IICQI @Kish
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Multi entanglement in a single-neutron system

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I. Introduction — Neutron optical experiments
   Neutron interferometry

II. Multi entanglement in single particle
   Spin-path entanglement
   Spin-path-energy entanglement
   Multi energy-level entanglement

III. Summary

Neutron interferometry

Neutrons

$m = 1.67 \times 10^{-27}$ kg

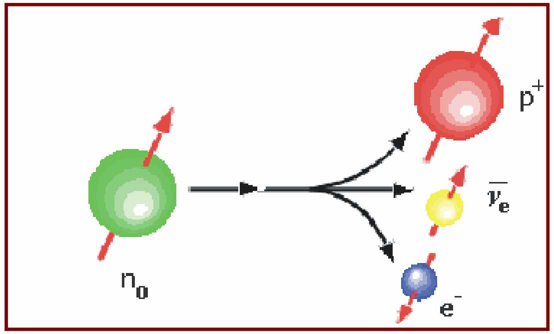
$s = \frac{1}{2}\hbar$

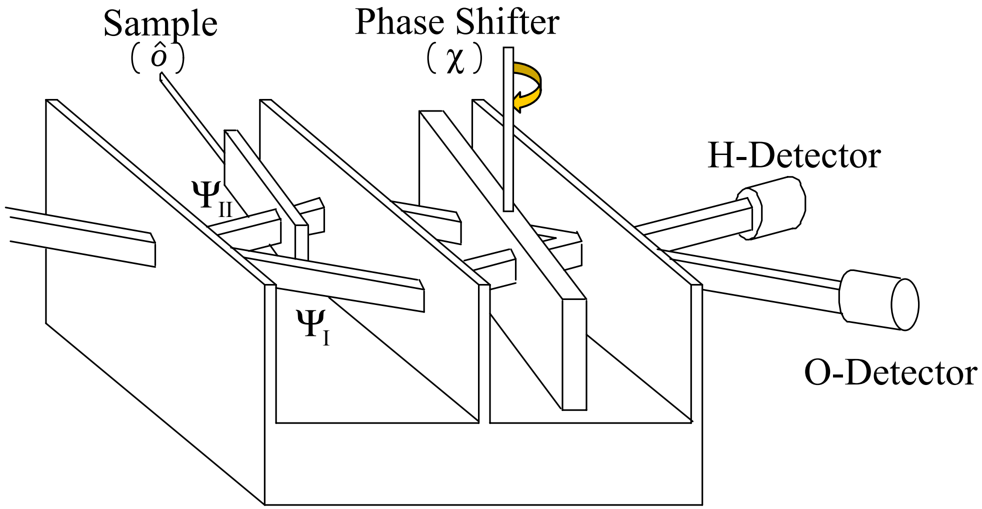
$\mu = -9.66 \times 10^{-27}$ J/T

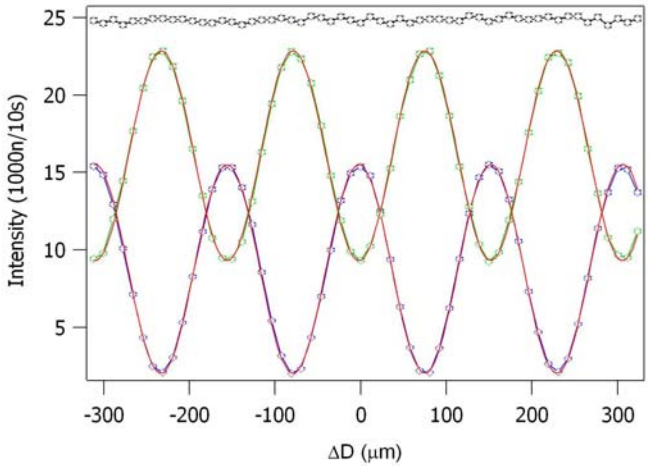
$\tau = 887$ s

$R = 0.7$ fm

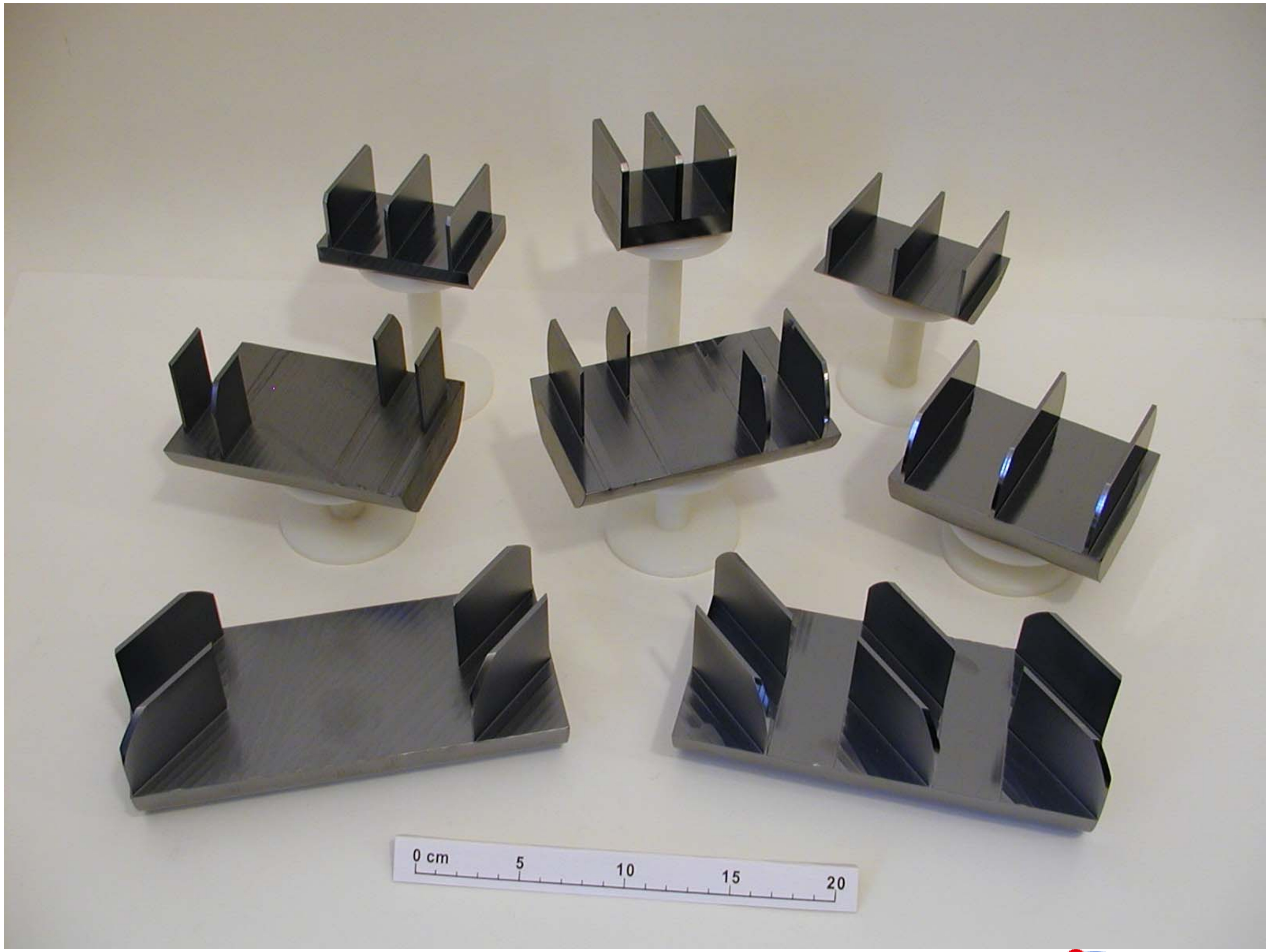
u–d–d quark structure







Neutron interferometers



Advantages of the use of neutrons

- Single-particle (events)
  Massive, composite system, no fine-structure
- Following Schrödinger-equation
- *Pure* single-events (of Ferminons)
- ~100% detector efficiency
- Week (controllable)-coupling with
  an environment ➔ decoherence
- Storable, e.g. neutron bottle

E_{photon}~1eV

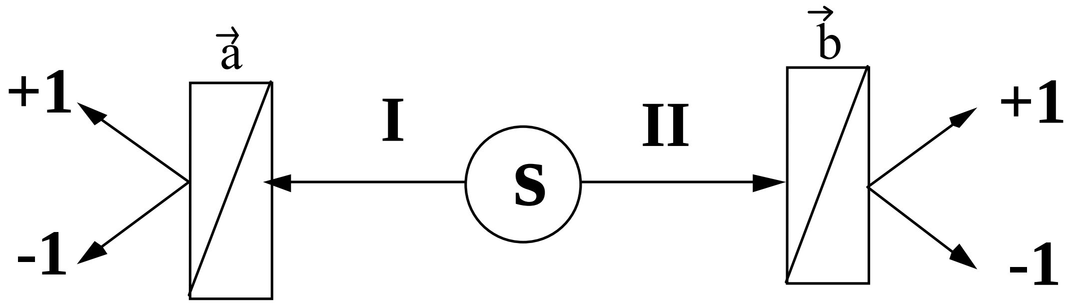
$E_{neutron}$~1GeV

Multi-entanglement in single-particle

Muti-entanglement in neutrons

- ☆ bi-entanglement: spin-path
- ☆ tri-entanglementl: spin-path-energy
- ☆ multi-entanglement: energy-levels

From two-particle to two-space entanglement



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_I \otimes |\downarrow\rangle_{II} + |\downarrow\rangle_I \otimes |\uparrow\rangle_{II}\}$$

⇒⇒⇒ Entanglement between *Two-Particles*

2-Particle Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_I \otimes |\downarrow\rangle_{II} + |\downarrow\rangle_I \otimes |\uparrow\rangle_{II}\}$$

