International Iranian Conference on Quantum Information, 7-10 Sept 2014, Esfahan, Iran

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10 things you always wanted to know about non-Markovian open quantum systems

1

What do you **exactly** mean with **non-Markovian** open quantum systems?

$$\rho(t) = \Lambda_t \rho(0)$$

dynamical map

$$\dot{\rho}(t) = L_t \rho(t)$$

master equation

$$\Lambda_t = \mathrm{T}\exp\left(\int_0^t L_\tau \, d\tau\right)$$

completely positive (CP) and trace preserving

$$\rho(t) = \Lambda_t \rho(0)$$

Φ is **completely positive (CP)** if

$\mathbb{1}_k \otimes \Phi$ is positive for all $k = 1, 2, \ldots$

$$\Lambda_t = V_{t,s}\Lambda_s$$

propagator

CP CP

$$\Lambda_t = V_{t,s}\Lambda_s$$

CP?


YES
NO
MAYBE

$$\Lambda_t = V_{t,s} \Lambda_s$$

CP

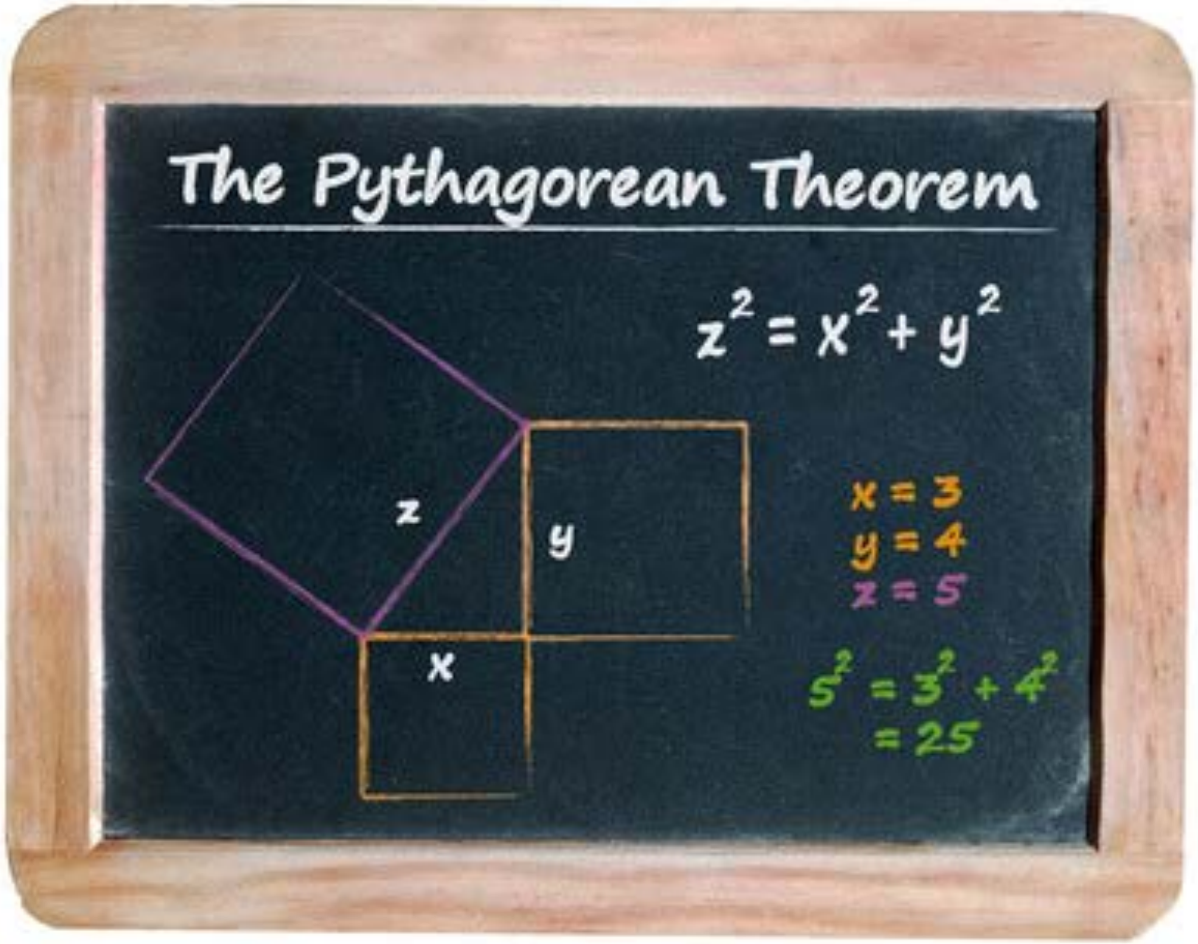
The dynamical map is (CP) divisible

$$\Lambda_t = V_{t,s} \Lambda_s$$

not CP

The dynamical map is (CP) non-divisible

The "Pythagorean theorem" of Open Quantum Systems theory



Λ_t is divisible if and only if L_t can be written in the Lindblad form

$$L_t(\rho) = -i[H(t), \rho] + \sum_k \left(A_k(t)\rho A_k^\dagger(t) - \frac{1}{2}\{A_k^\dagger(t)A_k(t), \rho\} \right)$$

$$\dot{\rho}(t) = L_t \rho(t)$$

V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, J. Math. Phys. 17, 821 (1976)

G. Lindblad, Comm. Math. Phys. 48, 119 (1976)

$$L_t(\rho) = -i[H(t), \rho] + \sum_k \left(A_k(t) \rho A_k^\dagger(t) - \frac{1}{2}\{A_k^\dagger(t) A_k(t), \rho\} \right)$$

$$L_t(\rho) = -i[H, \rho] + \sum_k \gamma_k(t) \left(A_k \rho A_k^\dagger - \frac{1}{2}\{A_k^\dagger A_k, \rho\} \right)$$

time-dependent decay rates

$$L_t(\rho) = -i[H,\rho] + \sum_k \gamma_k(t) \left(A_k \rho A_k^\dagger - \frac{1}{2}\{A_k^\dagger A_k, \rho\} \right)$$

$\gamma_k(t) \geq 0 \iff \Lambda_t$ **divisible**

MARKOVIAN

$$L_t(\rho) = -i[H,\rho] + \sum_k \gamma_k(t)\left(A_k \rho A_k^\dagger - \frac{1}{2}\{A_k^\dagger A_k, \rho\}\right)$$

$$\gamma_k(t) < 0 \iff \Lambda_t \text{ nondivisible}$$

NON-MARKOVIAN

Are the quantum and classical definitions of Markovian dynamics different?


Yes!

