Non-Markovianity through accessible information

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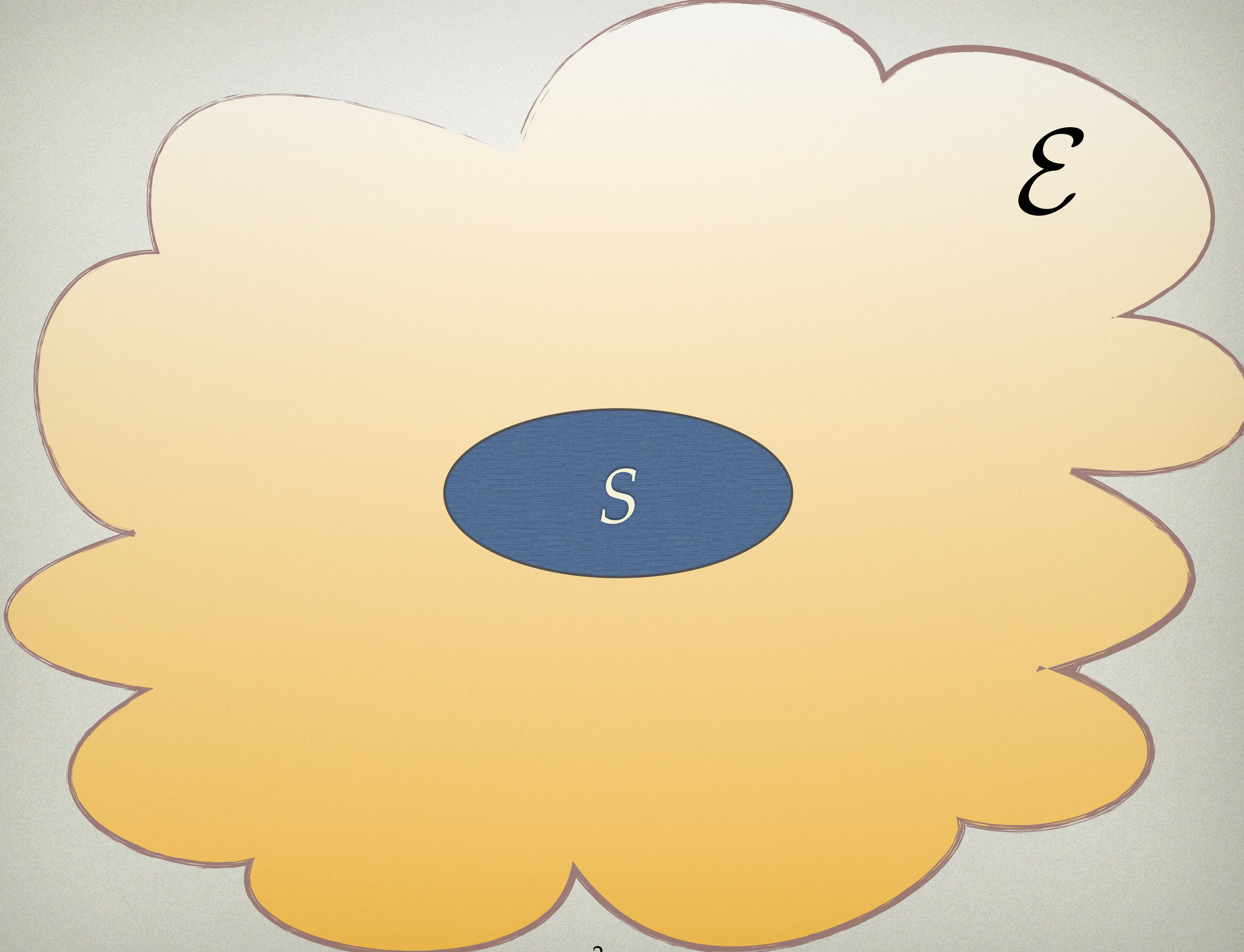
