Generation of Entanglement in a Quantum Wire ands applications to Single-Electron Transmittivity

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1. Entanglement of Two Impurities through Electron Scattering

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The General Problem

OBJECTIVE:

To generate bipartite entanglement between two spins (or other degrees of freedom) in mesoscopic solid state structures.

MOTIVATION:

Obtaining an important resource for quantum information processing, in solids; linking different registers of a solid state quantum computer; testing Bell's inequalities in solids...

CONTEXT:

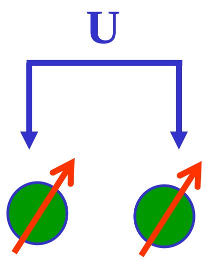
Recently there has been an increasing interest in this problem from both the quantum information and condensed matter communities, with a variety of schemes being proposed.

Previous proposals

- Quantum gate acting on two adjacent stationary spins:

  G. Burkard, D. Loss, D. DiVincenzo, PRB **59**, 2070 (1999)
  W. Oliver, F. Yamaguchi, Y. Yamamoto, PRL **88**, 037901 (2002)

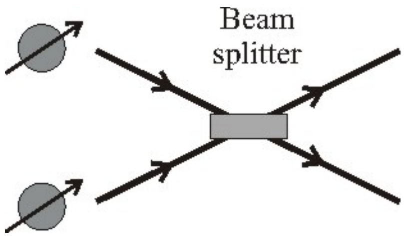
  How about distant spins/qubits?



- Interactions between mobile spins (and others):

  S. Bose, D. Home, PRL **88**, 050401 (2002)
  D. Saraga, D. Loss, PRL **90**, 166803 (2003)

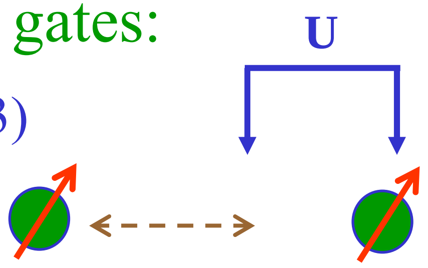
  How about distant stationary spins/qubits?



- Shuttling spins over a distance to precisely-timed gates:

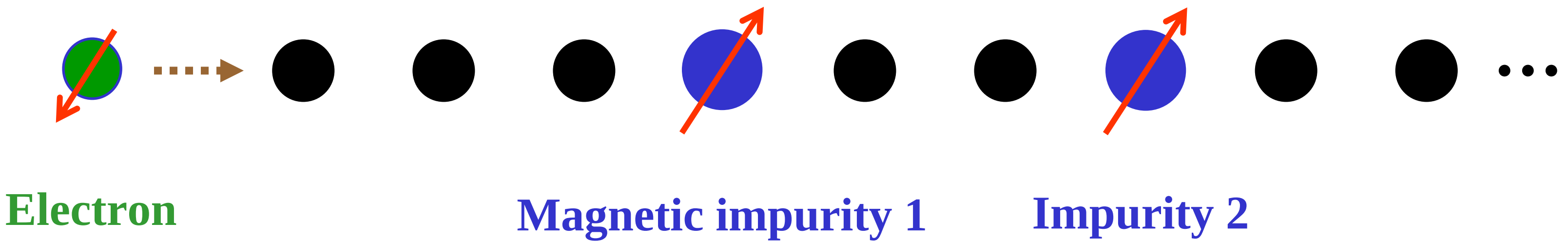
  A. Skinner, M. Davenport, B. Kane, PRL **90**, 087901 (2003)

  How about a low control/precision scheme?



Our Proposal

We propose a scheme to entangle two magnetic impurities (stationary spins 1/2) embedded in a 1-D solid state system, using a ballistic electron as an agent which scatters off the two impurities in succession and entangles them:



Solid: metallic chain of non-magnetic atoms (reduced cross section);

Qubits: two distant embedded spin-1/2 magnetic impurities;

Entangler: ballistic conduction electron, injected under low bias, to naturally scatter off the two impurities – no control!

See also D. Sagara *et al*, PRB **71**, 045338 (2005).

Our Model

Magnetic impurities embedded in a conduction electron sea are traditionally modeled by a *s-d* Hamiltonian.

In this model, the magnetic impurities are localized spins interacting with the conduction electrons via an exchange term.