Classical definition of Markovian stochastic process

$$\{x_i\}_{i\in\mathbb{N}}$$

$$p_{1|n}(x_n, t_n | x_{n-1}, t_{n-1}; ...; x_0, t_0) = p_{1|1}(x_n, t_n | x_{n-1}, t_{n-1})$$

$$t_n \geq t_{n-1} \geq ... \geq t_1 \geq t_0$$

n-time probabilities

Classical definition of Markovian stochastic process

$$\{x_i\}_{i\in\mathbb{N}}$$

$$p_{1|n}(x_n,t_n|x_{n-1},t_{n-1};\ldots;x_0,t_0)=p_{1|1}(x_n,t_n|x_{n-1},t_{n-1})$$

$$t_n \geq t_{n-1} \geq \ldots \geq t_1 \geq t_0$$

quantum conditional probabilities

MARKOVIAN

$$p_{1|n}(x_n, t_n | x_{n-1}, t_{n-1}; ...; x_0, t_0) = p_{1|1}(x_n, t_n | x_{n-1}, t_{n-1})$$

Chapman-Kolmogorov equation

$$p_{1|1}(x, t | y, s) = \sum_z p_{1|1}(x, t | z, \tau) p_{1|1}(z, \tau | y, s)$$

$$\mathbf{p}(t) = \Lambda(t, s)\mathbf{p}(s)$$

stochastic matrix

Is there a unique definition of non-Markovian open quantum systems

Non-Markovianity *definitions*

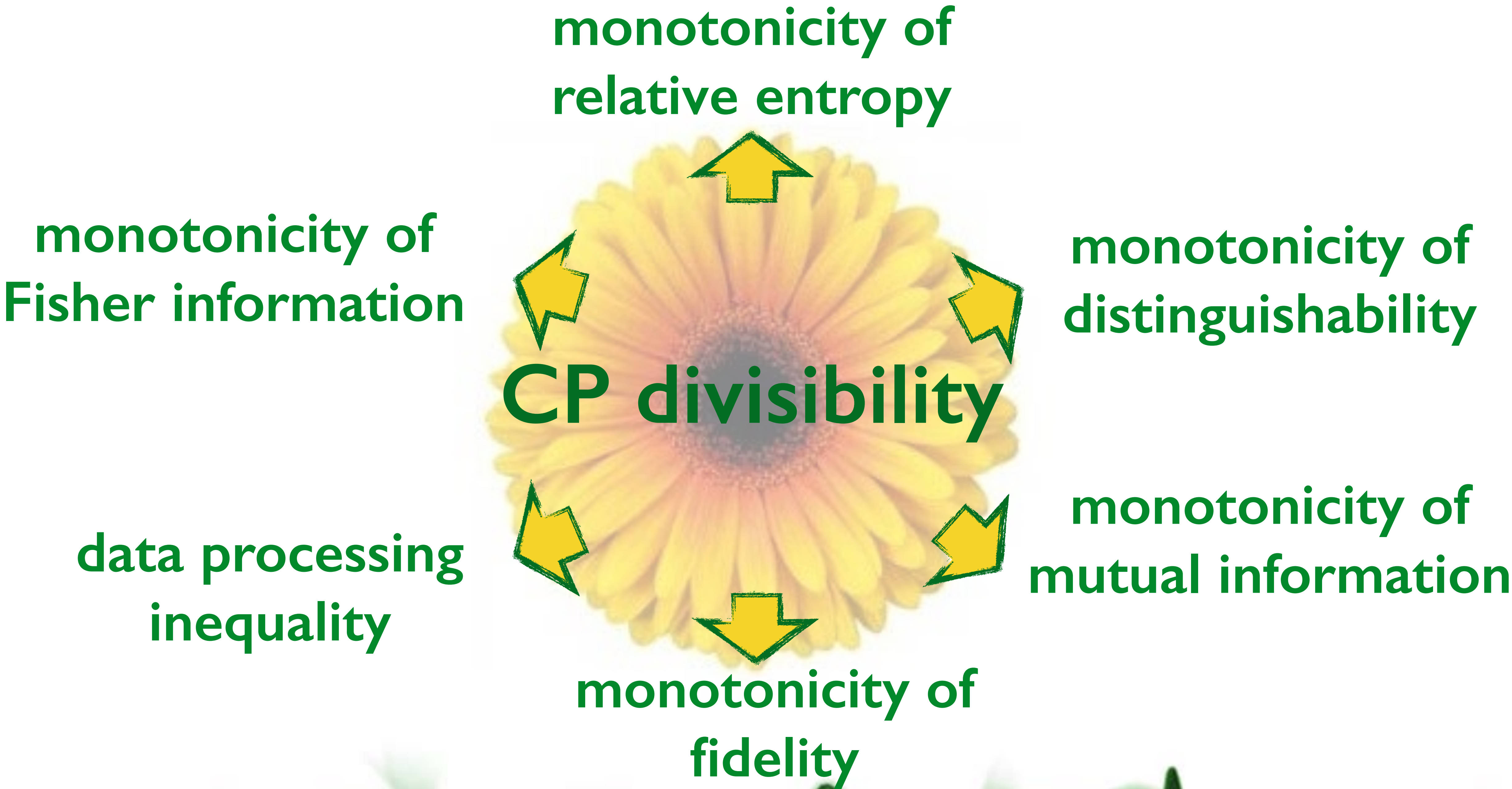
and corresponding measures

(this list is most likely to be incomplete)

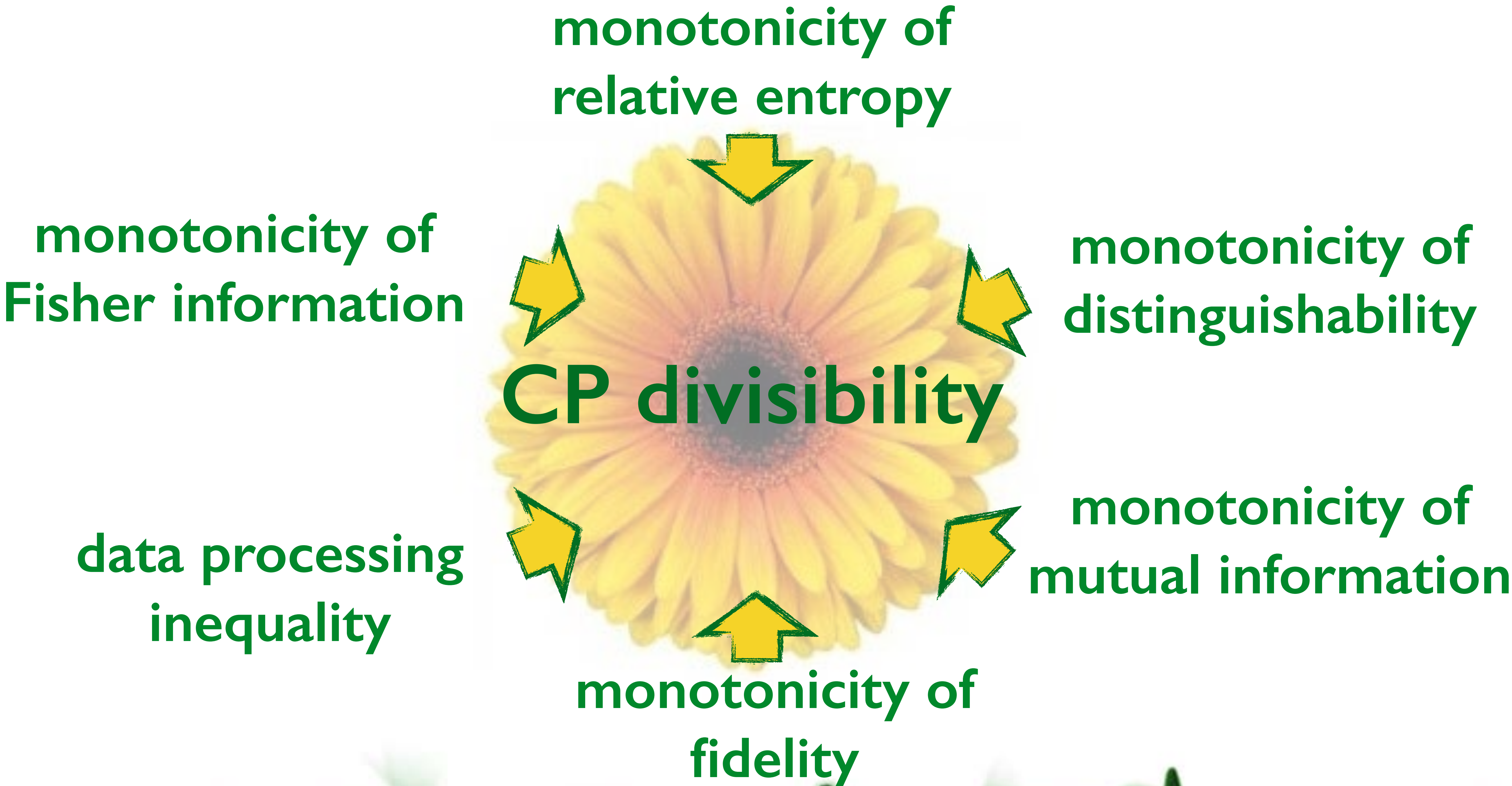
1. H.-P. Breuer, E.-M. Laine, J. Piilo, Phys. Rev. Lett., 210401 (2009)
2. A. Rivas, S.F. Huelga, and M.B. Plenio, Phys. Rev. Lett. 105, 050403 (2010)
3. Xiao-Ming Lu, Xiaoguang Wang, and C. P. Sun Phys. Rev. A 82 042103 (2010)
4. S. C. Hou, X. X. Yi, S. X. Yu, and C. H. Oh, Phys. Rev. A 83 062115 (2011)
5. R. Vasile, S. Maniscalco, M. G. A. Paris, H.-P. Breuer, and J. Piilo Phys. Rev. A 84 052118 (2011)
6. L. Mazzola, C. A. Rodríguez-Rosario, K. Modi, and M. Paternostro Phys. Rev. A 86, 010102(R) (2012)
7. S. Luo, S. Fu, and H. Song, Phys. Rev. A 86 044101 (2012)
8. S. Lorenzo, F. Plastina, and M. Paternostro, Phys. Rev. A 88 020102 (2013)
9. F. F. Fanchini, G. Karpat, B. Çakmak, L. K. Castelano, G. H. Aguilar, O. Jiménez Farías, S. P. Walborn, P. H. Souto Ribeiro, and M. C. de Oliveira Phys. Rev. Lett. 112, 210402 (2014)
10. B. Bylicka, D. Chruściński and S. Maniscalco, Scientific Reports 4, 5720 (2014).
11. S. Haseli, S. Salimi, arXiv:1406.0180
12. N. Lo Gullo, I. Sinayskiy, Th. Busch, F. Petruccione, arXiv:1401.1126.
13. F. Buscemi and N. Datta, 1408.7062v1.