IICQI 14 - 09- 2014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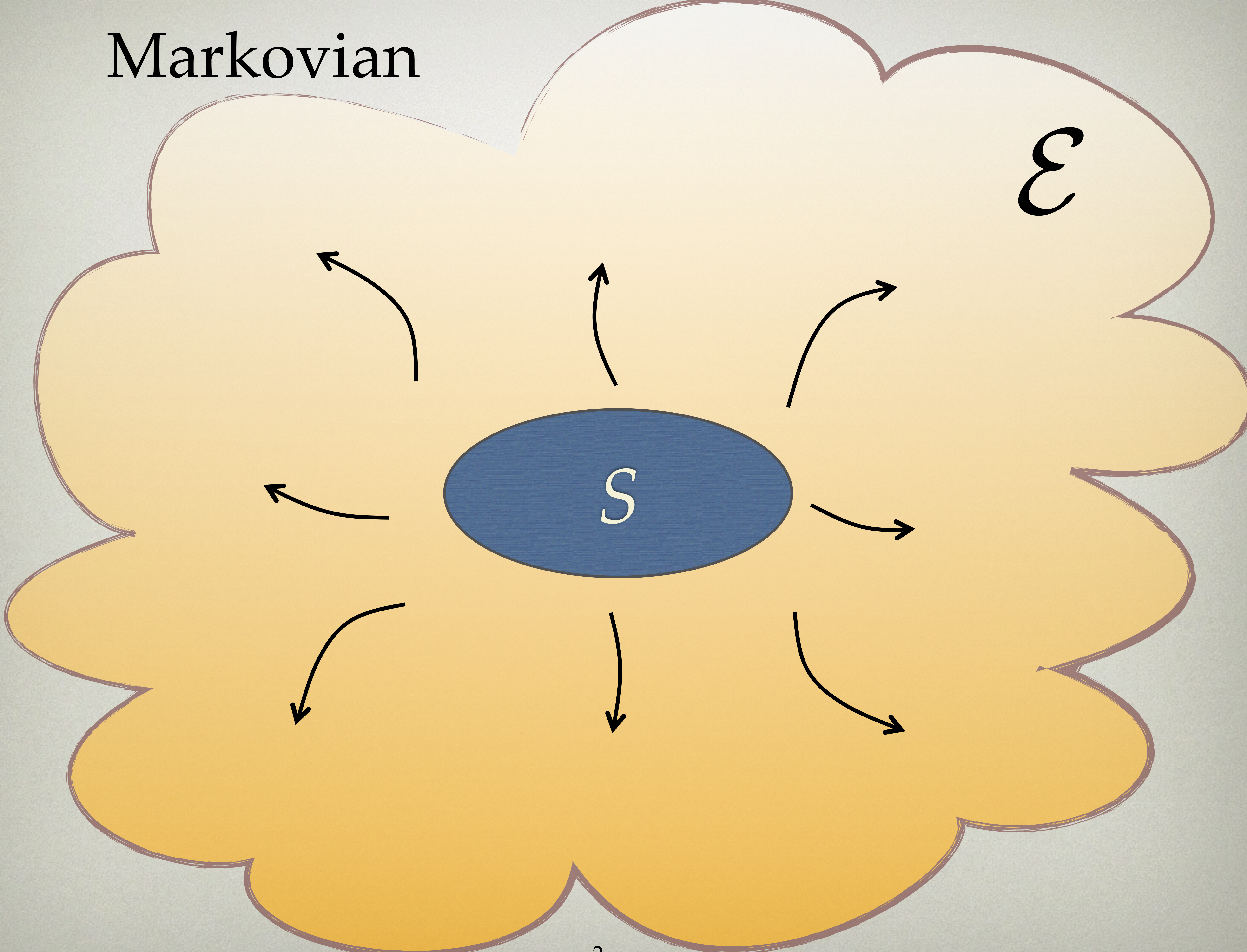
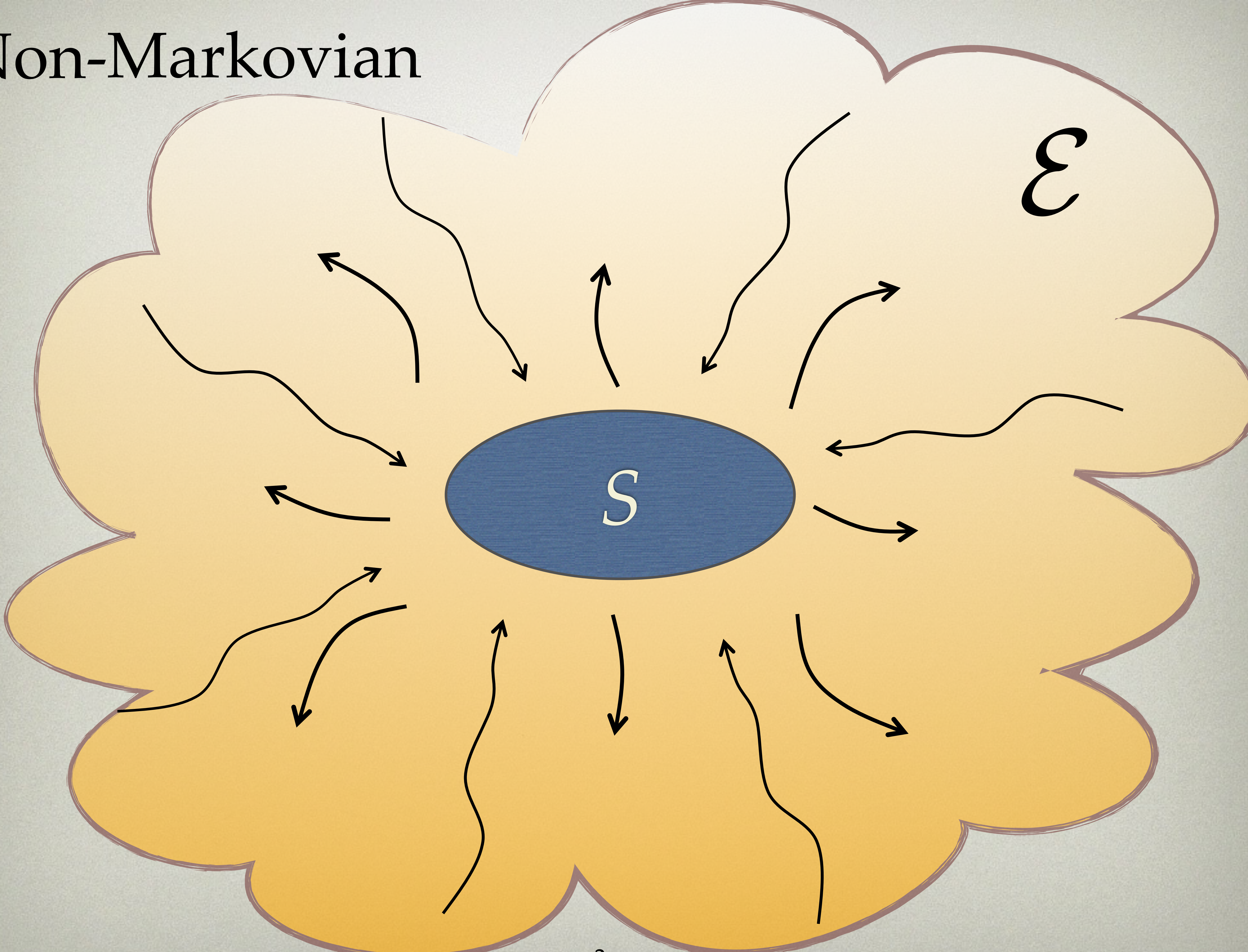
Markovian