The full Hamiltonian of a system with **one** impurity reads:

$$H = \sum_{k,\sigma} \varepsilon_k \hat{a}^\dagger_{k\sigma} \hat{a}_{k\sigma} + \sum_{kk'} J_{kk'} \vec{S}.\vec{s}_{kk'} \ ,$$

where $\vec{S}$ is the impurity spin operator, $a^\dagger_{k\sigma}$ creates an electron with wavevector k and spin σ and $\vec{s}_{kk'} = \hat{a}^\dagger \vec{\sigma} \hat{a}$, with $\hat{a} = \begin{bmatrix} a_{k\uparrow} \\ a_{k\downarrow} \end{bmatrix}$.

The *s-d* Hamiltonian is actually derived from the more fundamental Anderson Hamiltonian through the Schrieffer-Wolff transformation.

Consequently, the interaction strength *J* is related to the strength of the Coulomb interaction between electrons and the hybridization of narrow and conduction bands.

In our calculation we will adopt the usual assumption that *J* is independent of *k*,*k'*.

Our Calculations (1)

We want to find out how much entanglement may be generated by a conduction electron that is injected in the system and interacts with both magnetic impurities.

One may determine the system's final state by calculating the scattering matrix associated with each impurity and combining them together.

The result is a sequence of (infinitely many) scattering processes, in which the output of a scattering event is the input of the subsequent one.

The result of each individual scattering process is determined by use of Fermi's golden rule. The relevant T matrix is calculated to first order in the interaction.

Our Calculations (2)

If we consider that the conduction electron is being injected under low bias, its energy and wave vector will be the Fermi energy and Fermi wave vector of the system, respectively.

We thus assume a initial state of the form:

$$|\Psi_{in}\rangle = |k_F, \uparrow\rangle \otimes |\downarrow\downarrow\rangle$$

As a result of the multiple scatterings of the conduction electron by the two impurities, a final state is generated which is a superposition of states in which the conduction electron has been *reflected* or *transmitted*, and the latter component is:

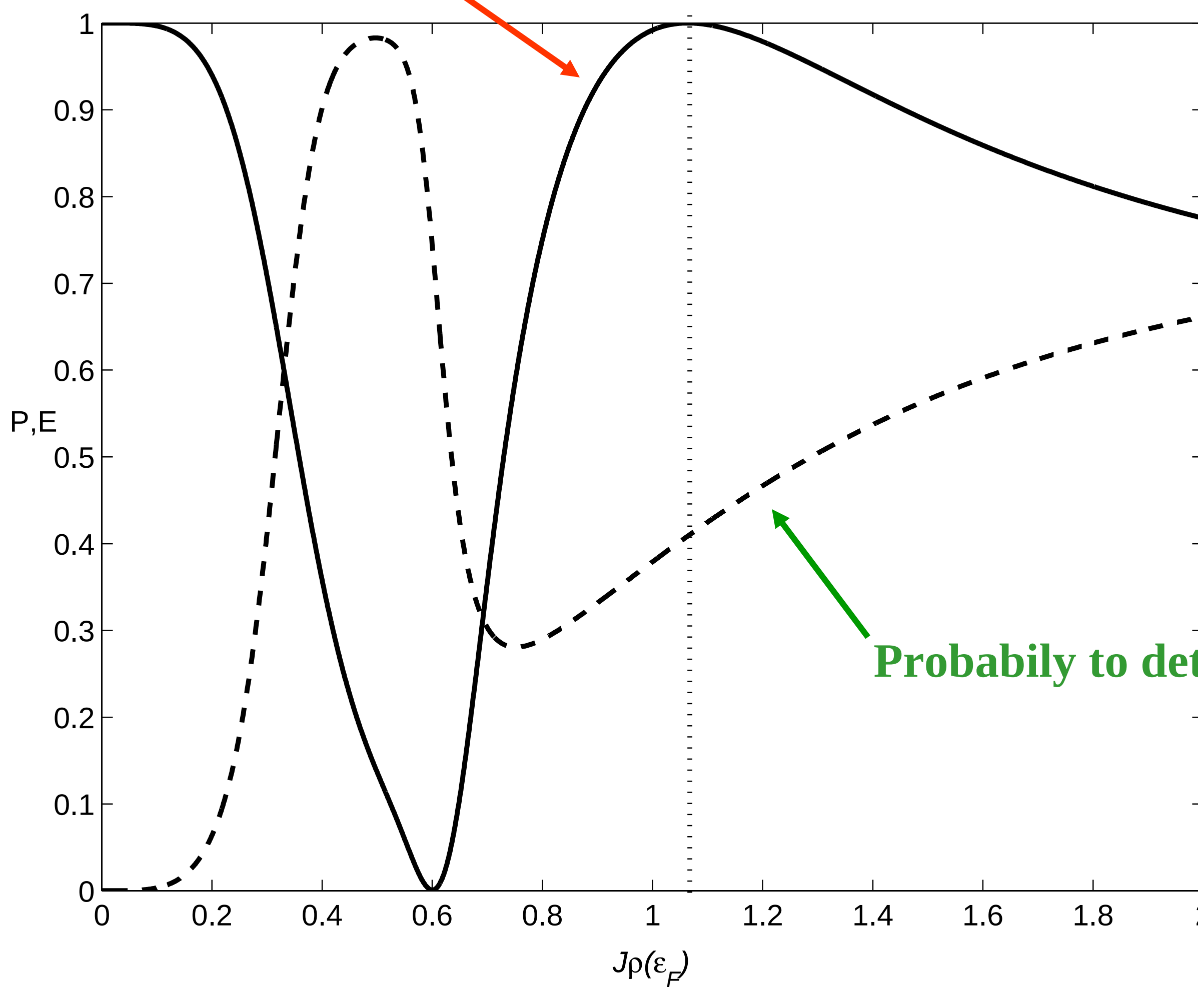
$$|\Psi^t_{out}\rangle = A|k_F, \uparrow\rangle \otimes |\downarrow\downarrow\rangle + B|k_F, \downarrow\rangle \otimes |\uparrow\downarrow\rangle + C|k_F, \downarrow\rangle \otimes |\downarrow\uparrow\rangle \,.$$