4 No… Really?

A common feature to all the measures



But the inverse is NOT true !

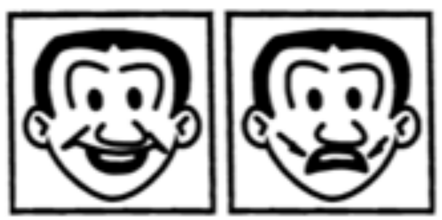


BLP Measure and information flow

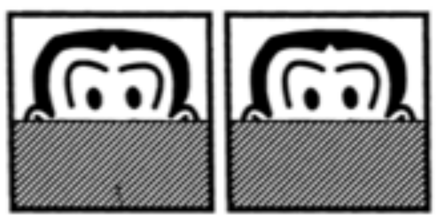
$$D(\rho_t^1, \rho_t^2) = \frac{1}{2}\mathrm{Tr}||\rho_t^1 - \rho_t^2||,$$

→ **distinguishability**

initial pair of states



information loss



states at time t

H.-P. Breuer, E.-M. Laine, J. Piilo, Phys. Rev. Lett., 210401 (2009)

BLP Measure and information flow

$$D(\rho_t^1, \rho_t^2) = \frac{1}{2}\mathrm{Tr}||\rho_t^1 - \rho_t^2||,$$

$$D(\Lambda_t\rho^1, \Lambda_t\rho^2) \leq D(\rho^1, \rho^2)$$

information flow $\sigma(t, \rho^1, \rho^2) = \frac{d}{dt}D(\Lambda_t\rho^1, \Lambda_t\rho^2)$

$\sigma(t, \rho^1, \rho^2) \leq 0$

information loss

$\sigma(t, \rho^1, \rho^2) > 0$

information backflow

H.-P. Breuer, E.-M. Laine, J. Piilo, Phys. Rev. Lett., 210401 (2009)

BLP Measure and information flow

information flow $\sigma(t,\rho^1,\rho^2) = \frac{d}{dt} D(\Lambda_t \rho^1, \Lambda_t \rho^2)$

$$\mathcal{N}_{BLP}(\Lambda_t) = \max_{\rho^1,\rho^2} \int_{\sigma>0} dt \sigma(t,\rho^1,\rho^2)$$

H.-P. Breuer, E.-M. Laine, J. Piilo, Phys. Rev. Lett., 210401 (2009)

information flow $\sigma(t, \rho^1, \rho^2) = \frac{d}{dt} D(\Lambda_t \rho^1, \Lambda_t \rho^2)$

CP-divisibility → $\sigma(t, \rho^1, \rho^2) \leq 0$

Markovianity

information loss

$\sigma(t, \rho^1, \rho^2) > 0$ → **non-divisibility**

information backflow

non-Markovianity

5 Is there a hierarchy among the different non-Markovianity measures?


YES
NO
MAYBE

Analogy between

Entanglement theory

&

Open system dynamics

↓

Degree of non-Markovianity

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

Schmidt rank $\mathrm{SR}(\psi)$

number of non vanishing Schmidt coefficients

$$\psi \in \mathcal{H} \otimes \mathcal{H} \qquad \psi = \sum_k s_k e_k \otimes f_k$$

Schmidt number

$$\mathrm{SN}(\rho) = \min_{p_k, \psi_k} \{\max_k \mathrm{SR}(\psi_k)\}$$

$$\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k|$$

$$\mathrm{SN}(\rho) = \min_{p_k, \psi_k} \{\max_k \mathrm{SR}(\psi_k)\}$$

set of states $S_k = \{\, \rho \,|\, \mathrm{SN}(\rho) \leq k \,\}$

$$S_1 \subset S_2 \subset \ldots \subset S_n$$

separable states (S_1)

all states (S_n)

duality between k-positive maps and bipartite states in S_k

Φ **k-positive** $\quad\quad$ $\mathbb{1}_k \otimes \Phi$ **positive**

$\forall \rho \in S_k$

$$[\mathbb{1}_k \otimes \Phi](\rho) \geq 0$$

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

$$\Lambda_t = V_{t,s}\Lambda_s$$

k-positive

The dynamical map is k-divisible

$$\Lambda_t = V_{t,s} \Lambda_s$$

CP

k-positive for all $k = 1, 2, \ldots$

The dynamical map is CP-divisible (*n*-divisible)

$$\Lambda_t = V_{t,s} \Lambda_s$$

positive $k = 1$

The dynamical map is P-divisible

non-Markovianity degree

analogue to Schmidt number

$\mathrm{NMD}[\Lambda_t] = k$ ⟺ Λ_t **is** $(n-k)$ **but not** $(n+1-k)$ **divisible**

$\mathrm{NMD}[\Lambda_t] = 0$ **MARKOVIAN**

$\mathrm{NMD}[\Lambda_t] = n$ **NON-MARKOVIAN**

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

non-Markovianity degree

analogue to Schmidt number

$$\mathcal{N}_k = \{ \Lambda_t \mid \mathrm{NMD}[\Lambda_t] \leq k \}$$

$$\mathcal{N}_0 \subset \mathcal{N}_1 \subset \ldots \subset \mathcal{N}_{n-1} \subset \mathcal{N}_n$$