Non-Markovian



Outline

- Markovianity and Non-Markovianity
- Measuring the degree of non-Markovianity Breuer, Laine and Pillo measure
  - Breuer, Laine and Pillo measure
  - Rivas, Huega and Plenio measure
- Non-Markovianity through accessible information.
  - Theory
  - Experiment

Markovian Master Equation

Lindblad form:

$$\frac{d\rho_s(t)}{dt} = -\frac{i}{\hbar}[H, \rho_s(t)]$$

$$+\sum_k \gamma_k \left(2L_k\rho_s(t)L_k^\dagger - \rho_s(t)L_k^\dagger L_k - L_k^\dagger L_k \rho_s(t)\right)$$

$\{\gamma_k\}:$ channel decay rate

$\{L_k\}:$ decay operator

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$\{\gamma_k\}:$ channel decay rate

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Non-Markovian: cannot be written in the Lindblad form

Time-local master equation

Time-local master equation

$$\frac{\partial}{\partial t}\rho_s(t) = \mathcal{L}(t)\rho_s(t)$$

$$\mathcal{L}(t)\rho_s(t) = -\frac{i}{\hbar}[H(t), \rho_s(t)]$$
$$+\sum_i \gamma_i(t)\left[A_i(t)\rho_s(t)A_i(t)^\dagger - \frac{1}{2}\left\{A_i(t)^\dagger A_i(t), \rho_s(t)\right\}\right]$$

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- time-dependent Markovian process iff $\gamma_i(t) \geq 0$

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- time-dependent Markovian process iff $\gamma_i(t) \geq 0$

Dynamical map: $\Lambda_{t,0} = T\exp[\int_0^t \mathcal{L}(t')dt']$

satisfies divisibility condition: $\Lambda_{t_2,0} = \Lambda_{t_2,t_1}\Lambda_{t_1,0}$

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CPTP

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CPTP

$\gamma_i(t) < 0$

~~CPTP~~

Non-Markovian process

- Environment correlation time

$$J(\omega) = \eta \, \frac{\omega^s}{{\omega_c}^{s-1}} \exp(-\omega/\omega_c)$$

$$\tau = 1/\omega_c$$

- Backflow of information
- Divisibility of the dynamical map
- Non-monotonical behavior of entanglement
- Non-monotonical behavior of mutual information

Breuer, Laine, and Piilo - PRL 103, 210401 (2009).

Backflow of information

During a Markovian process the distinguishability of the system densit matrix always reduce.

Trace distance:

$$D_{12}(t) = \frac{1}{2} Tr\{\rho_1(t) - \rho_2(t)\}$$

In a non-Markovian process the distinguishability between the system density matrix increase for some instant of time.

$$\frac{d}{dt} D_{12}(t) > 0$$

Breuer, Laine, and Piilo - PRL 103, 210401 (2009).

Backflow of information

Measure of non-Markovianity

$$\mathcal{N}_{BLP}(\Lambda) = \max_{\rho_1(0),\rho_2(0)} \int_{(dD_{12}(t)/dt)>0} \frac{d}{dt} D_{12}(t) dt$$

↑

maximum taken
over all pairs of initial states

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Information ?

Breuer, Laine, and Piilo - PRL 103, 210401 (2009).

Non-monotonical behavior of Entanglement

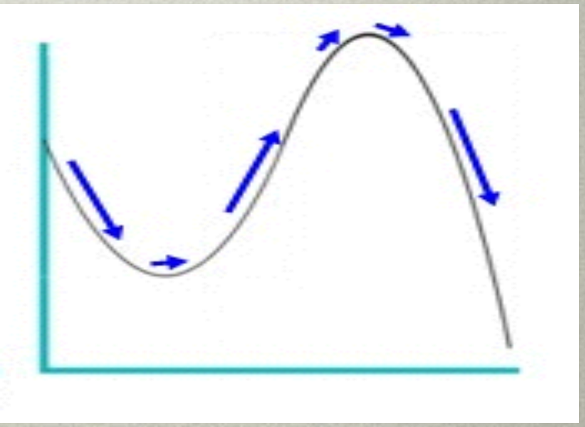
A quantum state in a Hilbert space H

An arbitrary ancilla system in H^a is introduced: $\rho^{sa} \in H \otimes H^a$

Quantum process $\Lambda(t)$: $\rho^{sa}(t) = (\Lambda(t) \otimes I)\rho^{sa}$

Since the entanglement shared by s and a
local operations, any entanglement measure has to monotonously decrease for all divisible processes.

$$\Lambda_{t_2,0} = \Lambda_{t_2,t_1}\Lambda_{t_1,0}$$



$E(\rho^{sa}(t))$ decays monotonically: Markovian

$d_t E(\rho^{sa}(t)) > 0$ ➡ Non-Markovian

Rivas, Huelga, and Plenio – PRL 105, 050403 (2010).

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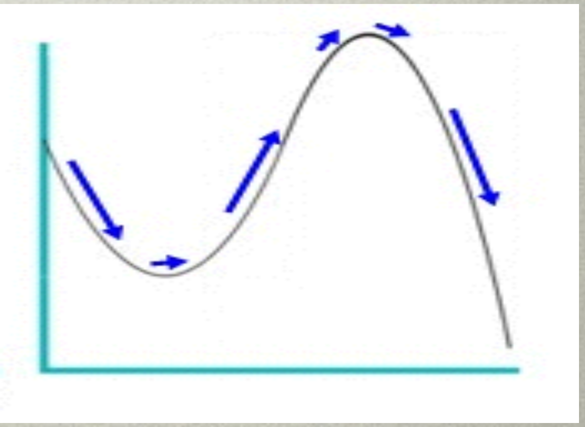

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Why?