The coefficients *A*, *B*, and *C* may be expressed as an infinite sum of powers of the product $J\rho(\varepsilon_F)$.

If the transmitted electron is down, the impurities are entangled!

Our Results

Amount of entanglement between the impurities



Probabily to detect the electron ↓

Conclusions and Future Directions

We have presented a scheme that allows for the generation of (maximal) entanglement between two distant stationary qubits in a 1-D solid simply by scattering, with *no control* required.

- → Improve probabilities?
- → Robustness to noise, temperature?
- → Generalization to more dimensions, multipartite entanglement?
- → Implementations (p/d)? Applications?

2. Entanglement Controlled Single-Electron Transmittivity

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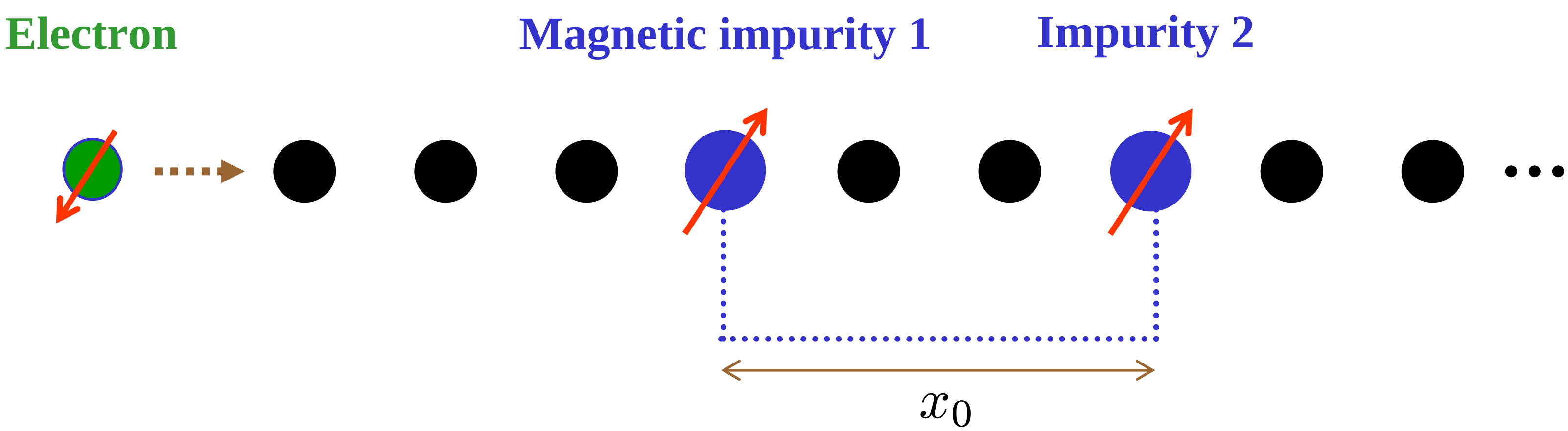
New J. Phys. 8, 214 (2006), quant-ph/0611025, ...