I, II represent 2-Particles

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_s \otimes |\,I\rangle_p + |\downarrow\rangle_s \otimes |\,II\rangle_p\}$$

s, p represent 2-Spaces, e.g., spin & path

Various two-level system

1/2-Spinor

$$|s\rangle = \begin{bmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{bmatrix} \qquad \hat{H}_{int} = -\boldsymbol{\mu}\cdot\boldsymbol{B}$$

Larmor precession

Two-level atom

$$|\phi_{\mathrm{atom}}\rangle = \begin{bmatrix} |e\rangle \\ |g\rangle \end{bmatrix} \qquad \hat{H}_{int} = i\hbar g_{\mathrm{k}}\{|e\rangle\langle g|\hat{a}_{\mathrm{k}} e^{i\mathbf{kR}} - |g\rangle\langle e|\hat{a}_{\mathrm{k}}^{\dagger} e^{-i\mathbf{kR}}\}$$

Rabi oscillation

Two-path interferometer

$$|\Psi\rangle = \begin{bmatrix} |\Psi_{\mathrm{I}}\rangle \\ |\Psi_{\mathrm{II}}\rangle \end{bmatrix} \qquad \hat{H}_{PS} = \begin{bmatrix} e^{+i\chi} & 0 \\ 0 & e^{-i\chi} \end{bmatrix}$$

Sinusouldal intensity oscillation

==>> Described by SU(2)

Two-particle vs. two-space entanglement

2-Particle Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_I \otimes |\downarrow\rangle_{II} + |\downarrow\rangle_I \otimes |\uparrow\rangle_{II}\}$$

I, II represent 2-Particles

Measurement on each particle

$$\begin{cases} \hat{A}^I(\vec{a}) = (+1)\cdot\hat{P}^I_{(\vec{a};+1)} + (-1)\cdot\hat{P}^I_{(\vec{a};-1)} \\ \hat{B}^{II}(\vec{b}) = (+1)\cdot\hat{P}^{II}_{(\vec{b};+1)} + (-1)\cdot\hat{P}^{II}_{(\vec{b};-1)} \end{cases}$$

where $\hat{P}_{(\vec{\xi};\pm 1)} = \frac{1}{2}(|\uparrow\rangle \pm e^{i\xi}|\downarrow\rangle)(\langle\uparrow| \pm e^{-i\xi}\langle\downarrow|)$

Then, $\left[\hat{A}^I, \hat{B}^{II}\right] = 0$

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_s \otimes |\,\mathrm{I}\rangle_p + |\downarrow\rangle_s \otimes |\,\mathrm{II}\rangle_p\}$$

s, p represent 2-Spaces, e.g., spin & path

Measurement on each property

$$\begin{cases} \hat{A}^s(\alpha) = (+1)\cdot\hat{P}^s_{(\alpha)} + (-1)\cdot\hat{P}^s_{(\alpha+\pi)} \\ \hat{B}^p(\chi) = (+1)\cdot\hat{P}^p_{(\chi)} + (-1)\cdot\hat{P}^p_{(\chi+\pi)} \end{cases}$$

where $\hat{P}_{(\phi)} = \frac{1}{2}(|\varphi\rangle + e^{i\phi}|\overline{\varphi}\rangle)(\langle\varphi| + e^{-i\phi}\langle\overline{\varphi}|)$

Then, $\left[\hat{A}^s, \hat{B}^p\right] = 0$

(Non-)Contextuality ==>> Bell-like inequality

(In)Dependent Results for commuting Observables

Non-Locality: $\mathbf{r}_I \neq \mathbf{r}_{II}$ for $\hat{P}^{I(\mathbf{r}_I)}$ & $\hat{P}^{II(\mathbf{r}_{II})}$

Contextuality in quantum mechanics

Non-contextuality:

Independent results: $v\left(\hat{\mathrm{A}}^{\mathrm{S}}(\alpha)\cdot\hat{\mathrm{B}}^{\mathrm{P}}(\chi)\right)=v\left(\hat{\mathrm{A}}^{\mathrm{S}}(\alpha)\right)\cdot v\left(\hat{\mathrm{B}}^{\mathrm{P}}(\chi)\right)$

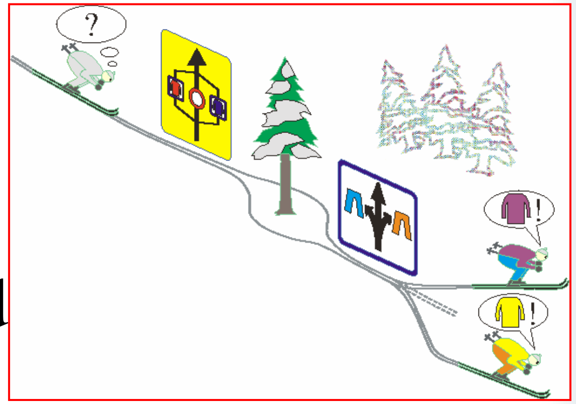
for measurements of the commuting observables, $\left[\hat{\mathrm{A}}^{\mathrm{S}}(\alpha),\hat{\mathrm{B}}^{\mathrm{P}}(\chi)\right]=0$.

➔➔Non-locality is one aspect of contextuality

$\left(\left[\hat{P}^{\mathrm{I}(\mathbf{r}_{\mathrm{I}})},\hat{P}^{\mathrm{II}(\mathbf{r}_{\mathrm{II}})}\right]=0\right.$, since $\left.\mathbf{r}_{\mathrm{I}}\neq\mathbf{r}_{\mathrm{II}}\right)$

In quantum muchanics:

$\left.\begin{matrix}\textit{Non–local}\\ \textit{Contextual}\end{matrix}\right\}$ *correlations* are expected



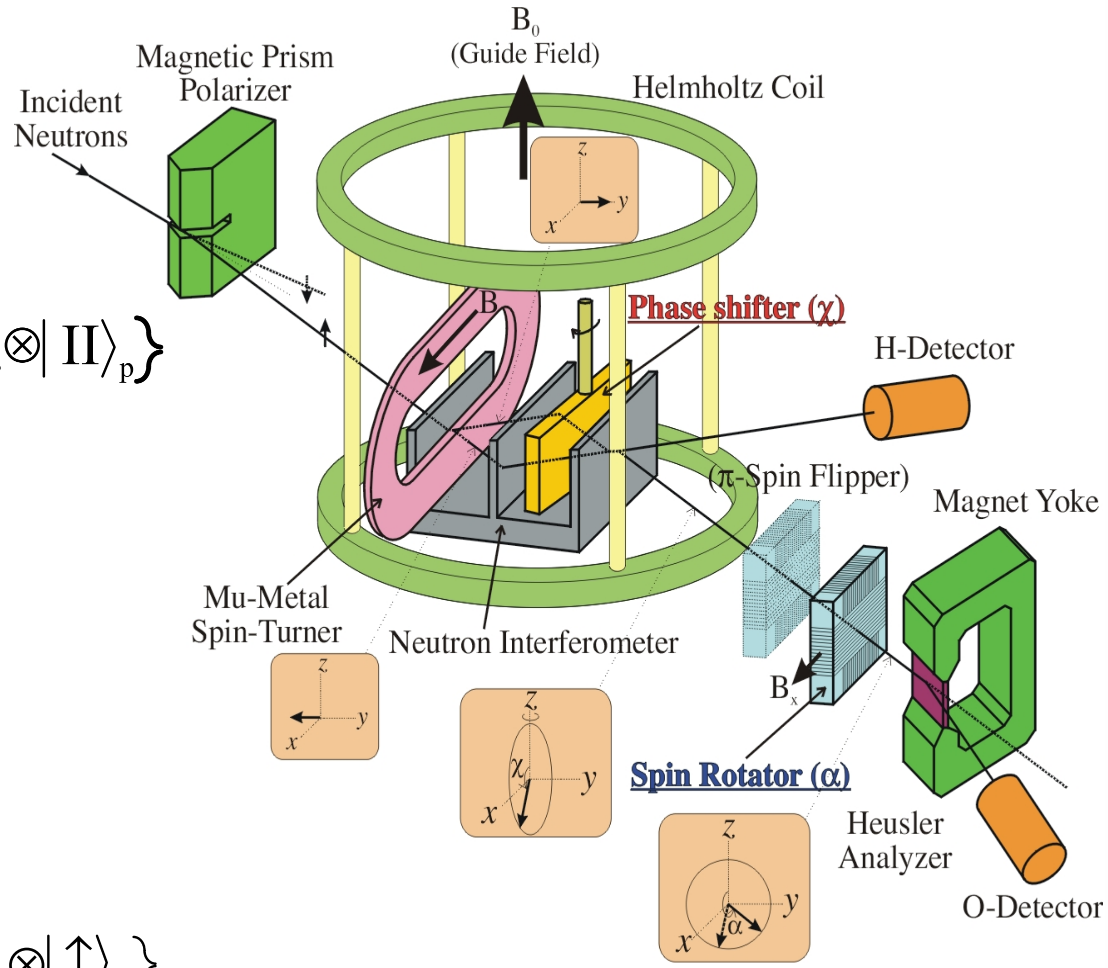
Entanglement between two-spaces

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_s \otimes |\,\mathrm{I}\rangle_p + |\downarrow\rangle_s \otimes |\,\mathrm{II}\rangle_p\}$$

