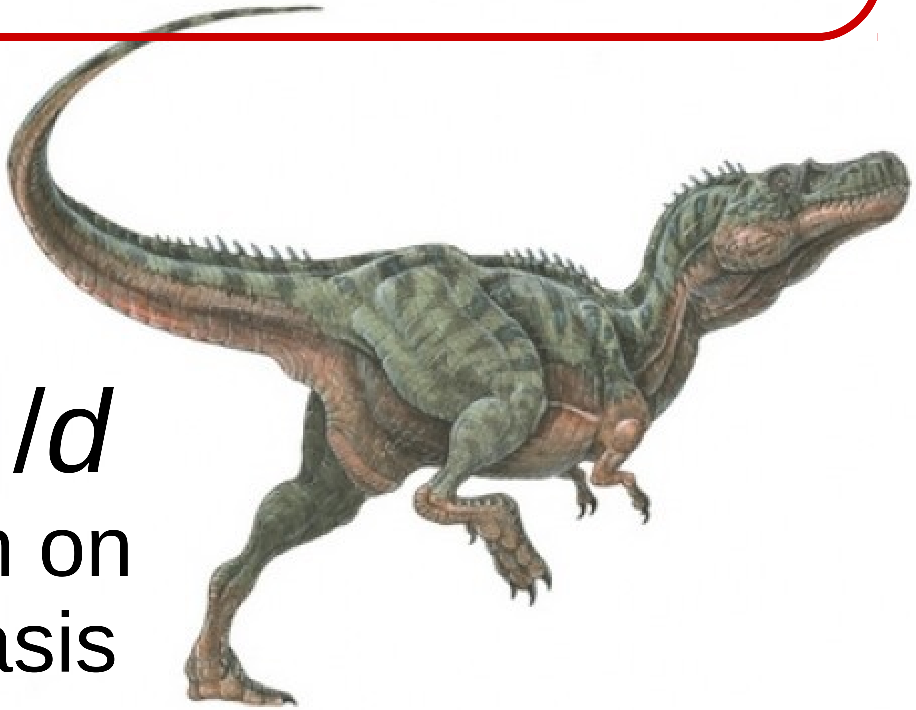

CC, CQ, QC states can be correlated only on **one** of the complem properties. S_{AB}

$$\frac{|00\rangle\langle 00| + |11\rangle\langle 11|}{2}$$

perfect correlation only on 0/1

Only CC states can have **perfect** correlation on one obs

CC, CQ, QC states can be correlated only on **one** of the complem properties. S_{AB}

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$$\sum_i p_{ij} |i\rangle\langle i| \otimes |j\rangle\langle j|$$ ← CC

$$\sum_i p_{ij} |i\rangle\langle i| \otimes \rho_j$$ ← CQ

$$\sum_i p_{ij} \rho_i \otimes |j\rangle\langle j|$$ ← QC

bases

What about QQ states?

$$\sum_i p_{ij} \rho_i \otimes \rho_j$$

QQ

What about QQ states?

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QQ

nonzero discord states can be **partially** correlated on **more** properties.

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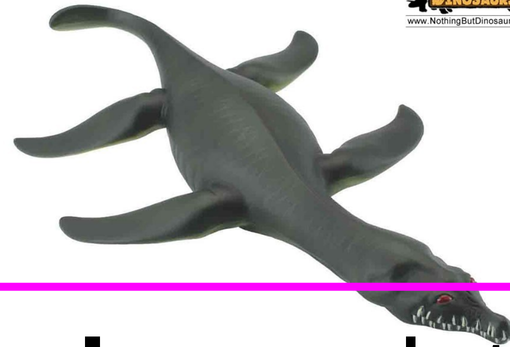
$$\frac{|00\rangle\langle 00| + |11\rangle\langle 11| + |++\rangle\langle ++| + |--\rangle\langle --|}{4}$$

$$p(0|0) = p(1|1) = p(+|+) = p(-|-) = 3/4$$

DISCORD:



- CC states can have **maximal** correlation only on one property
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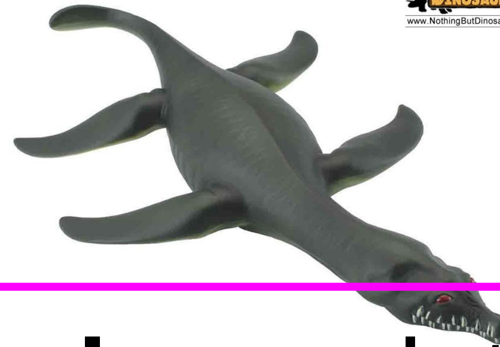

- CC states can have **maximal** correlation only on one property
- CQ states cannot have maximal correlation in **any** property
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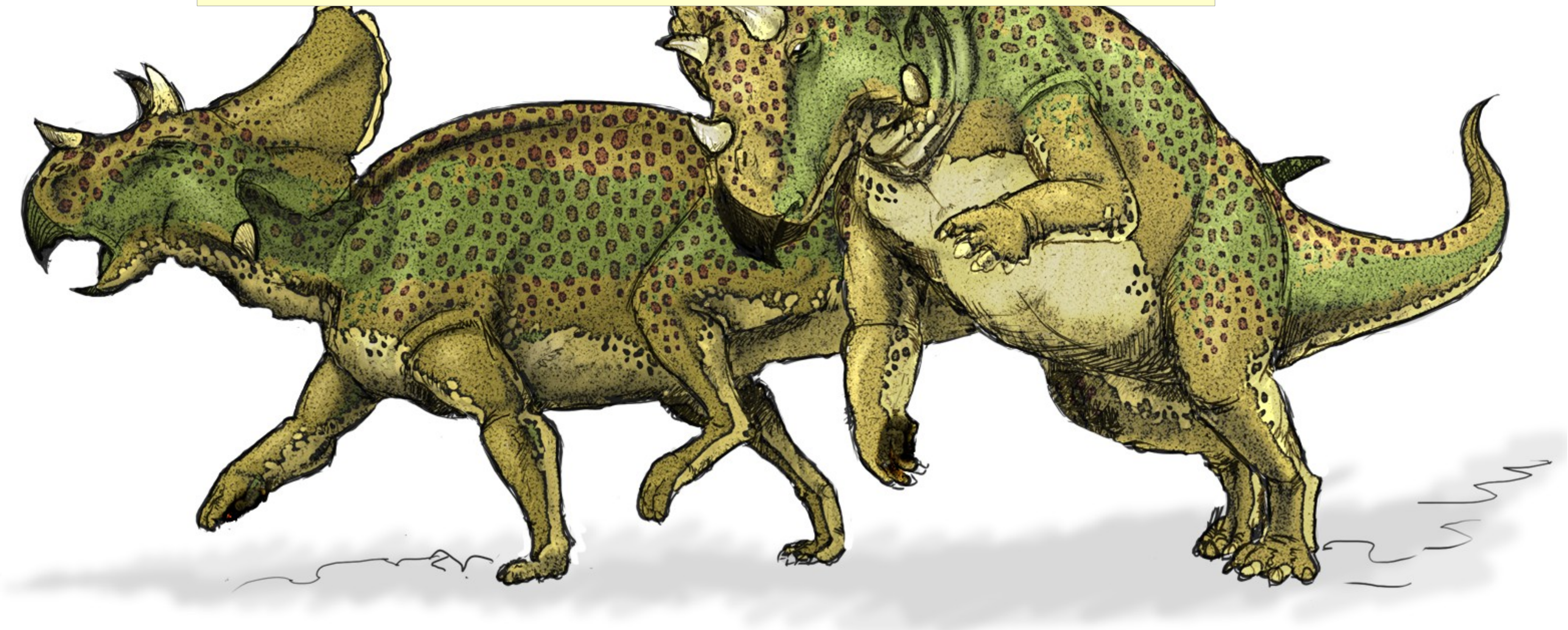
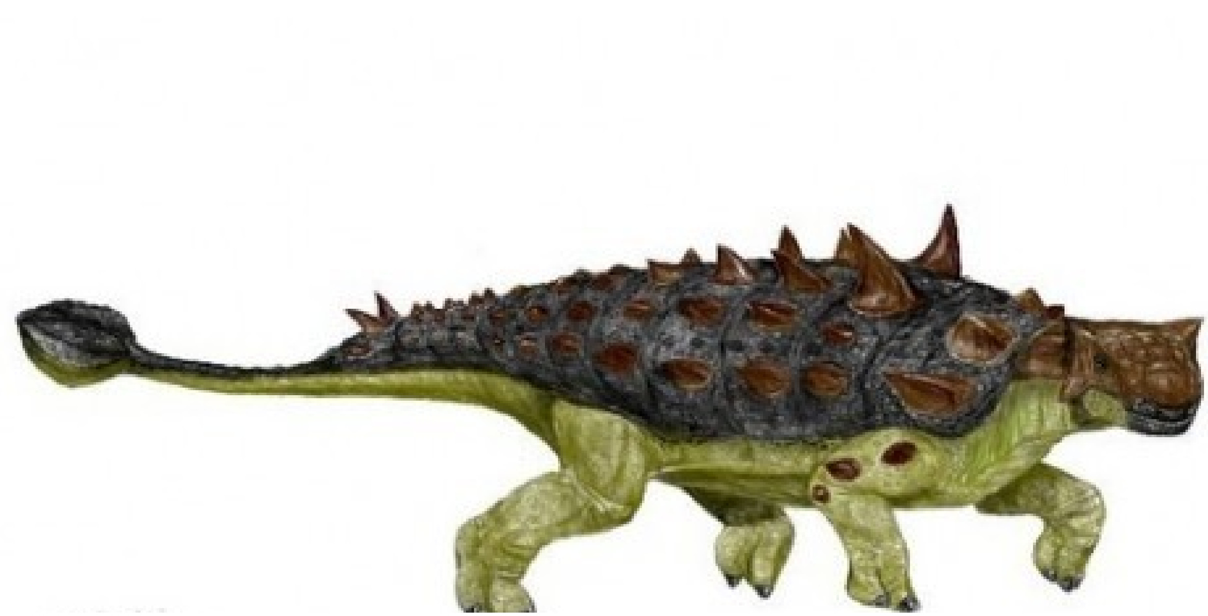
- **Only** pure, maximally entangled states have max correlations on more properties