Reversing the Problem!

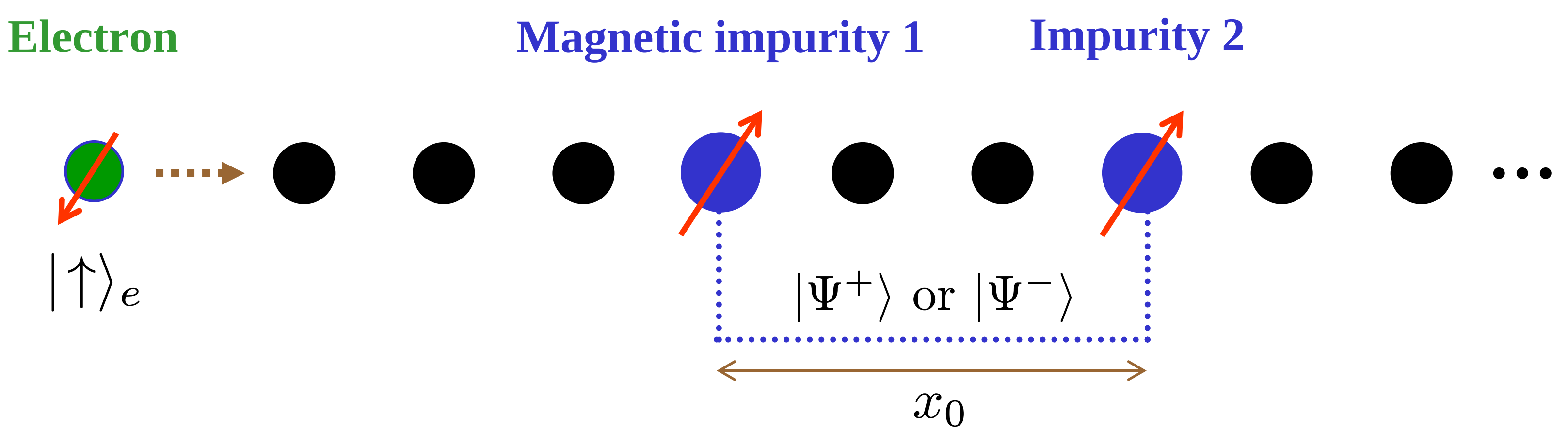
Now we want to investigate the transmittivity of sending one spin qubit through our quantum wire with already-entangled impurities.

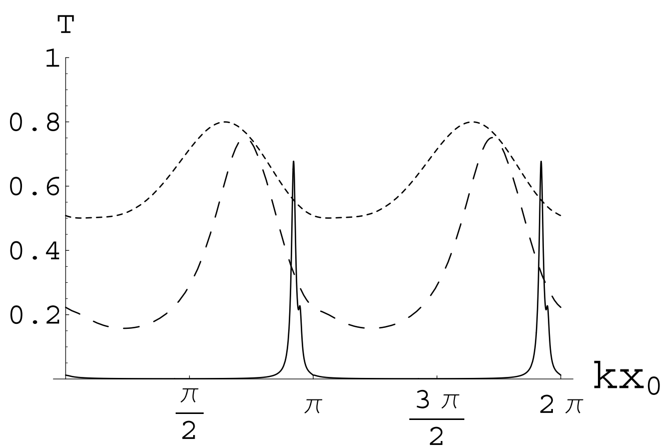


The transmittivity will be obtained as a function of the dimensionless quantities: kx_0 and $J\rho(\varepsilon)$.