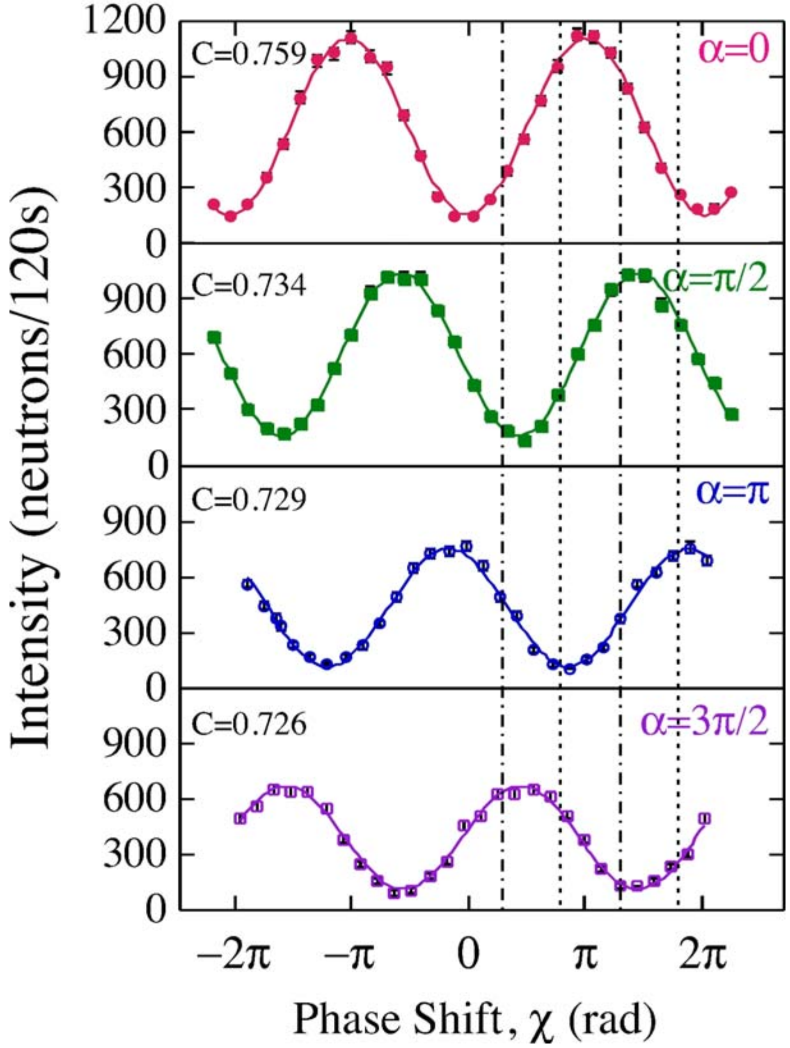
Subsystems
$H = H_1 \otimes H_2$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{\mathrm{I}} \otimes |\downarrow\rangle_{\mathrm{II}} + |\downarrow\rangle_{\mathrm{I}} \otimes |\uparrow\rangle_{\mathrm{II}}\}$$



Y. Hasegawa et al., Nature Vol. 425, Sept. 4, 2003

Violation of a Bell-like inequality



$$E'(\alpha,\chi) = \frac{N'_{++}(\alpha,\chi) + N'_{++}(\alpha+\pi,\chi+\pi) - N'_{++}(\alpha,\chi+\pi) - N'_{++}(\alpha+\pi,\chi)}{N'_{++}(\alpha,\chi) + N'_{++}(\alpha+\pi,\chi+\pi) + N'_{++}(\alpha,\chi+\pi) + N'_{++}(\alpha+\pi,\chi)}$$

where $N'_{++}(\alpha,\chi) = \langle\Psi|\hat{P}^{s}_{(\alpha)}\cdot\hat{P}^{p}_{(\chi)}|\Psi\rangle$

$$\begin{cases} E'(\alpha_1,\chi_1) = 0.542 \pm 0.007 \\ E'(\alpha_1,\chi_2) = 0.488 \pm 0.012 \\ E'(\alpha_2,\chi_1) = -0.538 \pm 0.006 \\ E'(\alpha_2,\chi_2) = 0.483 \pm 0.012 \end{cases} \quad \text{where} \begin{cases} \alpha_1 = 0 \\ \alpha_2 = 0.50\pi \\ \chi_1 = 0.79\pi \\ \chi_2 = 1.29\pi \end{cases}$$

===>>> $S' \equiv E'(\alpha_1,\chi_1) + E'(\alpha_1,\chi_2) - E'(\alpha_2,\chi_1) + E'(\alpha_2,\chi_2)$

$= \mathbf{2.051 \pm 0.019 \quad > 2}$

Cf. Max. violaion: S’=2.81>2

Y. Hasegawa et al., Nature Vol. 425, Sept. 4, 2003

Bell's inequality & Kochen-Specker theorem

Bell's inequality

$\left|S\right| \leq 2\ (classical)\quad or\quad \left|S\right| = 2\sqrt{2} \approx 2.828\ (quantum)$

where $S \equiv E(a_1, b_1) + E(a_1, b_2) - E(a_2, b_1) + E(a_2, b_2)$

Remark: statistical **violation**

Non-contextual assumption:

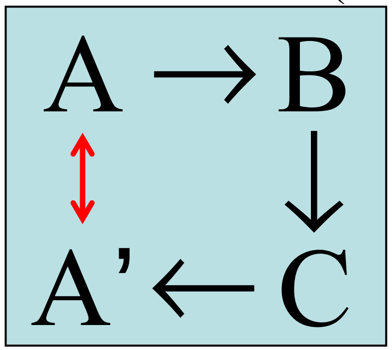
$v\left(\hat{A}\cdot\hat{B}\right) = v\left(\hat{A}\right)\cdot v\left(\hat{B}\right)$

$if\ \left[\hat{A}, \hat{B}\right] = 0$

Kochen Specker theorem

Contradiction between a hidden variable (HV) theory and the quantum theory:
namely, the HV theory assuming
(1) all observables have definite values of all time and
(2) the values of those variables are intrinsic and independent
of the measurement device, i.e., non-contextual hidden variables (NCHVs)

$$v\left(\hat{O}\right) = \pm 1\ \&\quad \begin{aligned} v\left(\hat{A}\right)\cdot v\left(\hat{B}\right) &= +1 \\ v\left(\hat{B}\right)\cdot v\left(\hat{C}\right) &= +1 \\ v\left(\hat{C}\right)\cdot v\left(\hat{A}\right) &= -1 \end{aligned}$$



Remark: no-go theorem, all vs nothing (AVN) results

Kochen-Specker experiment with neutrons

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Quantum Contextuality in a Single-Neutron Optical Experiment

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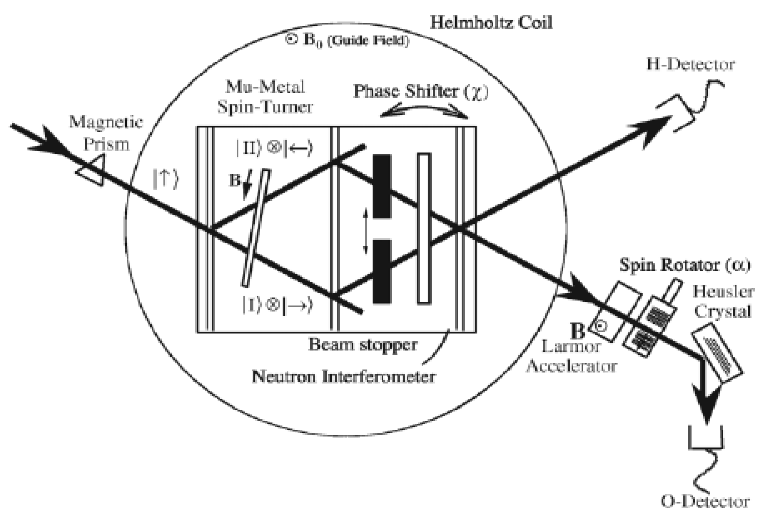
An experimental demonstration of quantum contextuality with neutrons is presented, which intended to exhibit a Kochen-Specker-like phenomenon. Since no perfect correlation is expected in practical experiments, inequalities are derived to distinguish quantitatively the obtained results from predictions by a noncontextual hidden variable theory. Experiments were accomplished with the use of a neutron interferometer combined with spinor manipulation devices. The results clearly violate the prediction of noncontextual theories.



(1) Contradiction

$$E_x \equiv \langle \hat{X}_1 \cdot \hat{X}_2 \rangle = -0.610$$

$$E_y \equiv \langle \hat{Y}_1 \cdot \hat{Y}_2 \rangle = -0.667$$

$$E_x \cdot E_y = 0.407$$

$$\xleftrightarrow{63\%} E' \equiv \langle \hat{X}_1 \hat{Y}_2 \cdot \hat{Y}_1 \hat{X}_2 \rangle = -0.861$$

(2) Violation with statistical probabilities

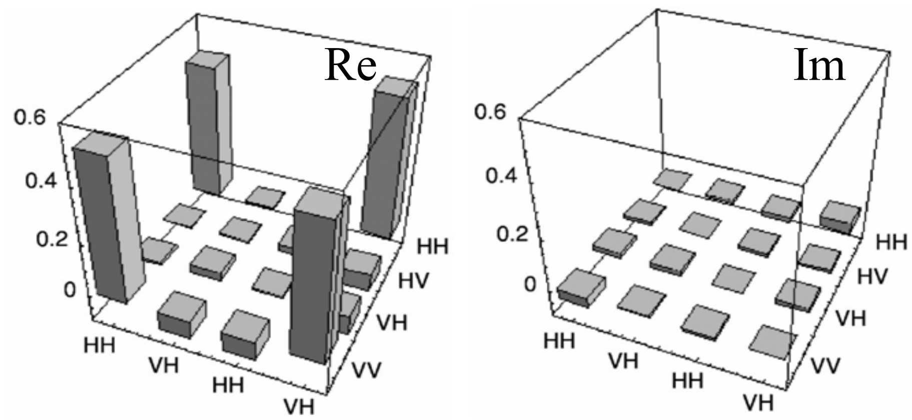
$$E' = -0.861 \geq \{1 - (p_x^+ + p_y^+)\} - (p_x^+ + p_y^+) = -E_x - E_y + 1 = 0.277$$