Markovian maps ($\mathcal{N}_0$)

all dynamical maps ($\mathcal{N}_n$)

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

non-Markovianity degree

analogue to Schmidt number

$$\mathcal{N}_0 \subset \mathcal{N}_1 \subset \ldots \subset \mathcal{N}_{n-1} \subset \mathcal{N}_n$$

Markovian maps ($\mathcal{N}_0$) — **all dynamical maps** ($\mathcal{N}_n$)

$$S_1 \subset S_2 \subset \ldots \subset S_n$$

separable states (S_1) — **all states** (S_n)

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

towards a hierarchy…

$$\mathcal{N}_0 \subset \mathcal{N}_1 \subset \ldots \subset \mathcal{N}_{n-1} \subset \mathcal{N}_n$$

Markovian maps — $\mathcal{N}_0$

all dynamical maps — $\mathcal{N}_n$

$$\mathcal{N}_{BLP}$$

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

Theorem

Λ_t **k-divisible**

$$\frac{d}{dt} ||[\mathbb{1}_k \otimes \Lambda_t](X)||_1 \leq 0$$

$$\forall X \in M_k \otimes \mathcal{B}(\mathcal{H})$$

$$\frac{d}{dt}||[\mathbb{1}_k \otimes \Lambda_t](X)||_1 \leq 0$$

special case

$$\frac{d}{dt}||\Lambda_t(X)||_1 \leq 0$$

$$X = \rho_1 - \rho_2$$

BLP condition

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

towards a hierarchy…

$$\mathcal{N}_0 \subset \mathcal{N}_1 \subset \ldots \subset \mathcal{N}_{n-1} \subset \mathcal{N}_n$$