What are these results good for, practically?

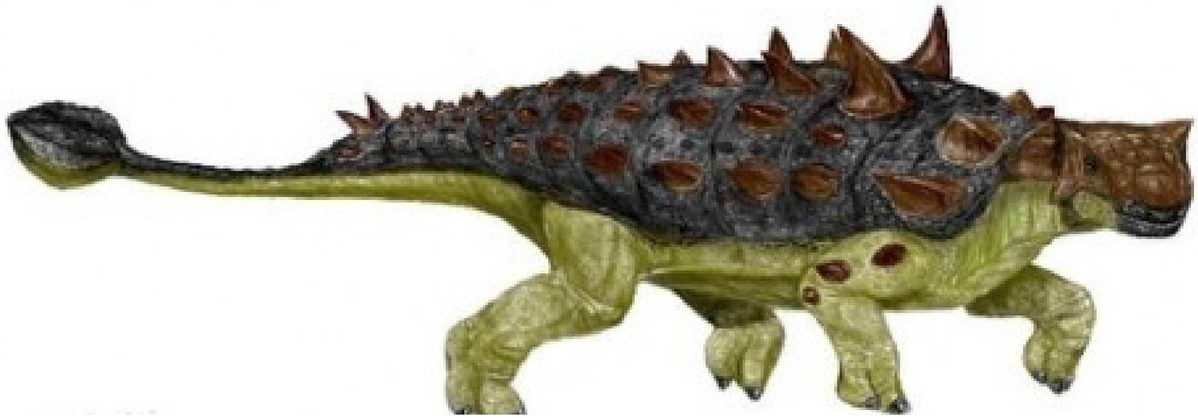


Simple criterion for entanglement detection!!