(3) Violation with a product observables

$$C' = 1 - E_x - E_y - E' = 3.138 \leq 2$$

Quantum state tomography of entangled 2-qubits

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle|H\rangle + |V\rangle|V\rangle\right) \rightarrow \rho = \begin{bmatrix} 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 \end{bmatrix}$$



$$\hat{\rho} = \begin{pmatrix} 0.4872 & -0.0042+i0.0114 & -0.0098-i0.0178 & 0.5192+i0.0380 \\ -0.0042-i0.0114 & 0.0045 & 0.0271-i0.0146 & -0.0648-i0.0076 \\ -0.0098+i0.0178 & 0.0271+i0.0146 & 0.0062 & -0.0695+i0.0134 \\ 0.5192-i0.0380 & -0.0648+i0.0076 & -0.0695-i0.0134 & 0.5020 \end{pmatrix}$$

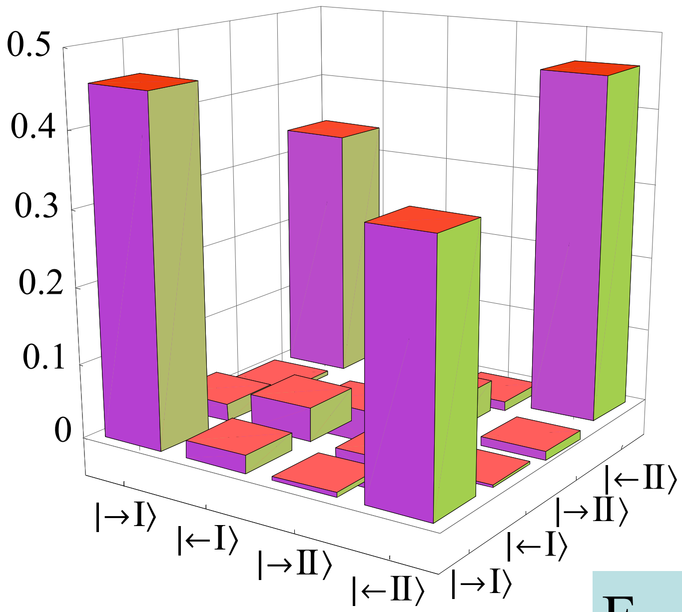
ν	Mode 1	Mode 2	h_1	q_1	h_2	q_2
1	$\lvert H\rangle$	$\lvert H\rangle$	45°	0	45°	0
2	$\lvert H\rangle$	$\lvert V\rangle$	45°	0	0	0
3	$\lvert V\rangle$	$\lvert V\rangle$	0	0	0	0
4	$\lvert V\rangle$	$\lvert H\rangle$	0	0	45°	0
5	$\lvert R\rangle$	$\lvert H\rangle$	22.5°	0	45°	0
6	$\lvert R\rangle$	$\lvert V\rangle$	22.5°	0	0	0
7	$\lvert D\rangle$	$\lvert V\rangle$	22.5°	45°	0	0
8	$\lvert D\rangle$	$\lvert H\rangle$	22.5°	45°	45°	0
9	$\lvert D\rangle$	$\lvert R\rangle$	22.5°	45°	22.5°	0
10	$\lvert D\rangle$	$\lvert D\rangle$	22.5°	45°	22.5°	45°
11	$\lvert R\rangle$	$\lvert D\rangle$	22.5°	0	22.5°	45°
12	$\lvert H\rangle$	$\lvert D\rangle$	45°	0	22.5°	45°
13	$\lvert V\rangle$	$\lvert D\rangle$	0	0	22.5°	45°
14	$\lvert V\rangle$	$\lvert L\rangle$	0	0	22.5°	90°
15	$\lvert H\rangle$	$\lvert L\rangle$	45°	0	22.5°	90°
16	$\lvert R\rangle$	$\lvert L\rangle$	22.5°	0	22.5°	90°

D.F. James et al., Phys. Rev. A **64** (2001) 052312.

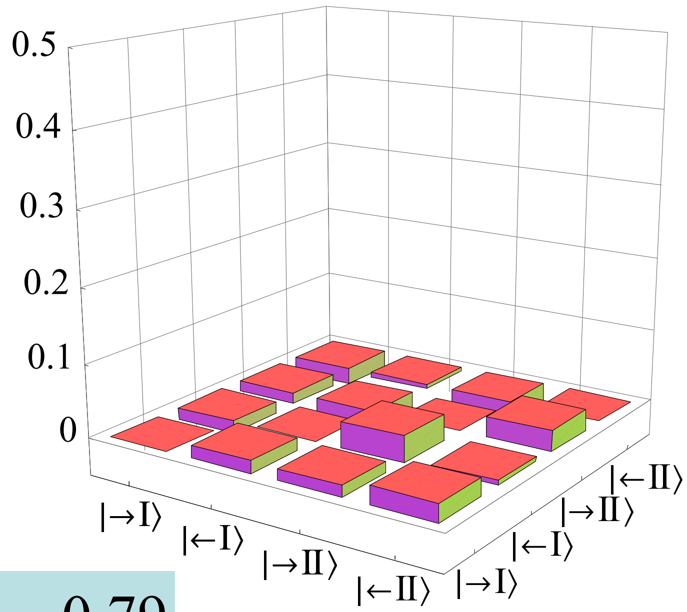
Quantum state tomography of neutron's Bell-state

$$|\Psi_1\rangle = |\rightarrow\rangle|\mathrm{I}\rangle + |\leftarrow\rangle|\mathrm{II}\rangle$$

real part

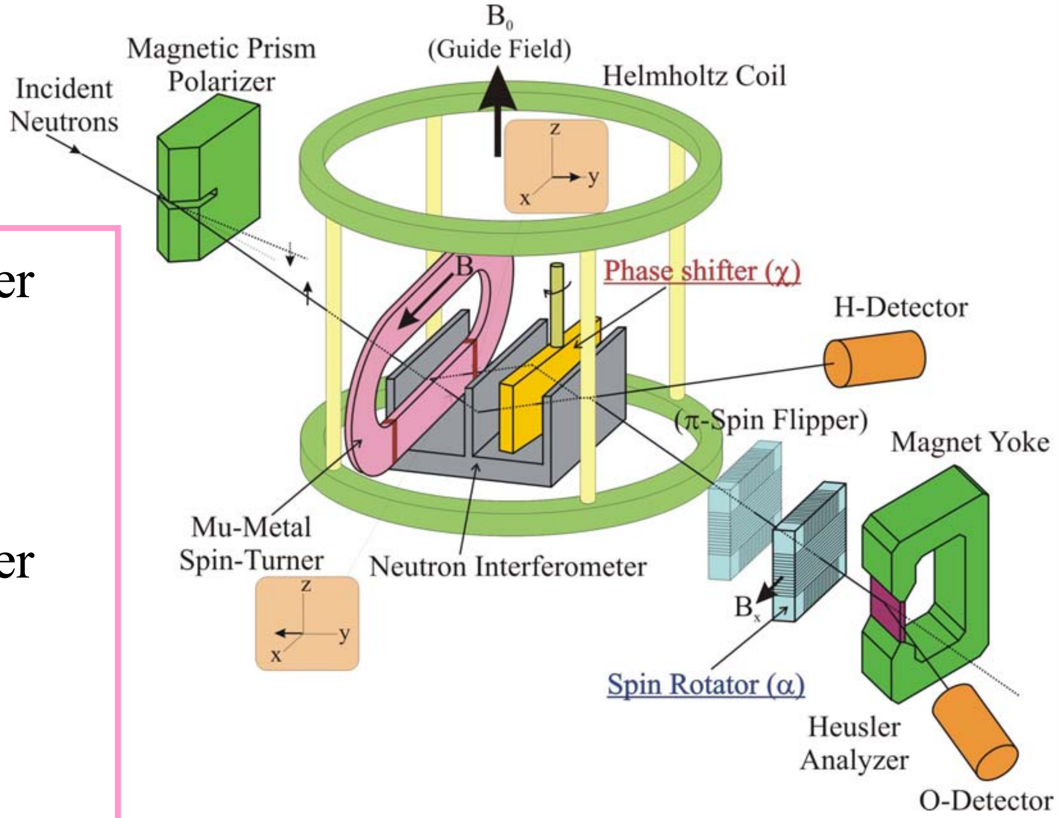


imaginary part



$$F = \langle\Psi|\rho|\Psi\rangle = 0.79$$

Schematic view of the experiment



◊ Incident $|\uparrow\rangle$, with spin-turner

$$|\Psi_1\rangle = |\rightarrow\rangle|\mathrm{I}\rangle + |\leftarrow\rangle|\mathrm{II}\rangle$$

◊ Incident $|\downarrow\rangle$, with spin-turner

$$|\Psi_2\rangle = |\leftarrow\rangle|\mathrm{I}\rangle + |\rightarrow\rangle|\mathrm{II}\rangle$$

◊ Incident $|\uparrow\rangle$, without spin-turner

$$|\Psi_0\rangle = |\uparrow\rangle|\mathrm{I}\rangle + |\uparrow\rangle|\mathrm{II}\rangle$$