Accessible Information

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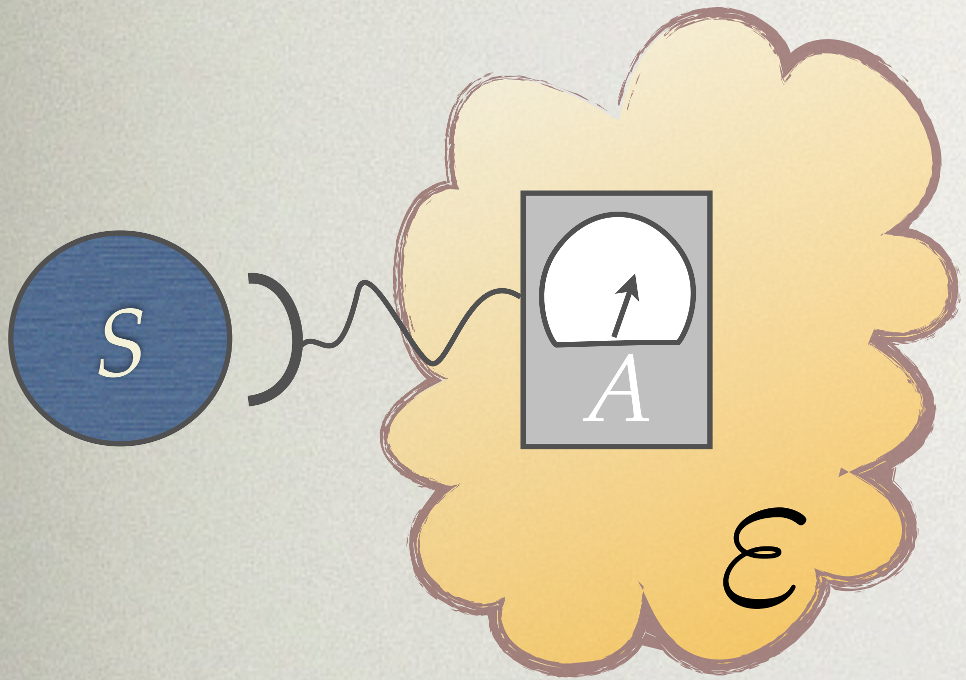

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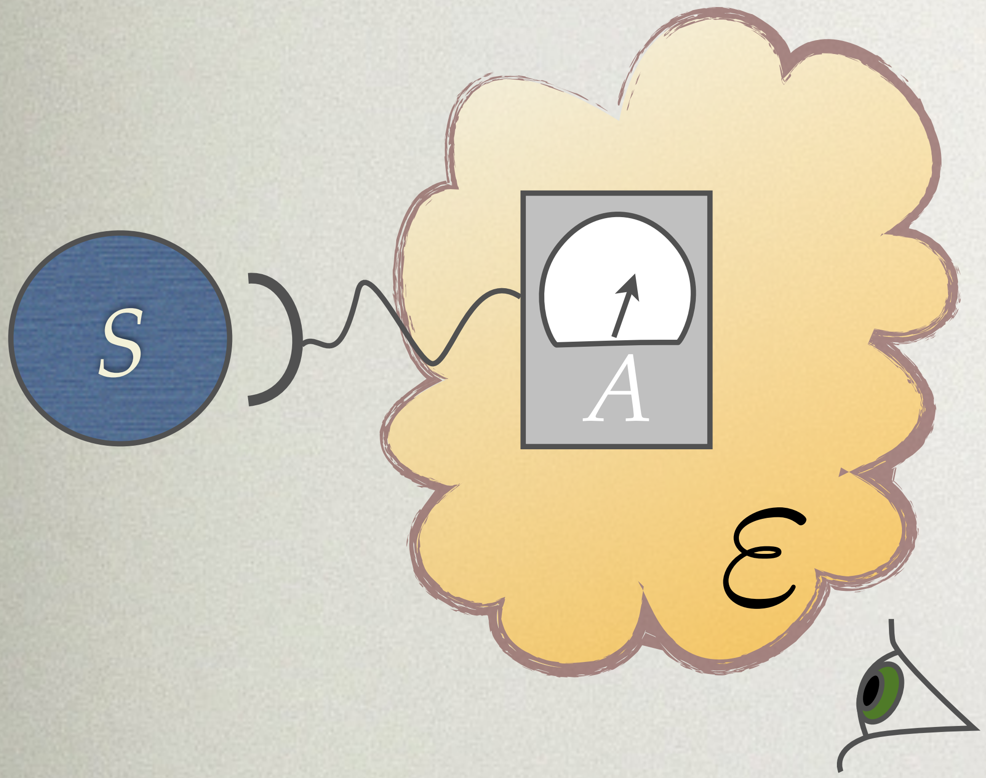