Simple criterion for entanglement detection!!

Just measure two complementary properties. Are the correlations greater than perfect correlation on one?



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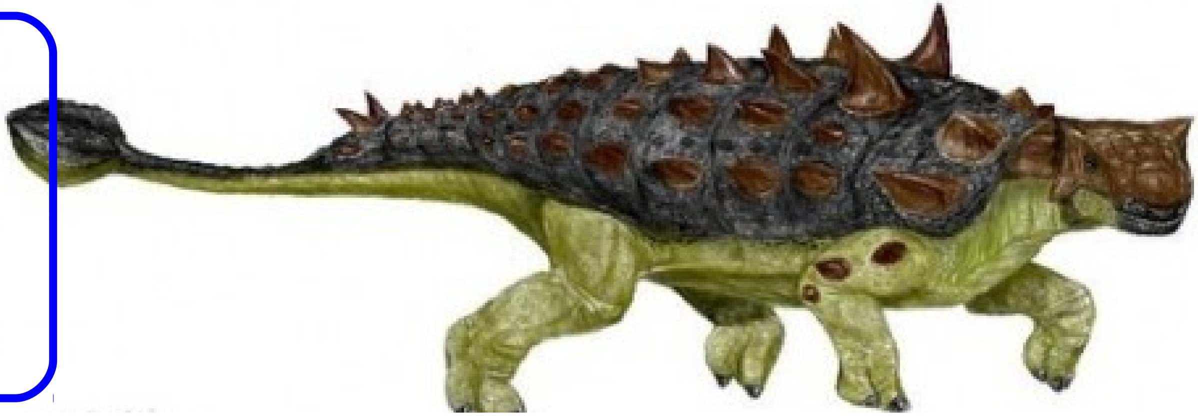

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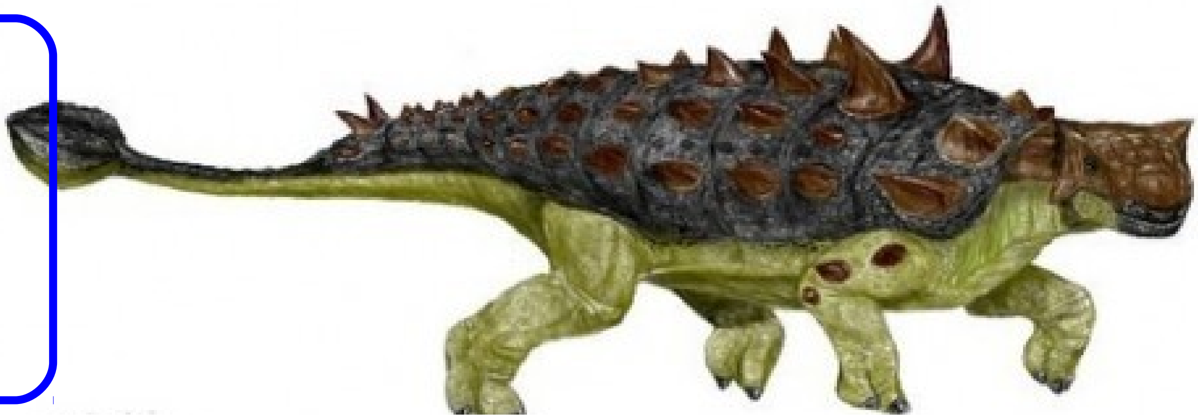

Simple to measure and simple to optimize.