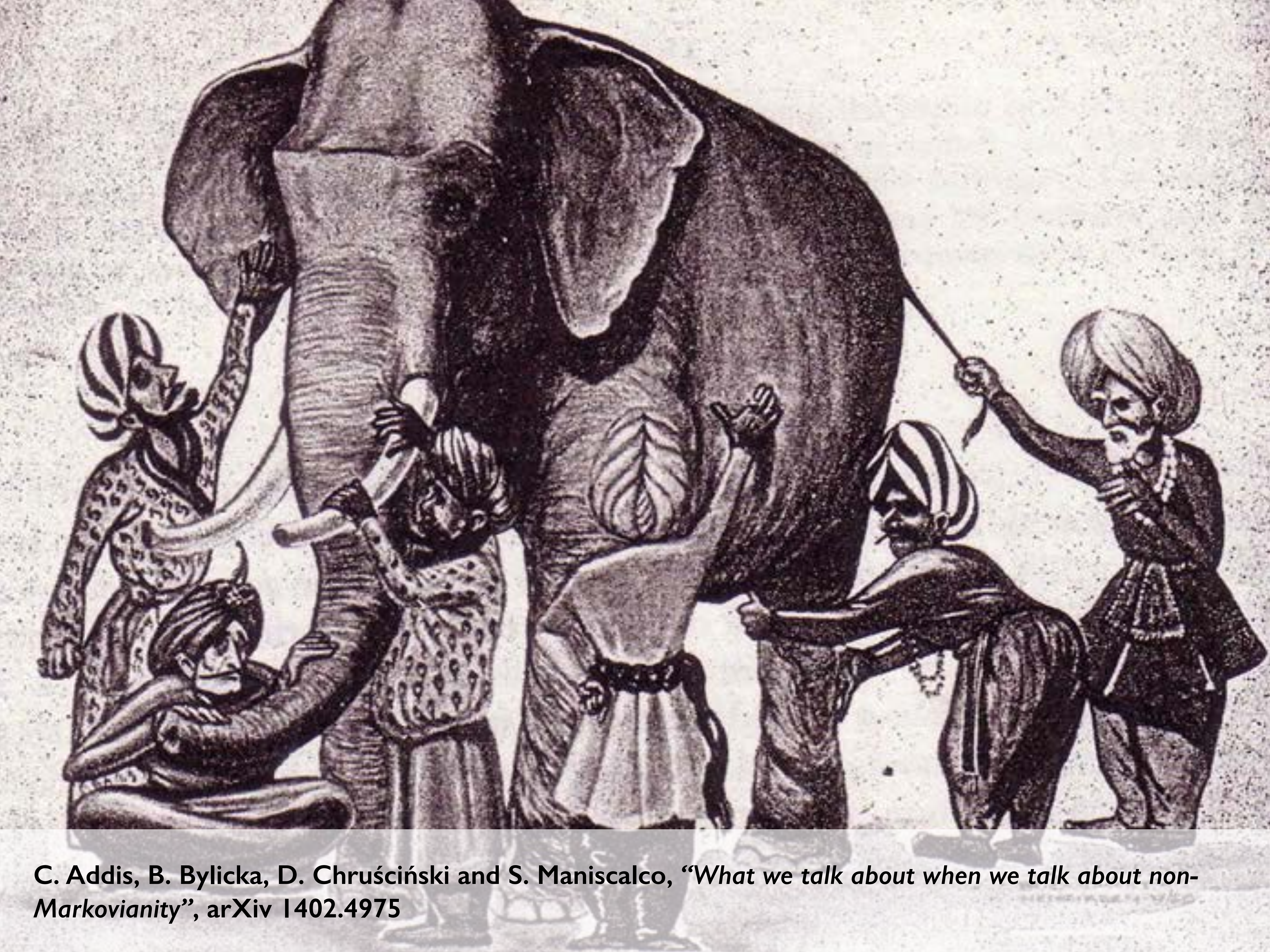
Markovian maps

$$\mathcal{N}_{BLP}$$

essentially non-Markovian maps

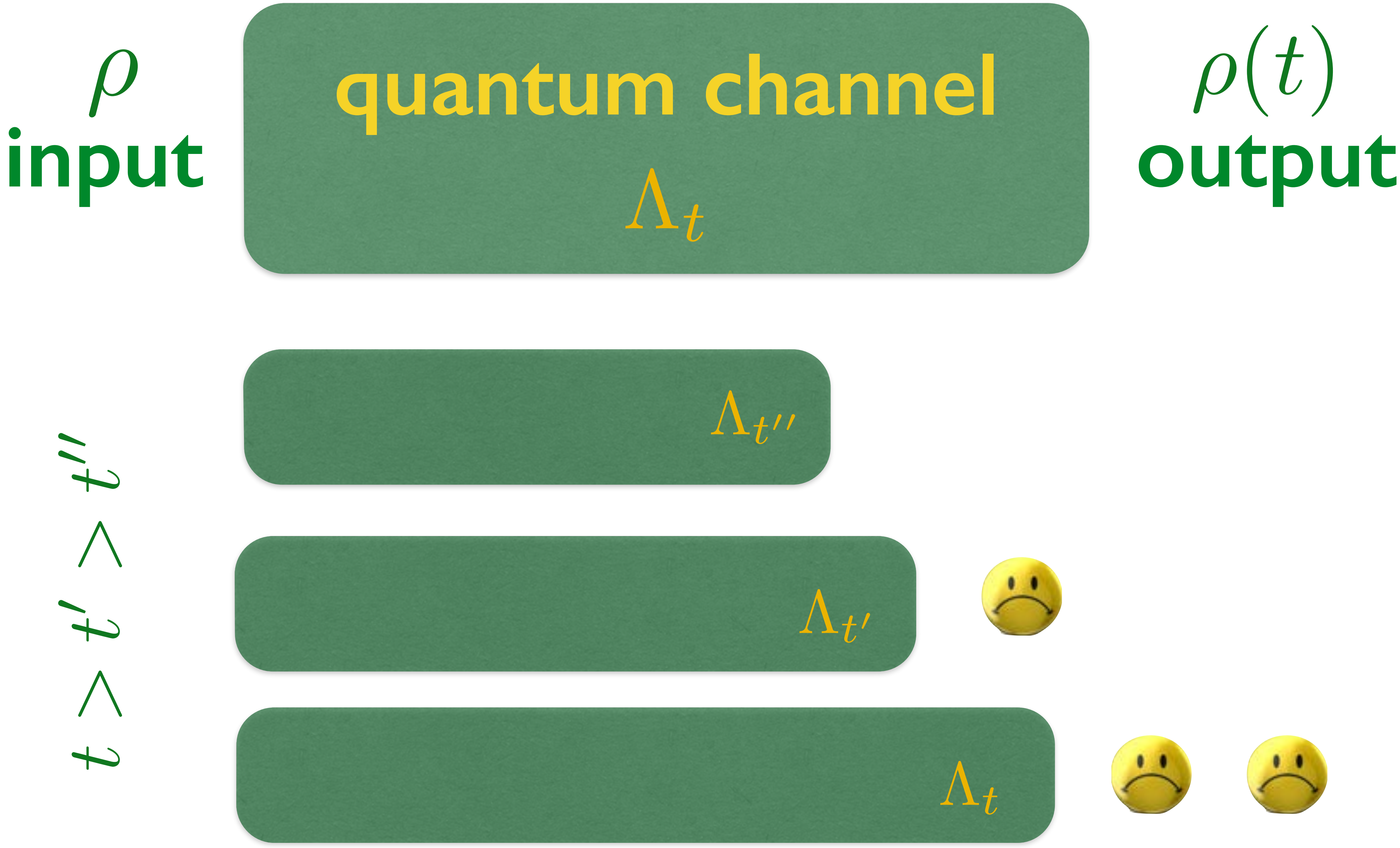
D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

Is there a physical interpretation of non-Markovianity? Or are there many?



C. Addis, B. Bylicka, D. Chruściński and S. Maniscalco, *“What we talk about when we talk about non-Markovianity”*, arXiv 1402.4975

7 Is non-Markovianity a resource for quantum technologies ?


ρ
input
quantum channel
Λ_t
$\rho(t)$
output
$t > t' > t''$
$\Lambda_{t''}$
$\Lambda_{t'}$
Λ_t