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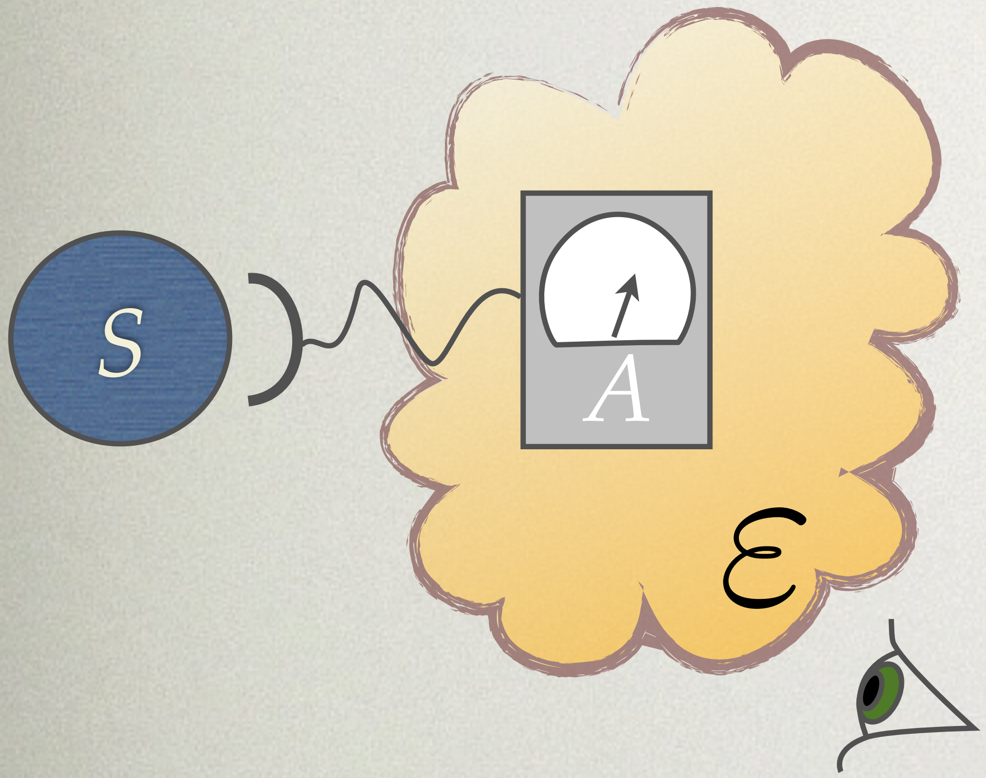

$$J^{\leftarrow}_{\mathcal{SE}} = S(\rho_{\mathcal{S}}) - \sum_i p_i S(\rho_{\mathcal{S}|i})$$

$$\rho_{\mathcal{S}|i} = \frac{Tr_{\mathcal{E}}\{\Gamma_i^{\mathcal{E}} \rho_{\mathcal{SE}} \Gamma_i^{\mathcal{E}\dagger}\}}{p_i}$$

$$p_i = Tr\{\Gamma_i^{\mathcal{E}\dagger} \Gamma_i^{\mathcal{E}} \rho_{\mathcal{SE}}\}$$

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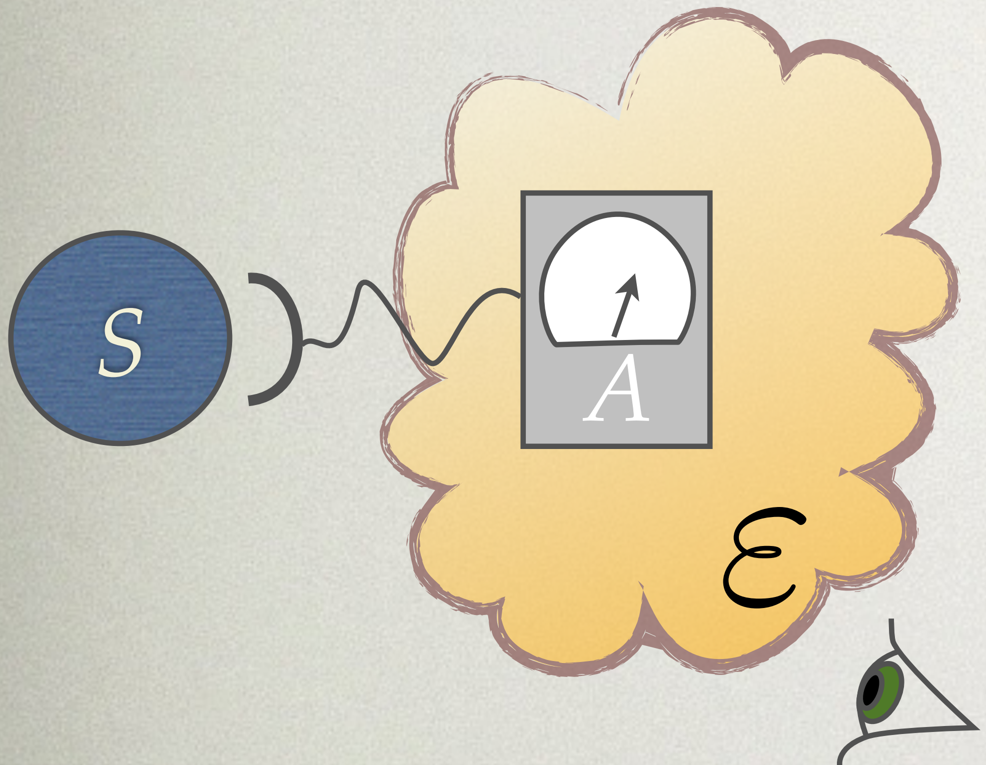

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Depends on chosen $\{\Gamma_i^{\mathcal{E}}\}$