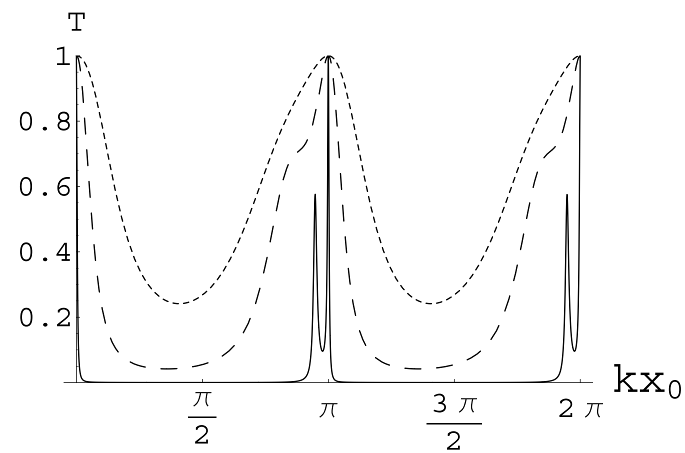
Again we assume that we can prepare and detect the single electron transmitted.

Our Results (1)





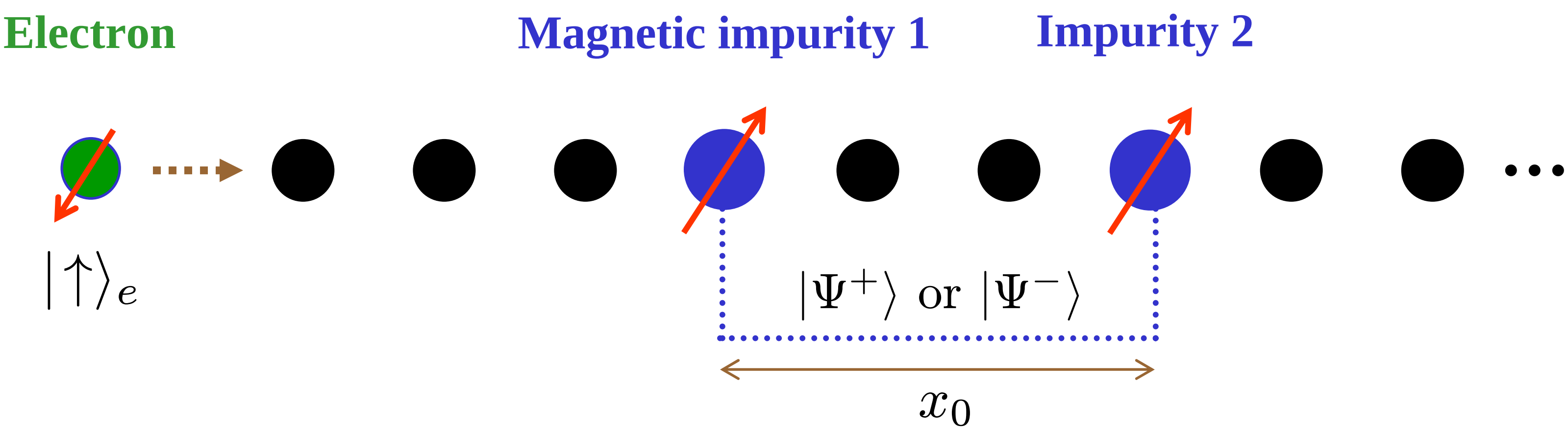
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

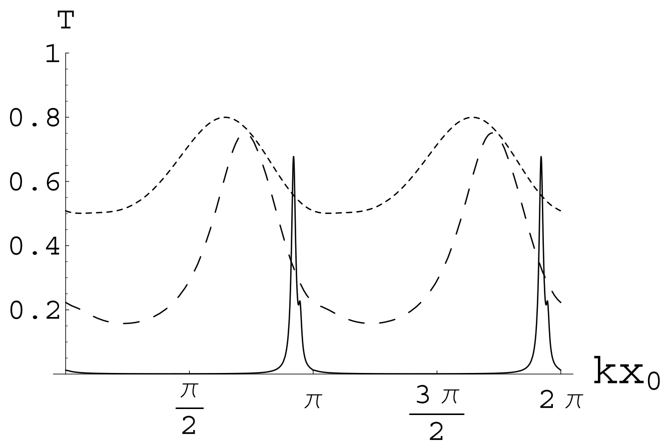


$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Dotted, dashed and solid lines stand for $J\rho(\varepsilon) = 1, 2, 10$, respectively.

Our Results (1)





$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

Let us assume an electron effective mass of 0.067 m_0 (as in *GaAs* quantum wires) and the impurities to be two quantum dots, each of size 1 *nm*.