ρ Λ_t $\rho(t)$

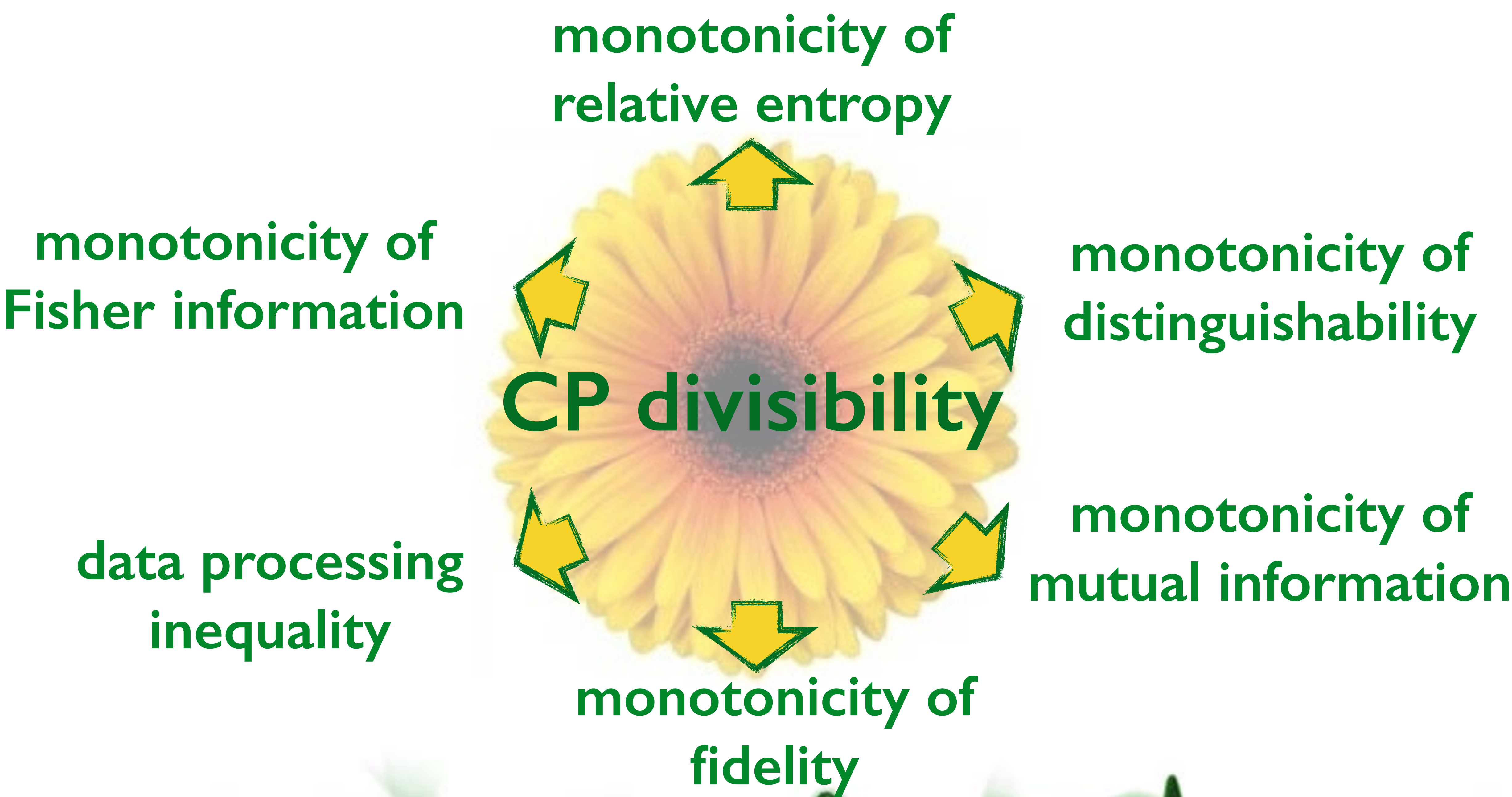
Quantum capacity

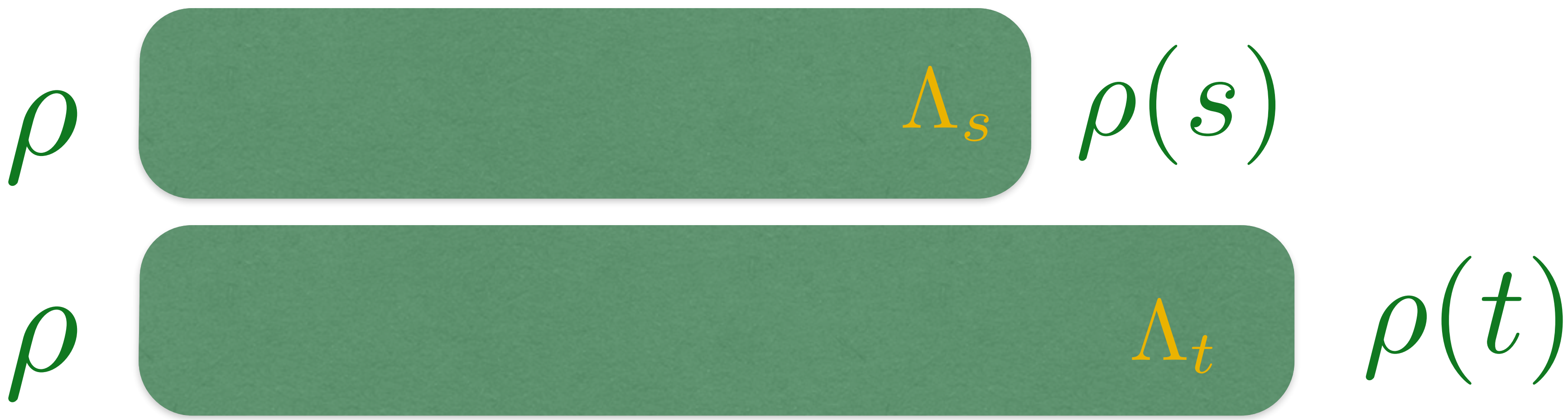
Bound on the maximum rate at which quantum information can reliably be transferred along a noisy quantum channel

$$Q(\Lambda_t) = \sup_{\rho} I_c(\rho, \Lambda_t)$$

$$I_c(\rho, \Lambda_t) = S(\Lambda_t \rho) - S(\rho, \Lambda_t)$$

A common feature to all the measures





CP-divisibility

$\Downarrow \; \forall s \leq t$

$$I_c(\rho, \Lambda_t) \leq I_c(\rho, \Lambda_s)$$

data processing inequality

$$Q(\Lambda_t) \leq Q(\Lambda_s)$$

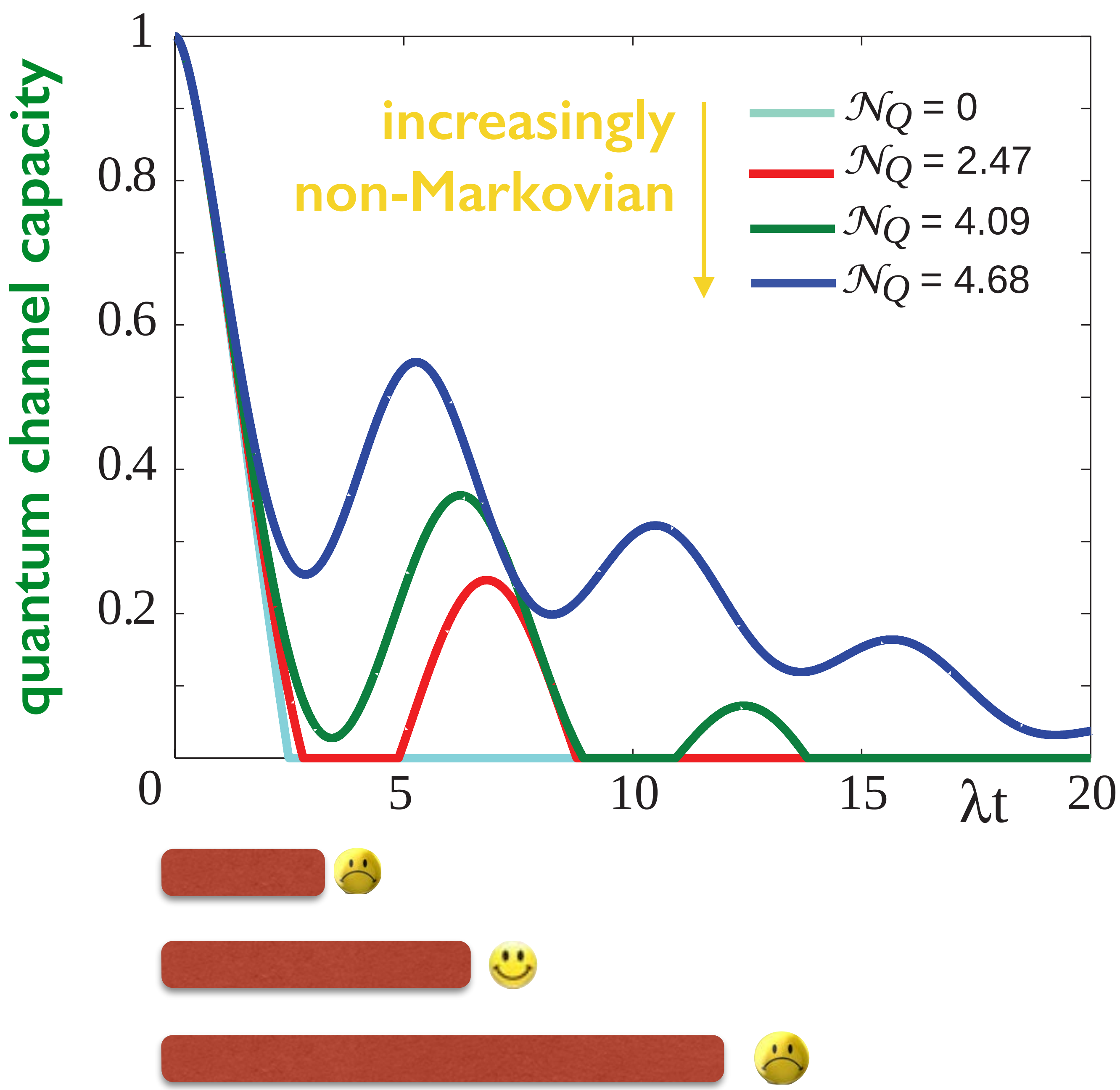
B. Bylicka, D. Chruściński and S. Maniscalco, Scientific Reports 4, 5720 (2014)

How about non-Markovian maps?