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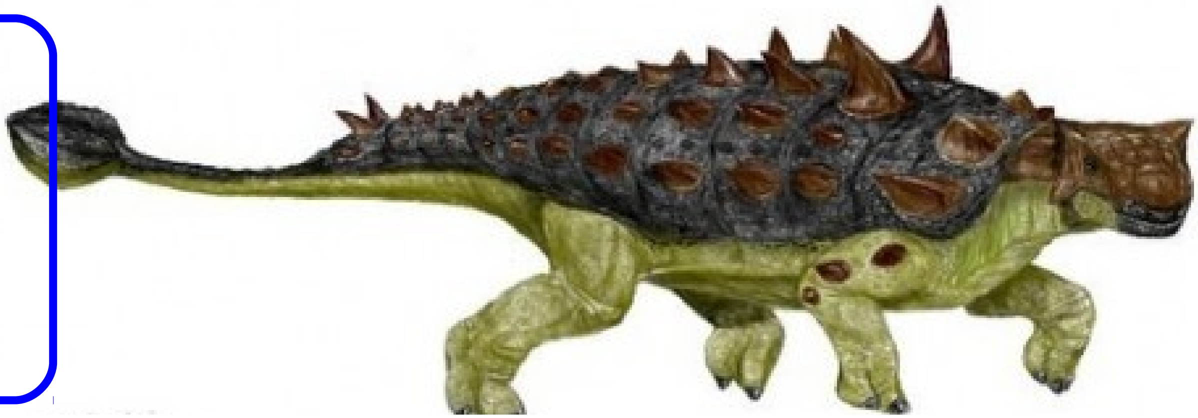
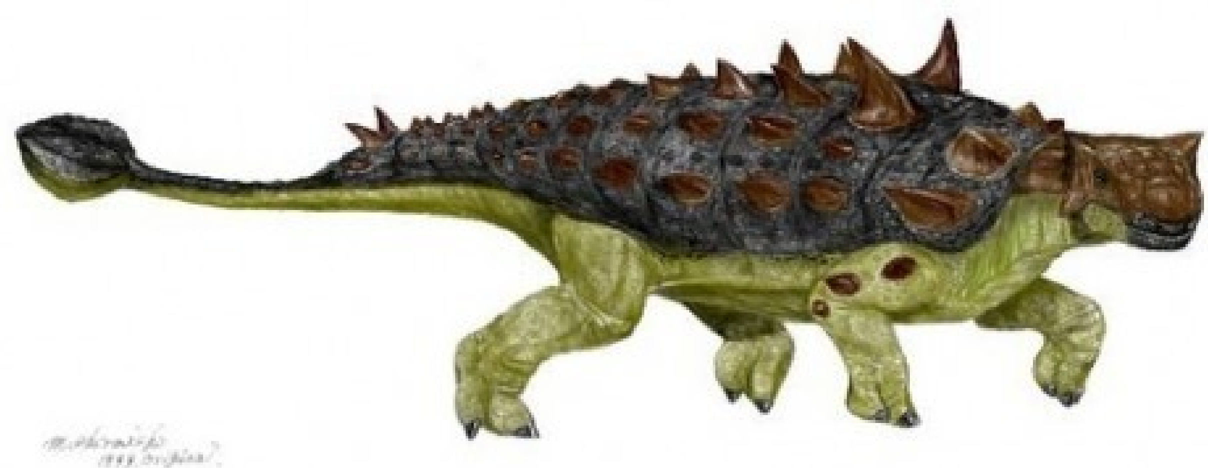


Simple to measure and simple to optimize.