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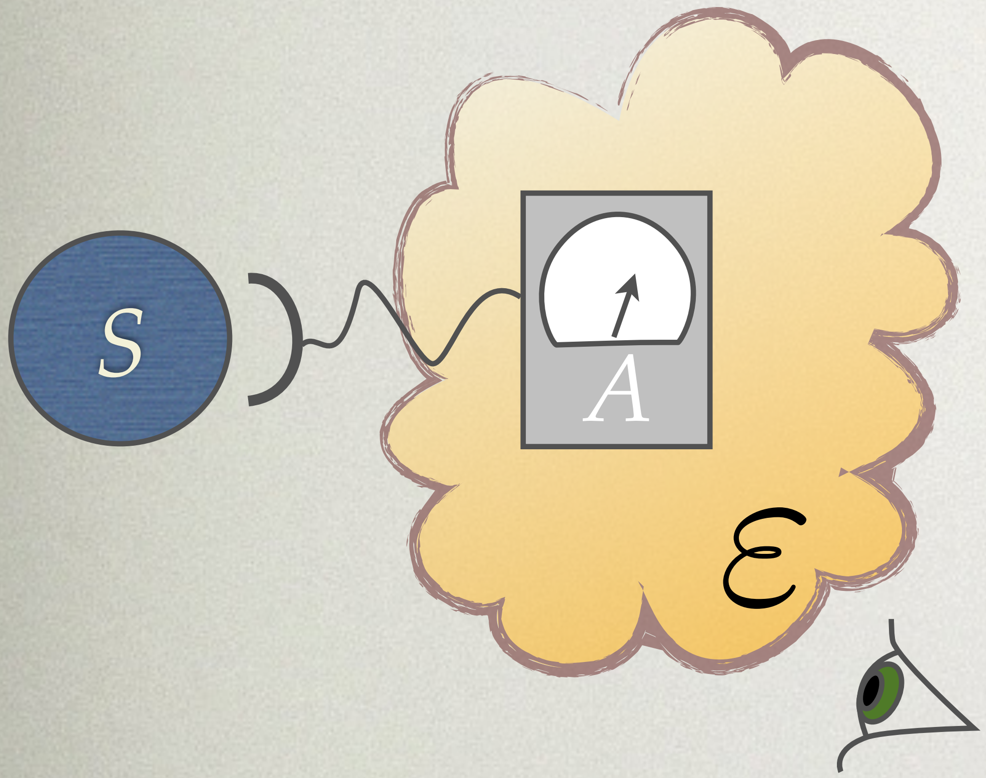

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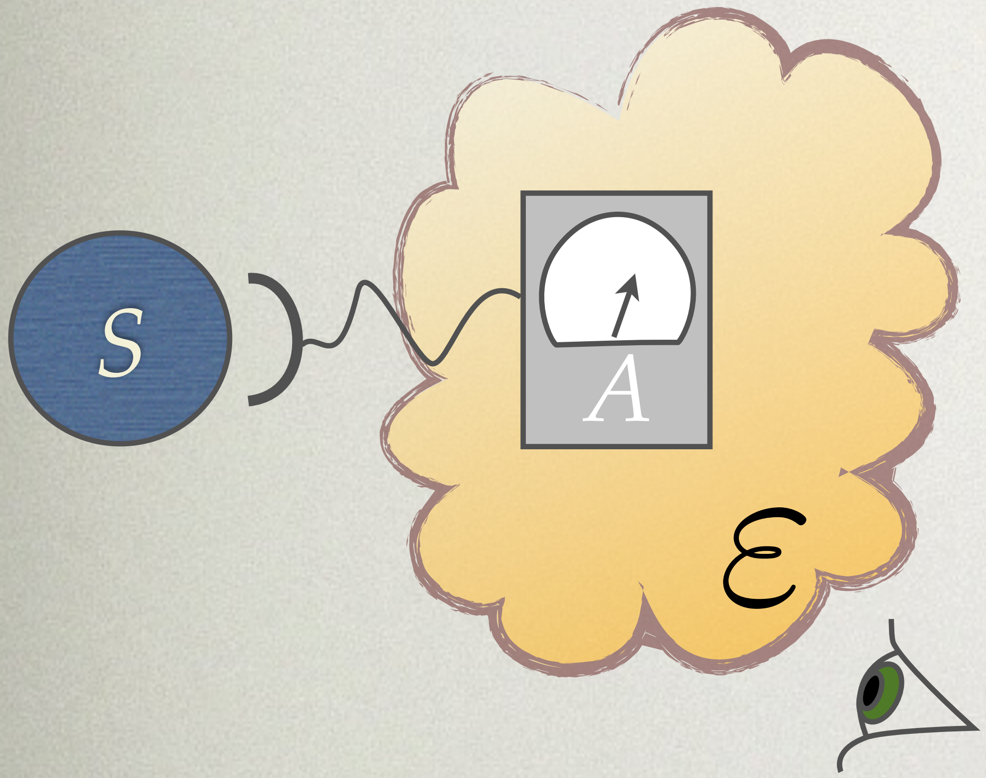
Depends on chosen $\{\Gamma_i^{\mathcal{E}}\}$

$$J^{\leftarrow}_{\mathcal{S}\mathcal{E}} = \max_{\{\Gamma_i^{\mathcal{E}}\}} \left[S(\rho_{\mathcal{S}}) - \sum_i p_i S(\rho_{\mathcal{S}|i}) \right]$$

Classical Correlation

L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

Accessible Information



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Depends on chosen $\{\Gamma_i^{\mathcal{E}}\}$