CP non-divisible

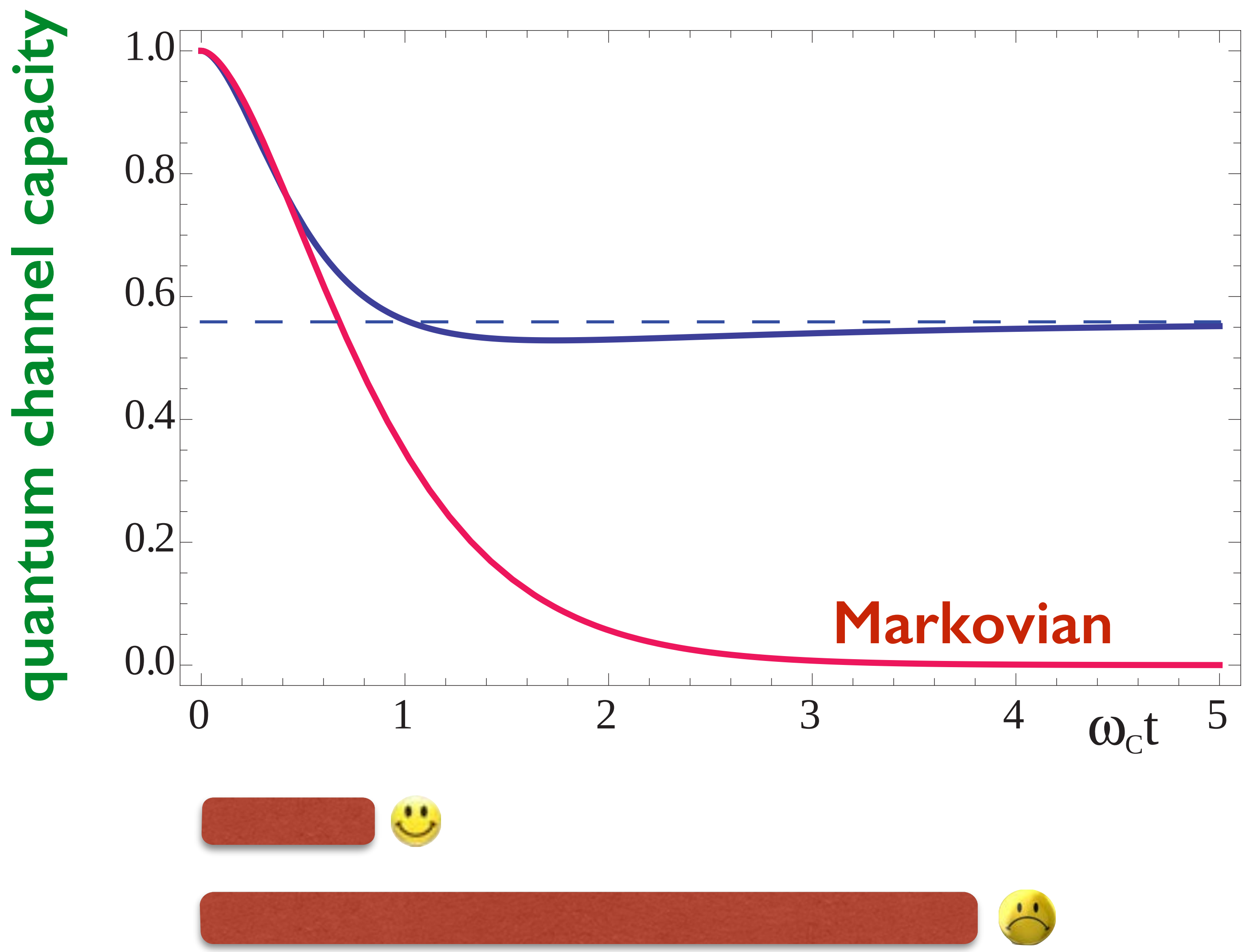
B. Bylicka, D. Chruściński and S. Maniscalco, Scientific Reports 4, 5720 (2014)

Exact amplitude damping channel

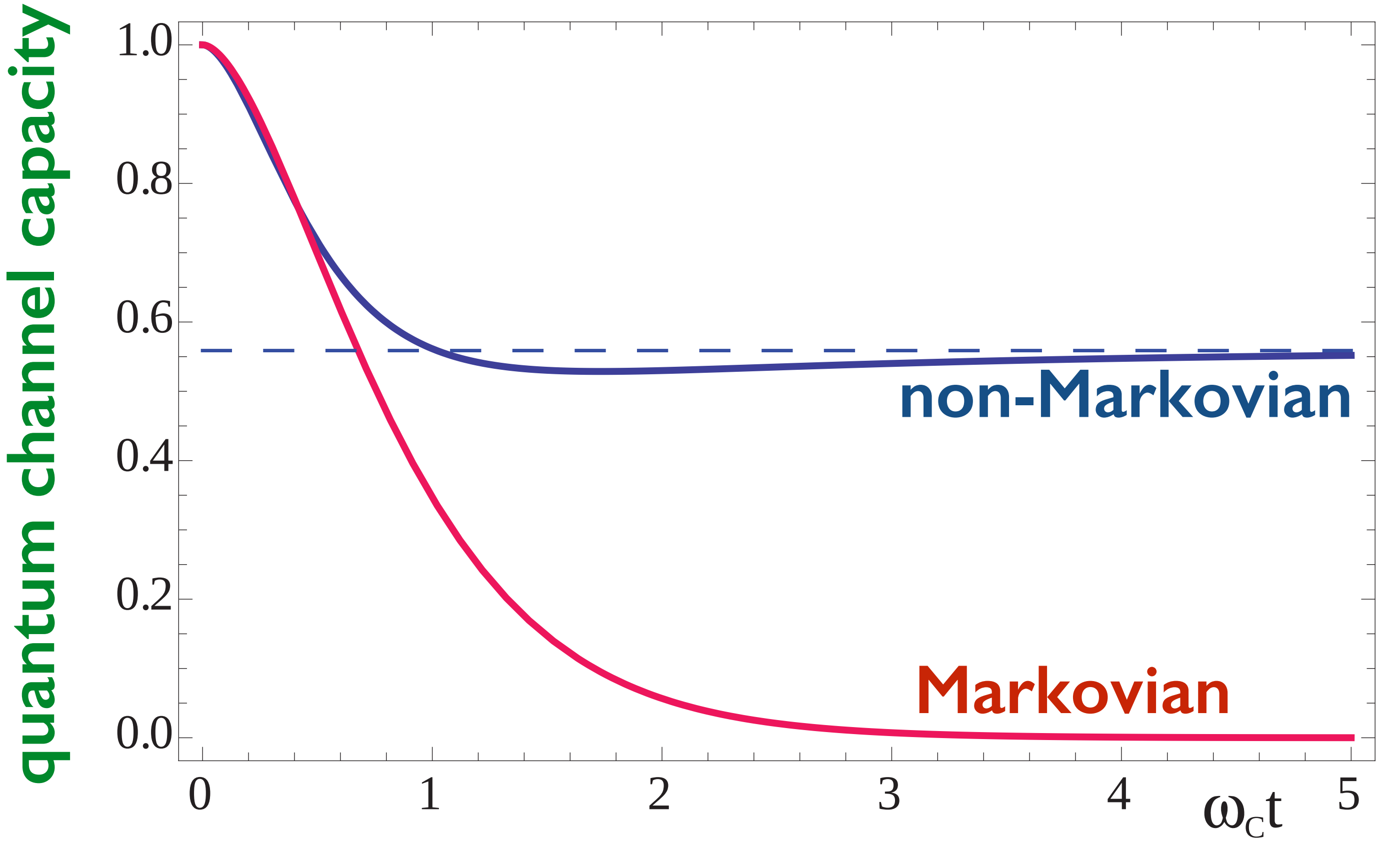


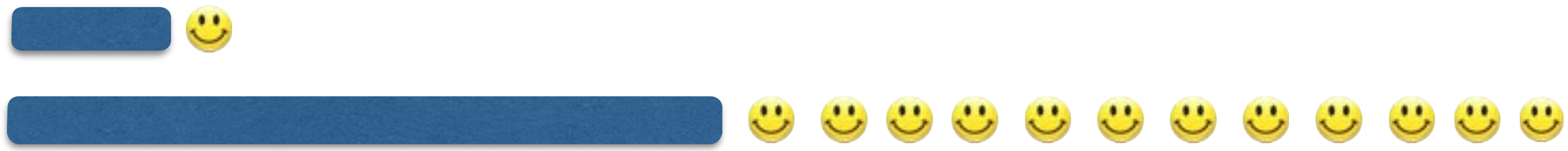
B. Bylicka, D. Chruściński and S. Maniscalco, Scientific Reports 4, 5720 (2014)

Exact dephasing channel



Exact dephasing channel





8

Yes, but isn't most of what is done in this field just Rabi oscillations business more or less?


YES
NO
MAYBE

What are the main open problems of open quantum systems theory?