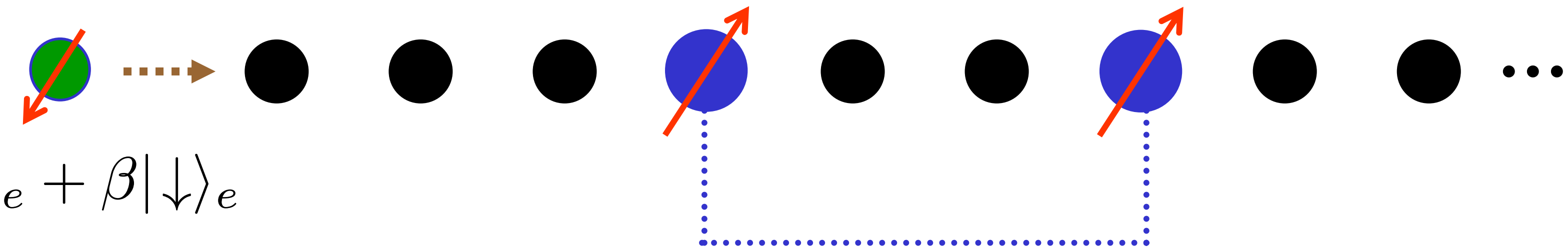
As a consequence, the maximum electron energy allowing to assume a contact electron-dot potential is around 2 *meV*.

In this case, for $J\rho(\varepsilon) = 1$, we obtain $J \simeq 1$ eVÅ which appears to be a reasonable value.

Dotted, dashed and solid lines stand for $J\rho(\varepsilon) = 1, 2, 10$, respectively.

Our Results (2)

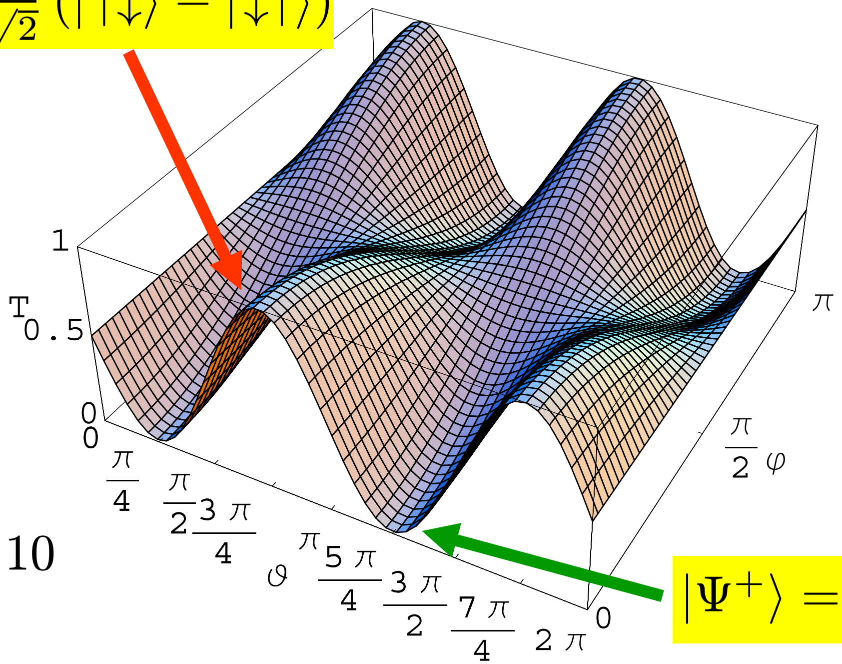
Electron **Magnetic impurity 1** **Impurity 2**



$\alpha|\uparrow\rangle_e + \beta|\downarrow\rangle_e$

$$|\psi\rangle = \cos\theta|\uparrow\downarrow\rangle + e^{i\phi}\sin\theta|\downarrow\uparrow\rangle$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