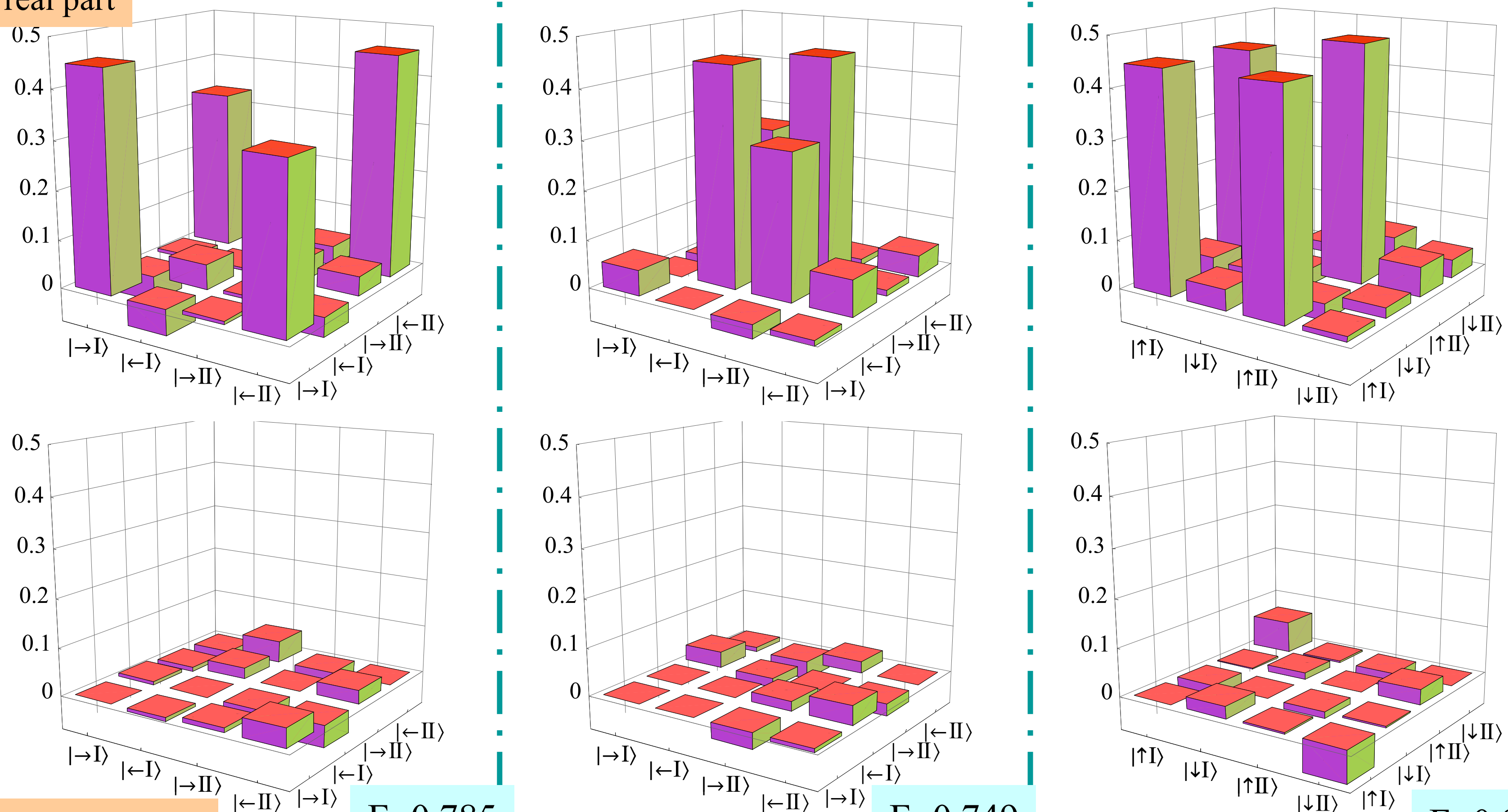
Quantum state tomography --- results

$|\Psi_1\rangle = |\rightarrow\rangle|I\rangle + |\leftarrow\rangle|II\rangle$

$|\Psi_2\rangle = |\leftarrow\rangle|I\rangle + |\rightarrow\rangle|II\rangle$

$|\Psi_0\rangle = |\uparrow\rangle|I\rangle + |\uparrow\rangle|II\rangle$

real part



imaginary part

F=0.785

F=0.749

F=0.908

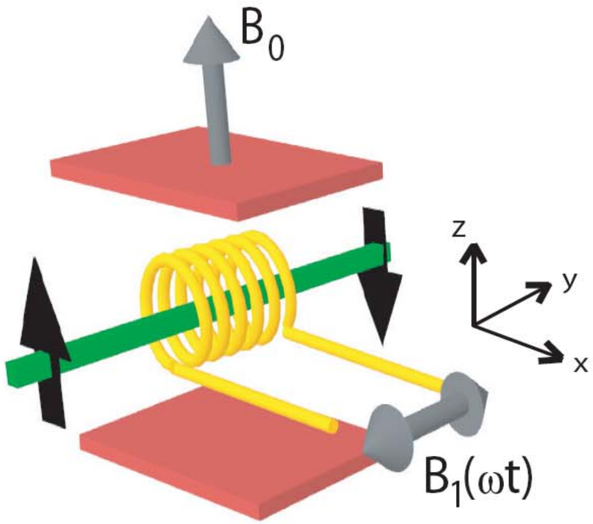
Multi-entanglement in single-particle

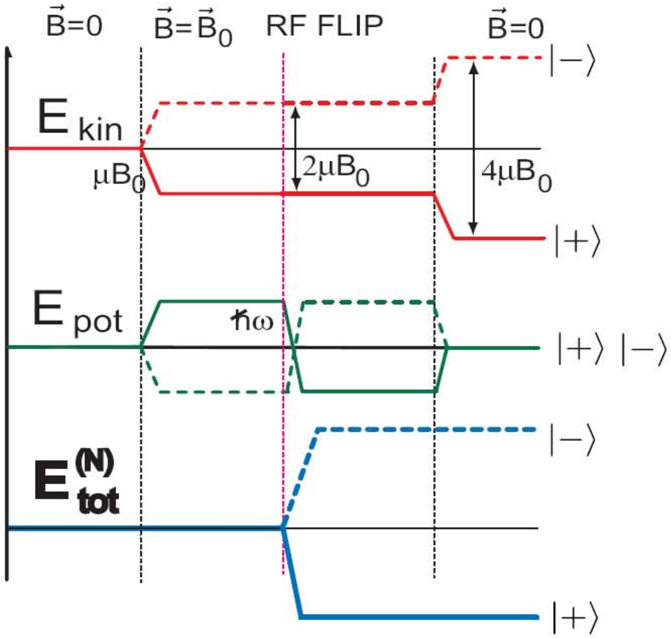
Muti-entanglement in neutrons

- ☆ bi-entanglement: spin-path
- ☆ tri-entanglementl: spin-path-energy
- ☆ multi-entanglement: energy-levels

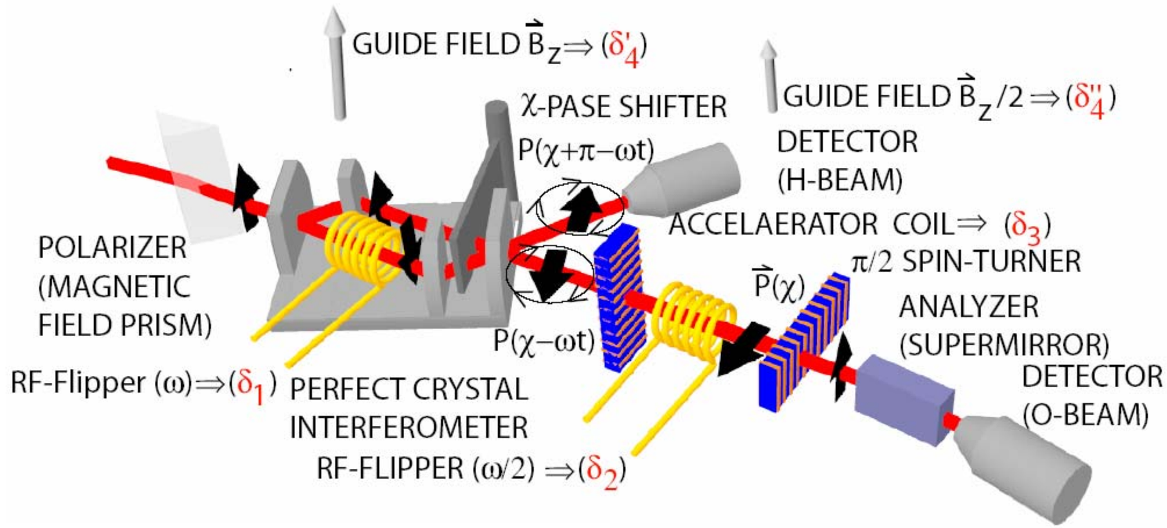
Radio-Frequency (RF) Spin-Flipper: energy transfer

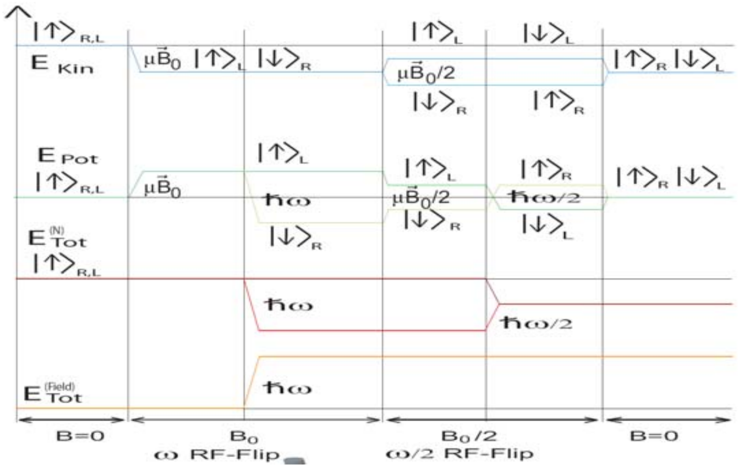
- Field Configuration:Static and time dependent Field:
  - $\vec{B}(\vec{r},t) = \vec{B}_1(t) + \vec{B}_0 = (B_1 \cos(\omega t), B_1 \sin(\omega t), B_0)$
- Rotating Coordinate Frame: $\vec{B}_{\text{eff}} = (B_1, 0, B_0 + \omega/\gamma)$
  - Frequency Resonace: $\omega = -\gamma B_0$
  - Amplitude Resonace: $\omega_L t = -\gamma B_1 t = \pi$