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Classical Correlation

$$J^{\leftarrow}_{\mathcal{SE}} = S(\rho_{\mathcal{S}}) - E_{\mathcal{SA}}$$

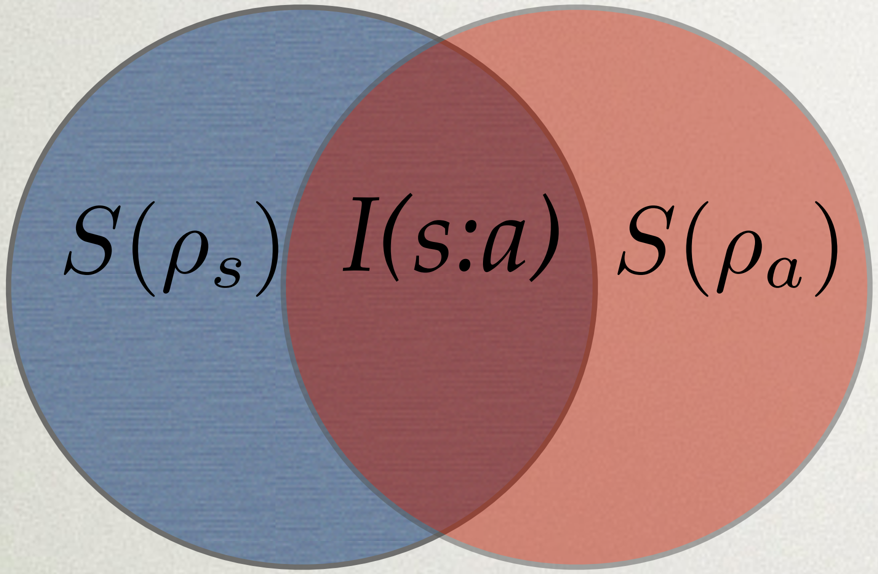
L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

Accessible Information

M. Koashi and A. Winter, PRA 69, 022309 (2004)

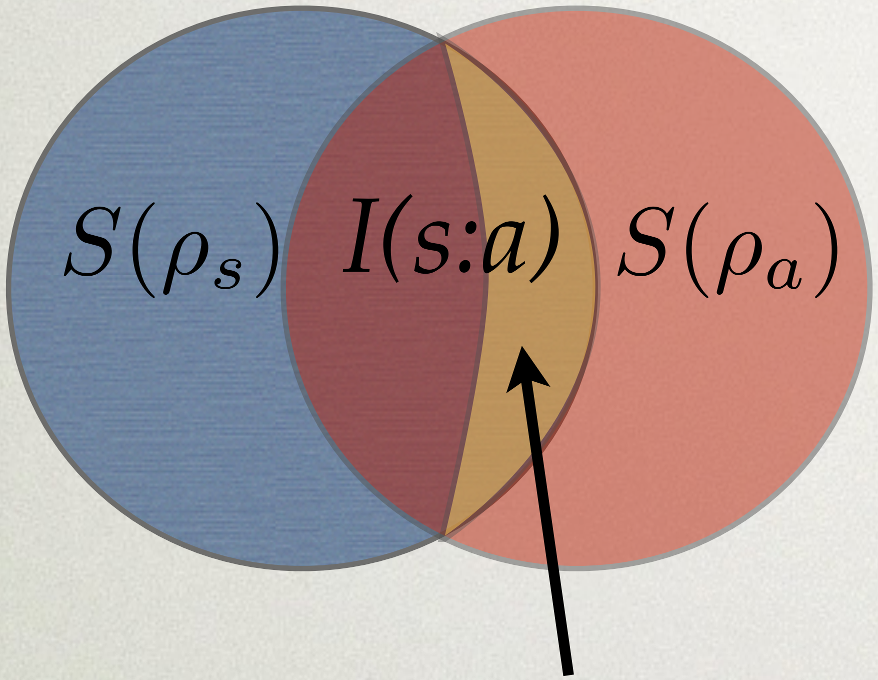
$$J^{\leftarrow}_{\mathcal{S}\mathcal{E}} = S(\rho_{\mathcal{S}}) - E_{\mathcal{S}\mathcal{A}}$$

Local accessible and Inaccessible information



$$I(s:a) = S(\rho_s) + S(\rho_a) - S(\rho_{sa})$$

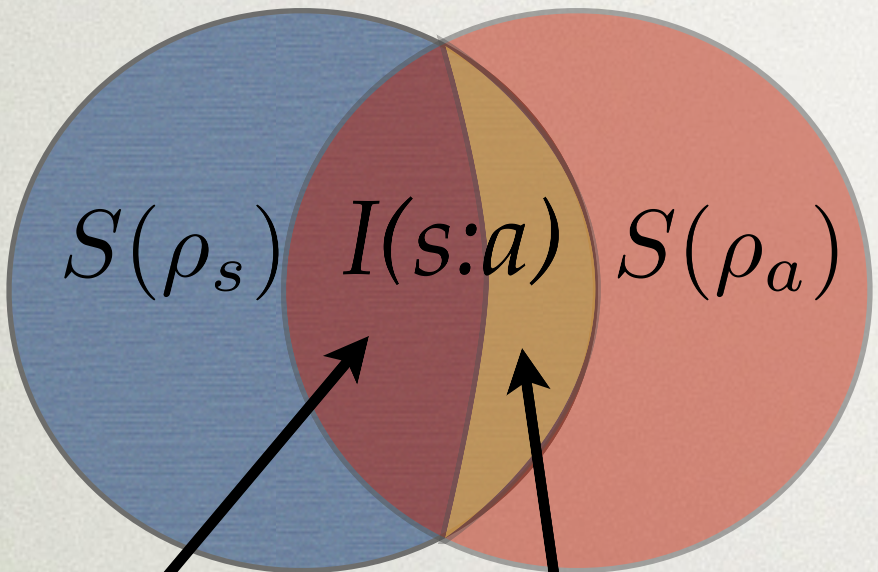
Local accessible and Inaccessible information



$$I(s:a) = S(\rho_s) + S(\rho_a) - S(\rho_{sa})$$

$$J^{\max}_{s|a}(\rho_{sa}) = \max_{\{\Gamma_i^a\}} \left[S(\rho_s) - \sum_i p_i S(\rho_s^i | \Gamma_i^a) \right] \quad \text{(C.C.)}$$

Local accessible and Inaccessible information



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$$\delta^{\leftarrow}_{AB} = I_{AB} - J^{\leftarrow}_{AB} \quad \text{(Quantum Discord)}$$