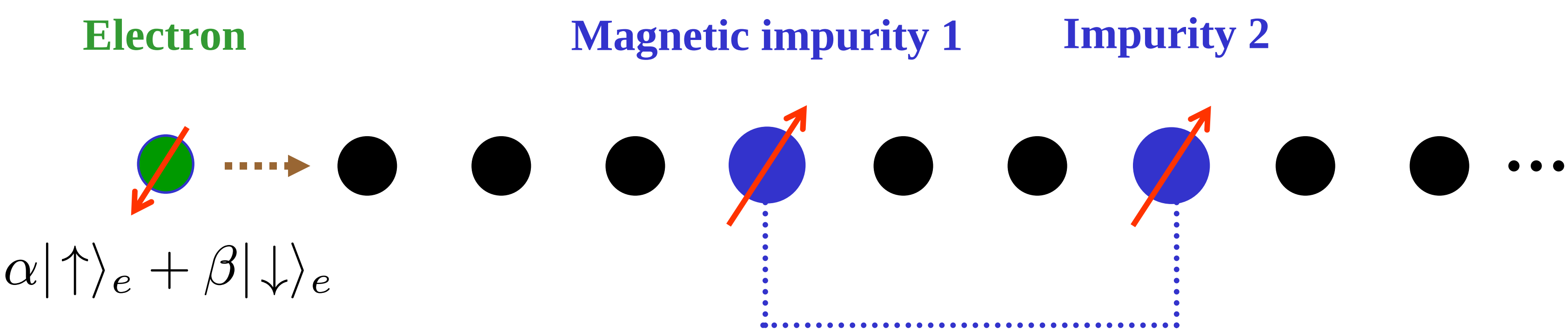


$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

$J\rho(\varepsilon) = 10$

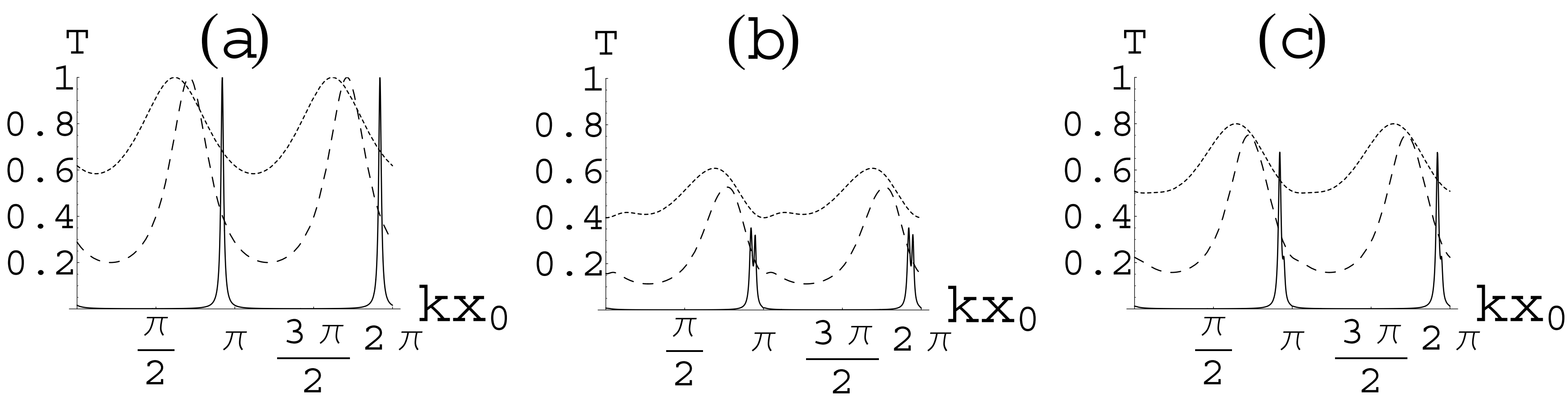
We observe perfect and zero transmittivity depending on the entangled state!

Our Results (3)



$$|\psi\rangle = \cos\theta|\uparrow\uparrow\rangle + e^{i\phi}\sin\theta|\downarrow\downarrow\rangle$$