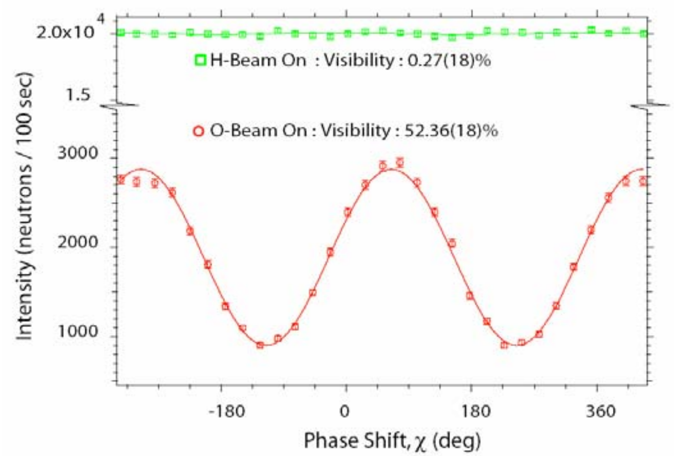




Stationary interference-pattern: energy compensation

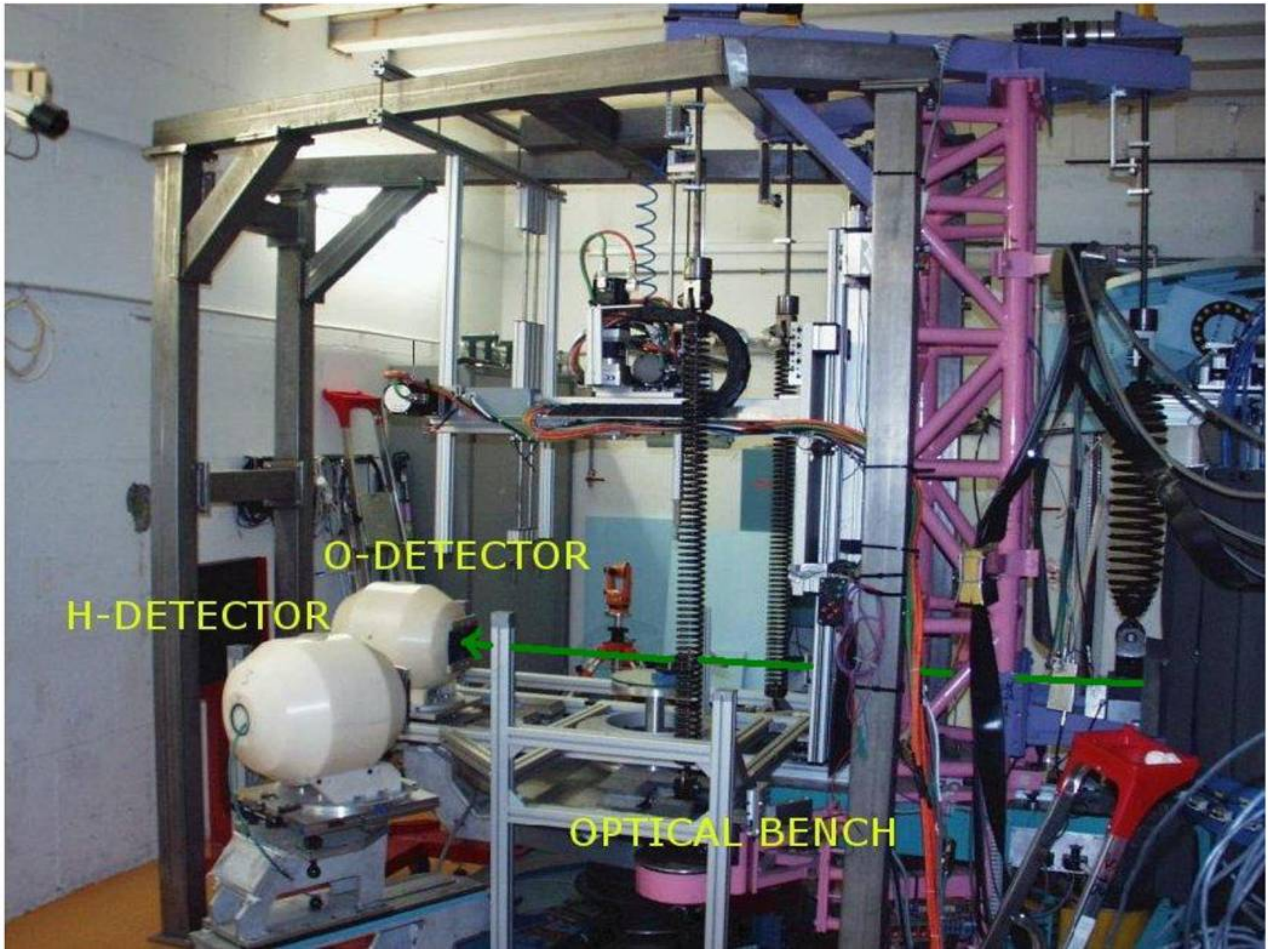




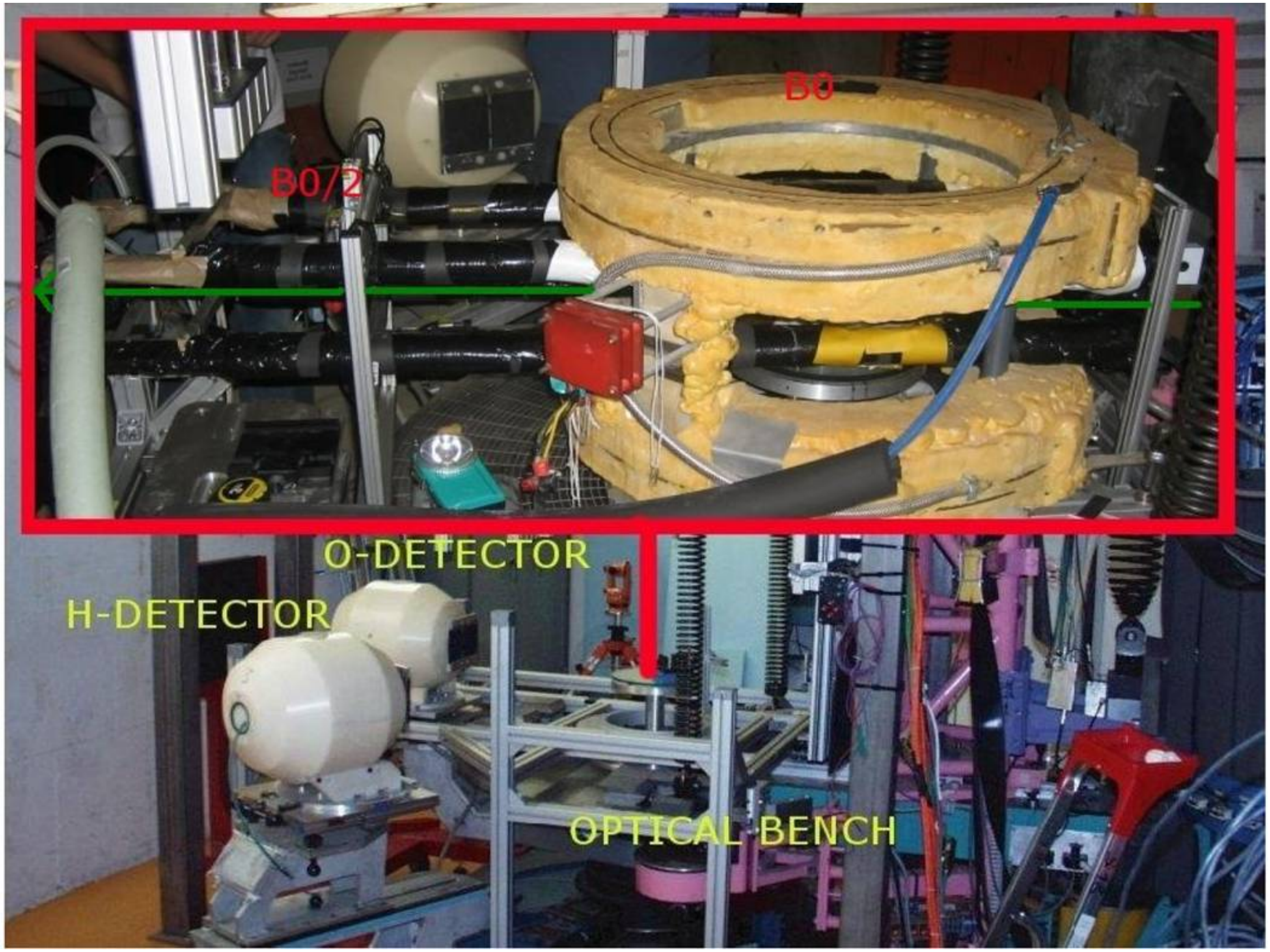


S. Sponar et al.

Experimental setup (1)



Experimental setup (2)