Witnesses for non-Markovianity

Resource theory for non-Markovianity

Admissible physical CP maps


10


THANK
YOU

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ρ Λ_t $\rho(t)$

Entanglement assisted capacity

Bound on the maximum rate at which classical information can be reliably transferred along a noisy quantum channel, when Alice and Bob share unlimited entanglement

$$C_{ea}(\Lambda_t) = \sup_{\rho} I(\rho, \Lambda_t)$$

$$I(\rho, \Lambda_t) = S(\rho) + S(\Lambda_t \rho) - S(\rho, \Lambda_t)$$

$$\mathbb{1}_k \otimes \Phi \qquad k = 1, 2, \ldots$$

Φ is **k-positive** if

$\mathbb{1}_k \otimes \Phi$ is positive

$$L_t(\rho) = -i[H(t), \rho] + \sum_k \left(A_k(t)\rho A_k^\dagger(t) - \frac{1}{2}\{A_k^\dagger(t)A_k(t), \rho\} \right)$$

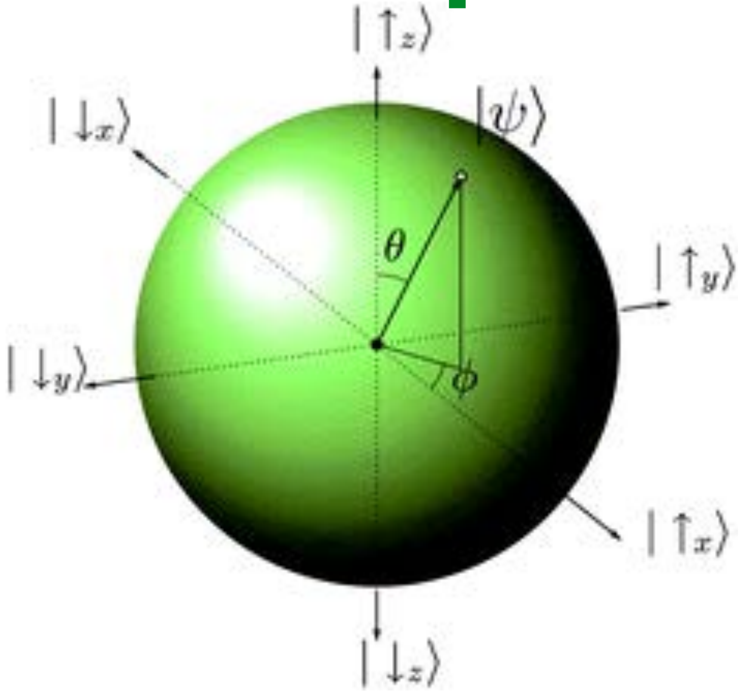
(Almost) **the most general form**

of master equation for open systems

Example qubit dynamics

$$L_t = \frac{\gamma_+(t)}{2}\left([\sigma_+, \rho\sigma_-] + [\sigma_+\rho, \sigma_-]\right) + \frac{\gamma_-(t)}{2}\left([\sigma_-, \rho\sigma_+] + [\sigma_-\rho, \sigma_+]\right)$$

Bloch sphere

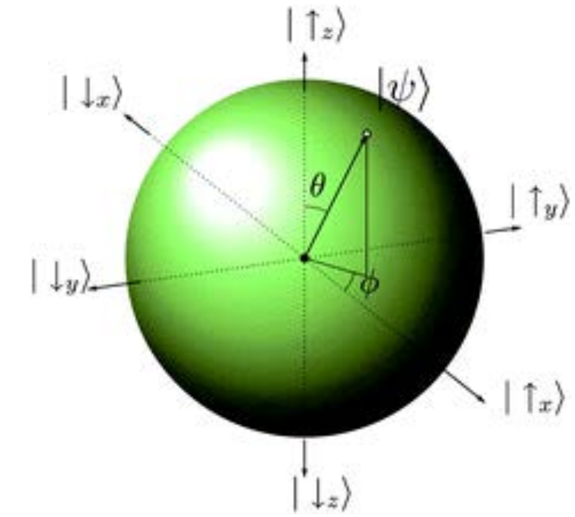


Bloch vector

$$x_k(t) = \mathrm{Tr}[\sigma_k \Lambda_t(\rho)] \quad k = 1, 2, 3$$

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

Bloch vector dynamics



$$\frac{d}{dt}x_k(t) = -\frac{1}{T_\perp(t)}x_k(t)\ , \quad k = 1, 2$$

TRANSVERSE RELAXATION TIME

$$T_\perp(t) = 2/[\gamma_-(t) + \gamma_+(t)]$$

$$\frac{d}{dt}x_3(t) = -\frac{1}{T_{||}(t)}x_3(t) + [\gamma_+(t) - \gamma_-(t)]$$

LONGITUDINAL RELAXATION TIME

$$T_{||}(t) = T_\perp(t)/2$$

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

TRANSVERSE RELAXATION TIME

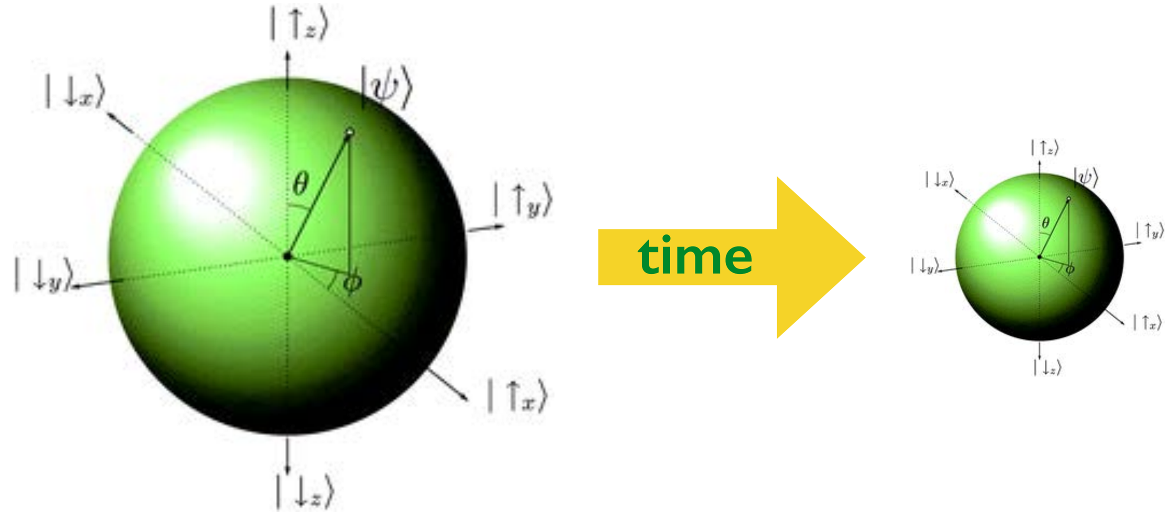
$$T_{\perp}(t) = 2/[\gamma_{-}(t) + \gamma_{+}(t)]$$

LONGITUDINAL RELAXATION TIME

$$T_{||}(t) = T_{\perp}(t)/2$$

P-divisibility = BLP Markovianity

$$T_{\perp}, T_{||}(t) \geq 0$$



D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

TRANSVERSE RELAXATION TIME

$$T_\perp(t) = 2/[\gamma_-(t) + \gamma_+(t)]$$

LONGITUDINAL RELAXATION TIME

$$T_{||}(t) = T_\perp(t)/2$$

CP-divisibility

$\Updownarrow$

$$\gamma_-(t), \gamma_+(t) \geq 0$$

D. Chruściński and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)