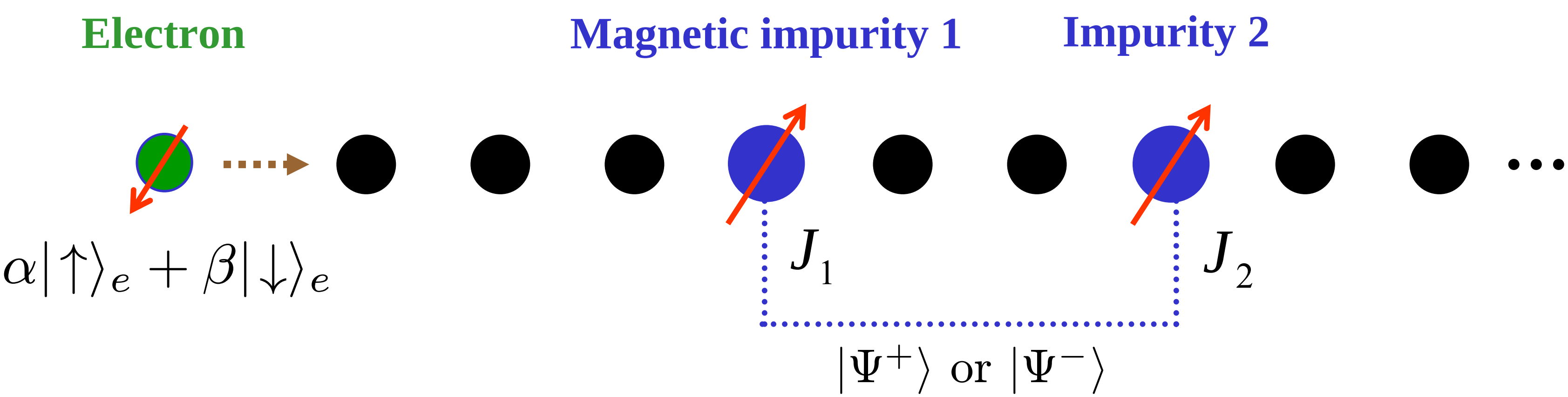
Just like a statistical mixture!



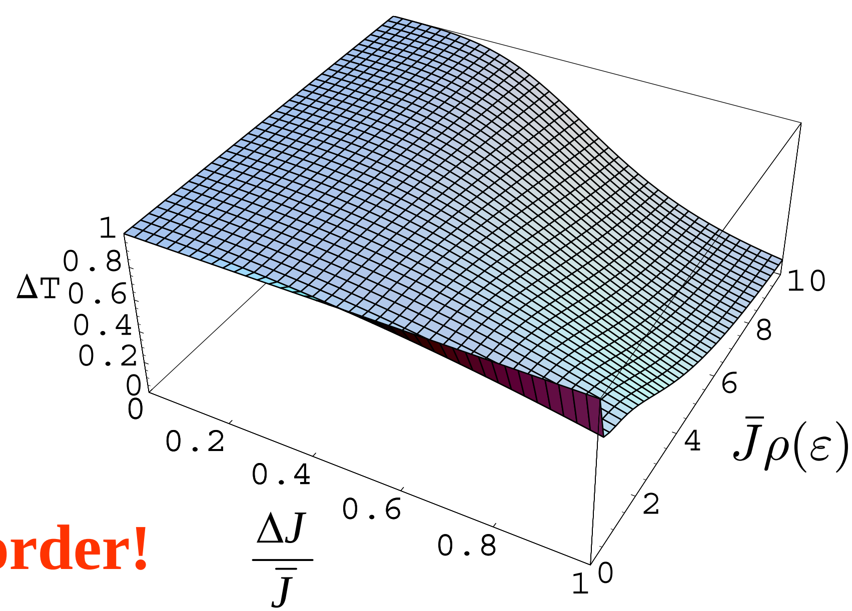
Initial state $|\uparrow\uparrow\rangle$ (a), $|\downarrow\downarrow\rangle$ (b) and $(|\uparrow\uparrow\rangle + e^{i\phi}|\downarrow\downarrow\rangle)/\sqrt{2}$ for arbitrary ϕ (c).

Dotted, dashed and solid lines stand for $J\rho(\varepsilon) = 1, 2, 10$, respectively.

What if the impurities are different?



$$\begin{cases} \Delta J \equiv J_2 - J_1 \\ \bar{J} \equiv \dfrac{J_1 + J_2}{2} \\ \Delta T \equiv T_{\Psi^-} - T_{\Psi^+} \end{cases}$$



Robust against static disorder!

Conclusions and Future Directions

We have shown that the entanglement between the impurities can have a strong influence in the transmittivity of a spin qubit in a quantum wire.

In fact, it can completely inhibit or transmit this spin. Such effect are due to destructive and constructive interference, just like in a Fabry-Perot inteferometer.

→ Generalization to other scenarios?

→ Implementations (preparation/detection)?

→ Applications to entanglement detection?

Summary

1. Generation of (maximal) entanglement between two distant stationary qubits in a 1-D solid.

2. Transmission of single electrons in a quantum wire controlled by entanglement.

→ Robustness to noise, temperature?

→ Generalization to more dimensions, multipartite entanglement?

→ Implementations? Applications?