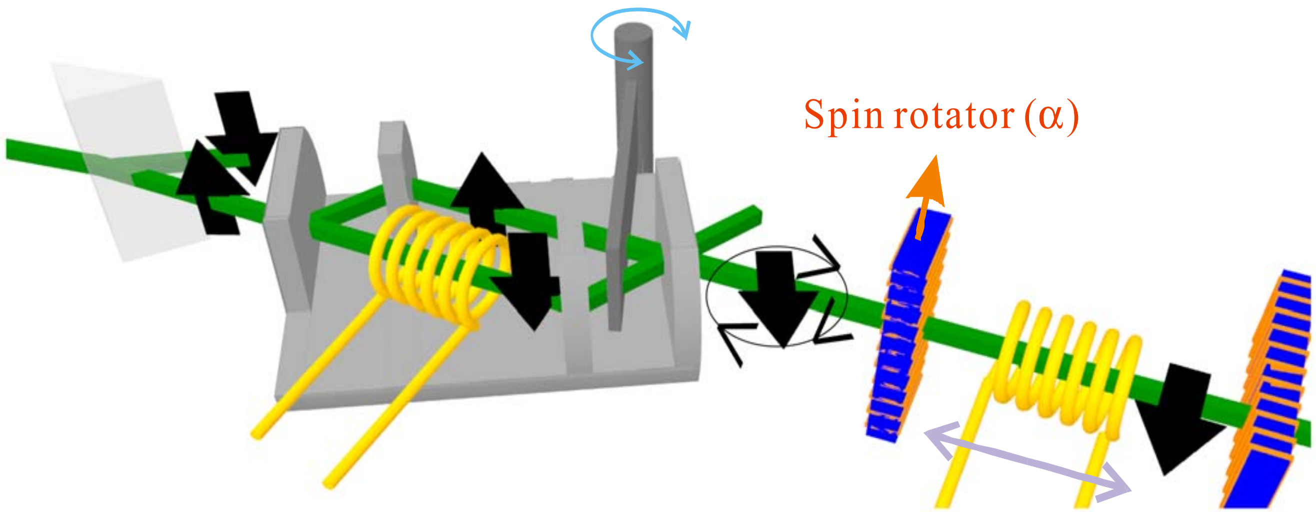


Experimental setup (3)

Experimental setup (4)

Experimental setup (5)

Multi entanglement (GHZ state)



$$\left|\Psi_{\text{Neutron}}\right\rangle = \left\{ \left|\Psi_{\text{I}}\right\rangle \otimes \left|\uparrow\right\rangle \otimes \left|\Psi(E_0)\right\rangle + \left(e^{i\chi}\left|\Psi_{\text{II}}\right\rangle\right) \otimes \left(e^{i\alpha}\left|\downarrow\right\rangle\right) \otimes \left(e^{i\gamma}\left|\Psi(E_0+\hbar\omega_r)\right\rangle\right) \right\}$$

where

$e^{i\chi} = N\, b_c\, \lambda\, D$: *phase shifter*

$e^{i\alpha} = \Delta\omega_L\, t$: *spin rotator*

$e^{i\gamma} = \omega_r\, t$: *zero field precession*

Mermin’s inequality for GHZ state

Neutron's GHZ-state

$$\left|\Psi_{GHZ}\right\rangle = \left\{\left|\Psi_{\mathrm{I}}\right\rangle \otimes \left|\uparrow\right\rangle \otimes \left|\Psi(E_0)\right\rangle + \left|\Psi_{\mathrm{II}}\right\rangle \otimes \left|\downarrow\right\rangle \otimes \left|\Psi(E_0+\hbar\omega_r)\right\rangle\right\}$$

Relative phases are manipulated:

$$\left|\Psi_{Neutron}\right\rangle = \left\{\left|\Psi_{\mathrm{I}}\right\rangle \otimes \left|\uparrow\right\rangle \otimes \left|\Psi(E_0)\right\rangle + \left(e^{i\chi}\left|\Psi_{\mathrm{II}}\right\rangle\right) \otimes \left(e^{i\alpha}\left|\downarrow\right\rangle\right) \otimes \left(e^{i\gamma}\left|\Psi(E_0+\hbar\omega_r)\right\rangle\right)\right\}$$

where

$$e^{i\chi} = N\, b_c\, \lambda\, D : \textit{phase shifter}$$

$$e^{i\alpha} = \Delta\omega_L\, t \quad : \textit{spin rotator}$$

$$e^{i\gamma} = \omega_r\, t \quad : \textit{zero field precession}$$

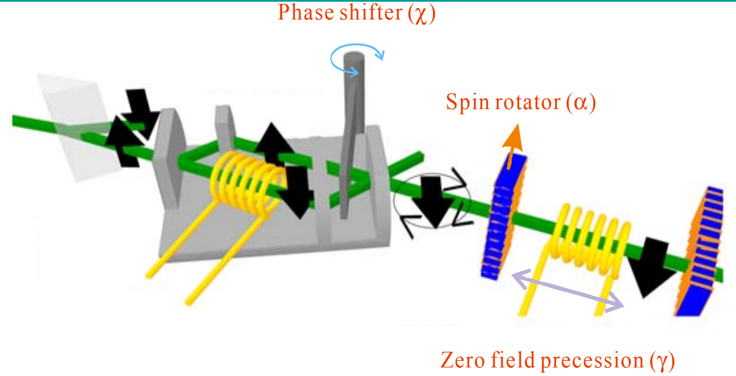
Mermin's inequality for GHZ-state

$\left|M_{NC}\right| \leq 2$ according to *non-contextual theory*

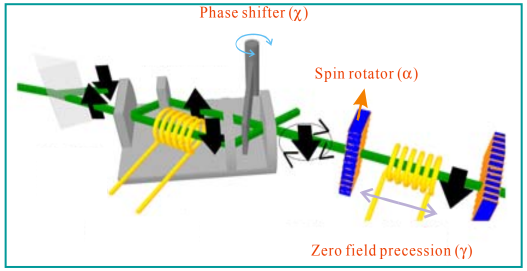
where $M \equiv E\ \sigma_x^p \cdot \sigma_x^s \cdot \sigma_x^e \ - E\ \sigma_x^p \cdot \sigma_y^s \cdot \sigma_y^e \ - E\ \sigma_y^p \cdot \sigma_x^s \cdot \sigma_y^e \ - E\ \sigma_y^p \cdot \sigma_y^s \cdot \sigma_x^e$

In contrast quantum theory predicts

$M_{Quantum} = 4 \ \ for \left|\Psi_{GHZ}\right\rangle$



Mermin's inequality: result 1



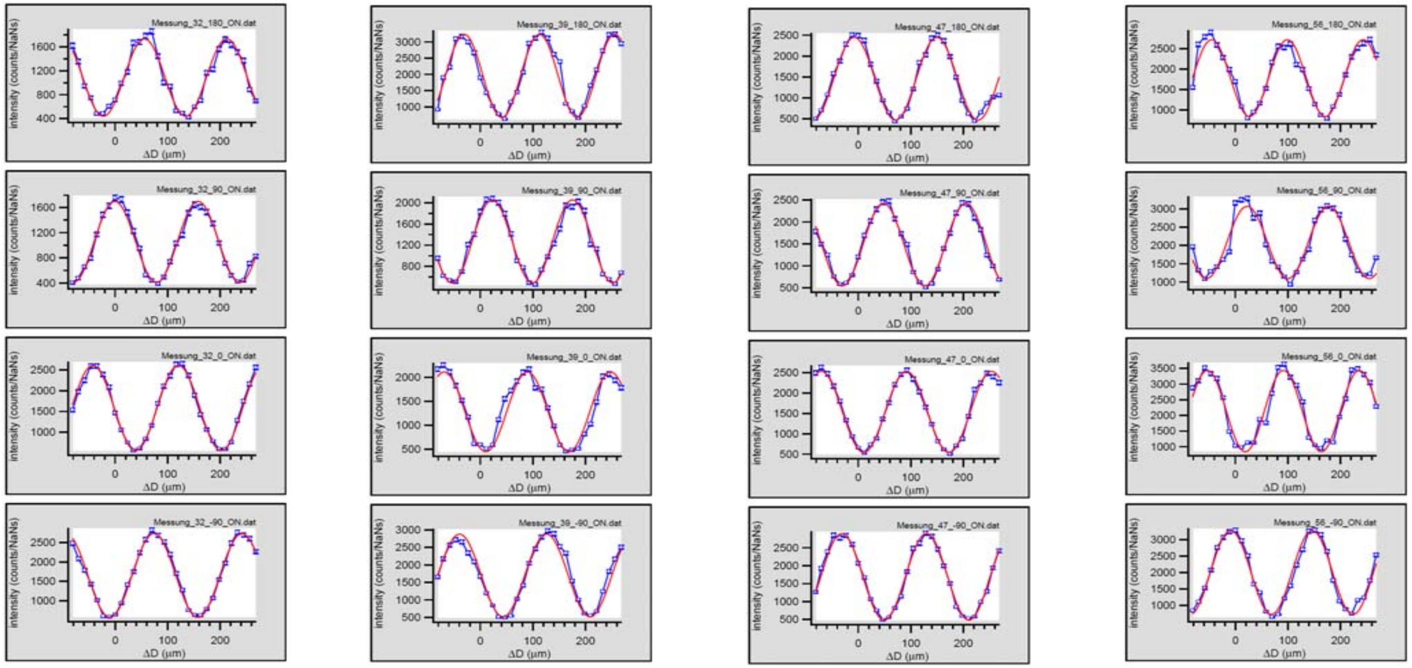
$$|\Psi_{\mathrm{Neutron}}\rangle = \{ |\Psi_{\mathrm{I}}\rangle \otimes |\uparrow\rangle \otimes |\Psi(E_0)\rangle + \left(e^{i\chi}|\Psi_{\mathrm{II}}\rangle\right) \otimes \left(e^{i\alpha}|\downarrow\rangle\right) \otimes \left(e^{i\gamma}|\Psi(E_0+\hbar\omega_r)\rangle\right) \}$$

where

$e^{i\chi} = N b_c \lambda D$: *phase shifter*

$e^{i\alpha} = \Delta\omega_L t$: *spin rotator*

$e^{i\gamma} = \omega_r t$: *zero field precession*



α=0

α=π/2

α=π

α=3π/2

γ=0 γ=π/2 γ=π γ=3π/2

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Mermin's inequality: result 2