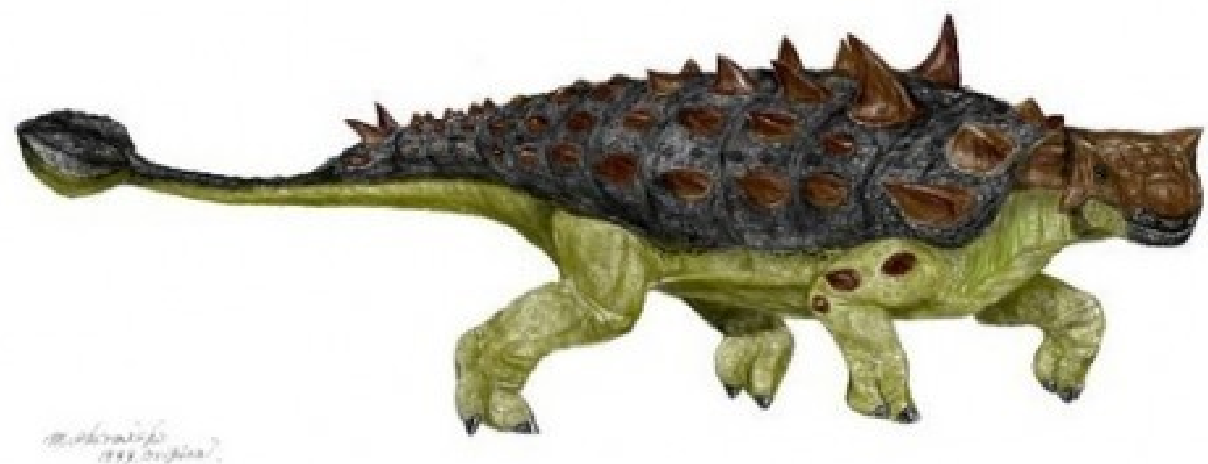
Unfortunately: not very effective in finding entanglement in random states

Simple and *effective* criterion for **maximal** entanglement detection!!



Simple and *effective* criterion for **maximal** entanglement detection!!

Just measure two complementary properties. Are the correlations maximal on both properties?

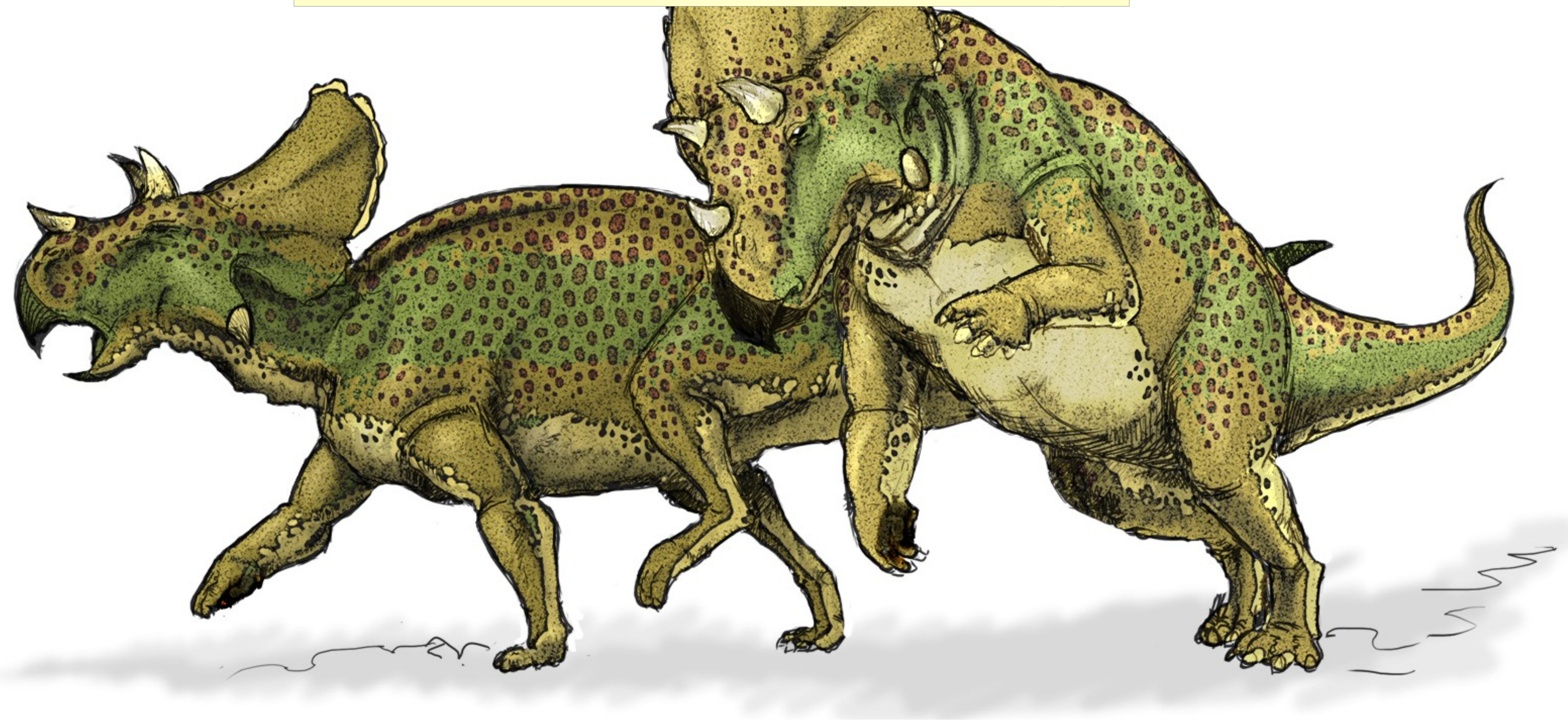


Simple and *effective* criterion for **maximal** entanglement detection!!

Just measure two complementary properties. Are the correlations maximal on both properties?

$\Rightarrow$ The state is maximally entangled!

Conclusions



What did I say?!?

- Entanglement as correlation among complementary observables

- Using different measures of correlation:
  - Mutual info
  - Pearson correlation
  - Sum of conditional prob

- Some theorems and some conjectures

- Role of discord

What did I say?!?

Results:

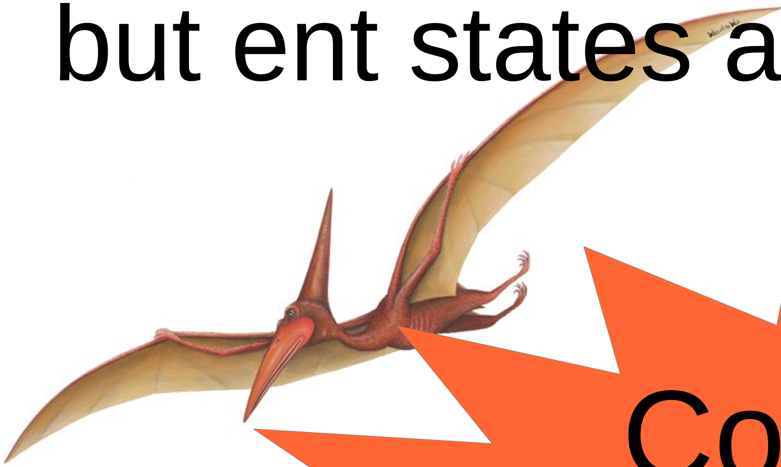
- necessary and sufficient conditions for maximal entanglement

- necessary conditions for entanglement

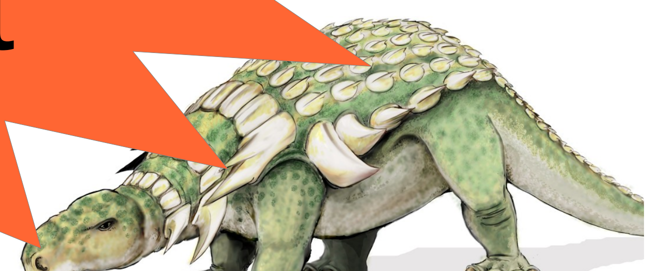
- discord:
  - mutual info: states on the boundary have no discord!
  - correlation properties of CC, CQ, QC, and QQ states.

Take home message

The most correlated states are entangled
but ent states are not the most correlated



Correlations on
complementary prop.
help understanding
entanglement



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