Mermin's inequality for tri-GHZ-state

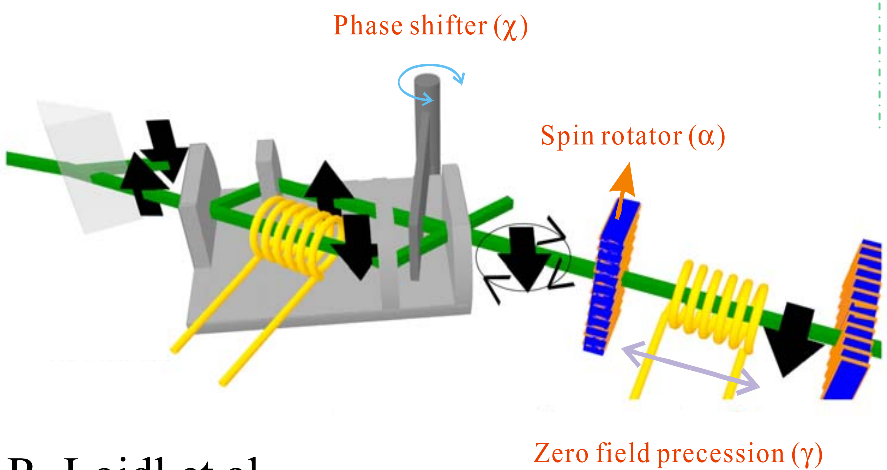
$|M_{NC}| \leq 2$ according to *non – contextual theory*

where $M \equiv E \;\; \sigma_x^p \cdot \sigma_x^s \cdot \sigma_x^e \;\; - E \;\; \sigma_x^p \cdot \sigma_y^s \cdot \sigma_y^e$

$$- E \;\; \sigma_y^p \cdot \sigma_x^s \cdot \sigma_y^e \;\; - E \;\; \sigma_y^p \cdot \sigma_y^s \cdot \sigma_x^e$$

In contrast quantum theory predicts

$M_{Quantum} = 4 \;\; for \, |\Psi_{GHZ}\rangle$



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We obtained the values:

$$E \;\; \sigma_x^p \cdot \sigma_x^s \cdot \sigma_x^e \;\; = \;\; 0.652$$

$$E \;\; \sigma_x^p \cdot \sigma_y^s \cdot \sigma_y^e \;\; = -0.663$$

$$E \;\; \sigma_y^p \cdot \sigma_x^s \cdot \sigma_y^e \;\; = -0.642$$

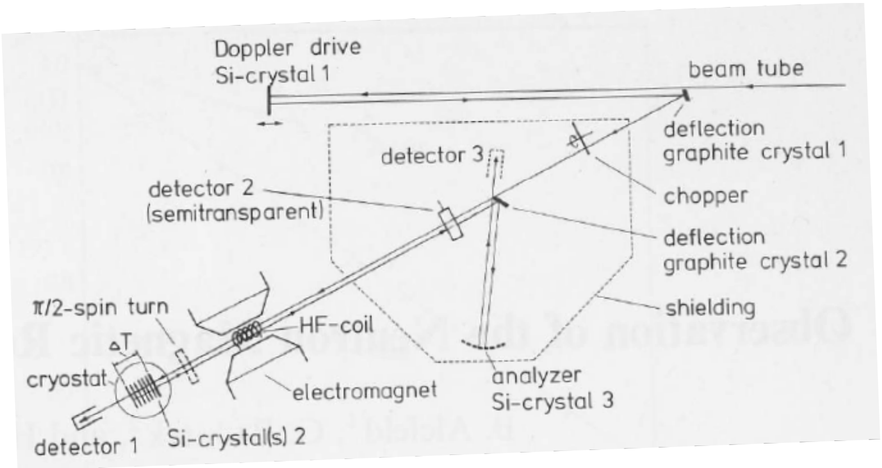
$$E \;\; \sigma_y^p \cdot \sigma_y^s \cdot \sigma_x^e \;\; = -0.664$$

Finally,

$$M_{Measured} = 2.62 \pm 0.08 \quad > 2$$

Preliminary!

Multi entanglement: discussions1



Energy shifts:

$$E_{shift}(f = 57\,\text{MHz}) = 0.47\,\mu\text{eV}$$

$$\Updownarrow$$

$$E_{shift}(f = 58\,\text{kHz}) = 0.48\,\text{neV}$$

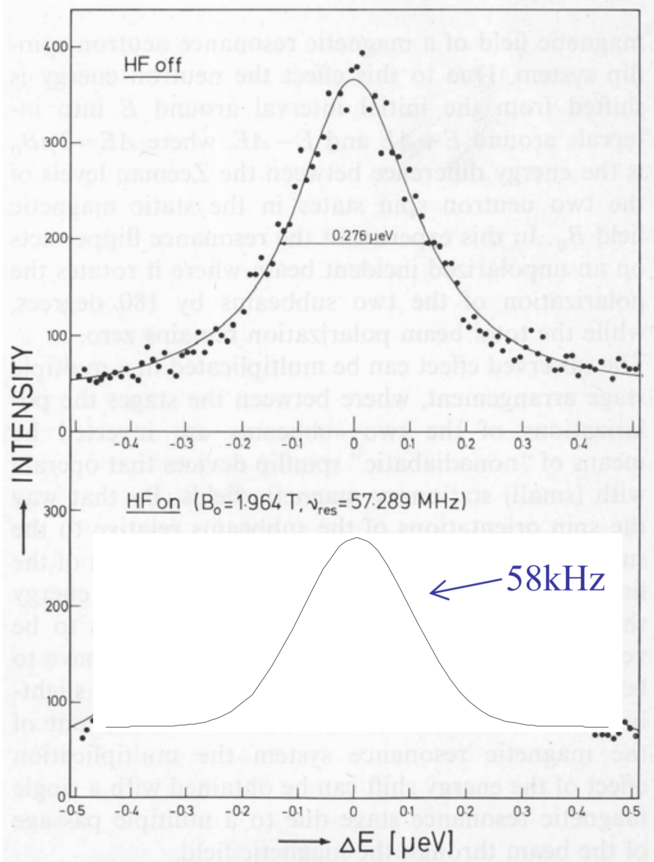


Fig. 4. HF-induced change of energy distribution of backscattered intensity. Data collection time was about 10 hours. The solid lines correspond to least square fitted Lorentzians. The observed energy splitting of 0.482±0.015 μeV is in excellent agreement with the theoretically expected value of 0.474 μeV

B.Alefeld et al. Z. Phys B41 (1981) 231.

Multi entanglement: discussions2

Multi degrees-of-freedoms entanglement

$$|\Psi_{\text{Neutron}}\rangle = |\Psi_{\text{Path}}\rangle \otimes |\Psi_{\text{Spin}}\rangle \otimes |\Psi_{\text{Energy}}\rangle$$

Bell's inequality

$$|\Psi_{\text{Path}}\rangle = \{|\Psi_{\text{I}}\rangle, |\Psi_{\text{II}}\rangle\}$$

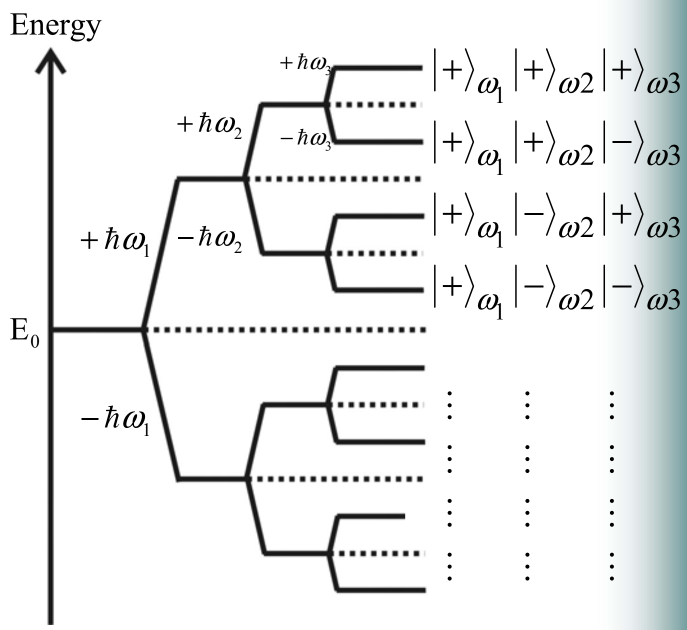
where $|\Psi_{\text{Spin}}\rangle = \{|\uparrow\rangle, |\downarrow\rangle\}$

$$|\Psi_{\text{Energy}}\rangle = \{|\Psi(E_0)\rangle, |\Psi(E_0 + \hbar\omega_r)\rangle\}$$

Multi energy-level entanglement

$$|\Psi\rangle = |\Psi_{\text{Energy}}\rangle_{\omega_1} \otimes |\Psi_{\text{Energy}}\rangle_{\omega_2} \otimes \dots \otimes |\Psi_{\text{Energy}}\rangle_{\omega_n}$$

where $|\Psi_{\text{Energy}}\rangle_{\omega_1} = \{|\Psi(E_0 - \hbar\omega)\rangle, |\Psi(E_0 + \hbar\omega)\rangle\}$



Investigations with neutrons: entanglement

Muti-entanglement in neutrons

- ☆ bi-entanglement: spin-path
- ☆ tri-entanglement: spin-path-energy
- ☆ multi-entanglement: energy-levels

Co-workers: S.Sponar, J.Klepp, R.Loidl, S.Filipp
H. Rauch

Fin !