Some results

PHYSICAL REVIEW A **84**, 012313 (2011)

Conservation law for distributed entanglement of formation and quantum discord

Felipe F. Fanchini,[1,*] Marcio F. Cornelio,[2] Marcos C. de Oliveira,[2] and Amir O. Caldeira[2]
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(Received 30 August 2010; revised manuscript received 3 May 2011; published 13 July 2011)

PHYSICAL REVIEW A **87**, 032317 (2013)

Why entanglement of formation is not generally monogamous

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PRL **107,** 020502 (2011) PHYSICAL REVIEW LETTERS week ending 8 JULY 2011

Entanglement Irreversibility from Quantum Discord and Quantum Deficit

Marcio F. Cornelio,[1,*] Marcos C. de Oliveira,[1,†] and Felipe F. Fanchini[1,2]
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New Journal of Physics
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Locally inaccessible information as a fundamental ingredient to quantum information

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PRL 112, 210402 (2014) **arXiv:1402.5395**

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PRL 112, 210402 (2014) **arXiv:1402.5395**

Non-monotonical behavior of Entanglement

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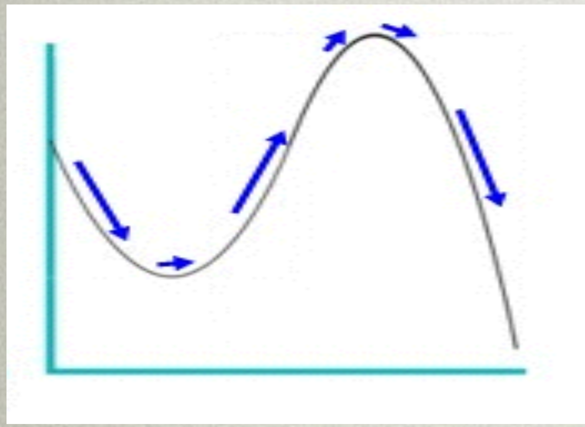

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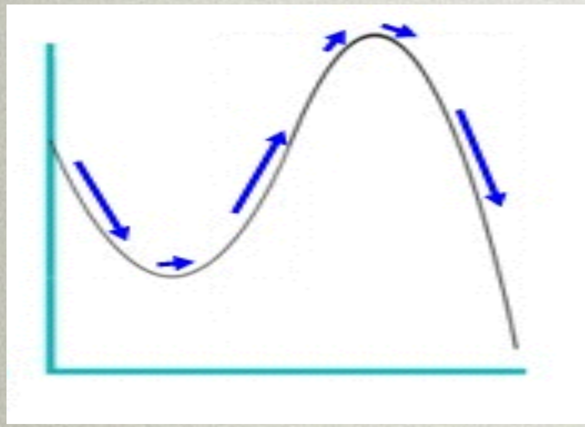

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→ *backflow of information*

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Since:

- System *S* does not interact with the environment,
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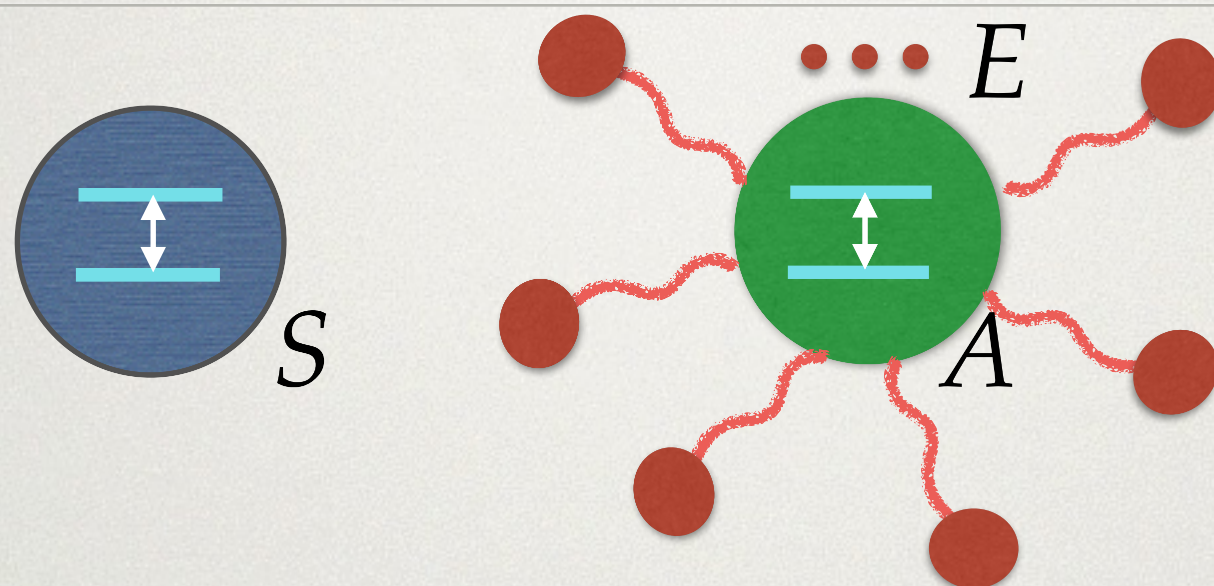
Example



$$H_{\mathcal{A}\mathcal{E}} = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k a_k^\dagger a_k + (\sigma_+ B + \sigma_- B^\dagger)$$

$$B = \sum_k g_k a_k$$

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_0 - \omega)^2 + \lambda^2}$$

γ_0 : related to the system reservoir coupling

τ_R : system relaxation time

$$\tau_R \approx 1/\gamma_0$$

λ : spectral width of the coupling

τ_B : bath correlation time

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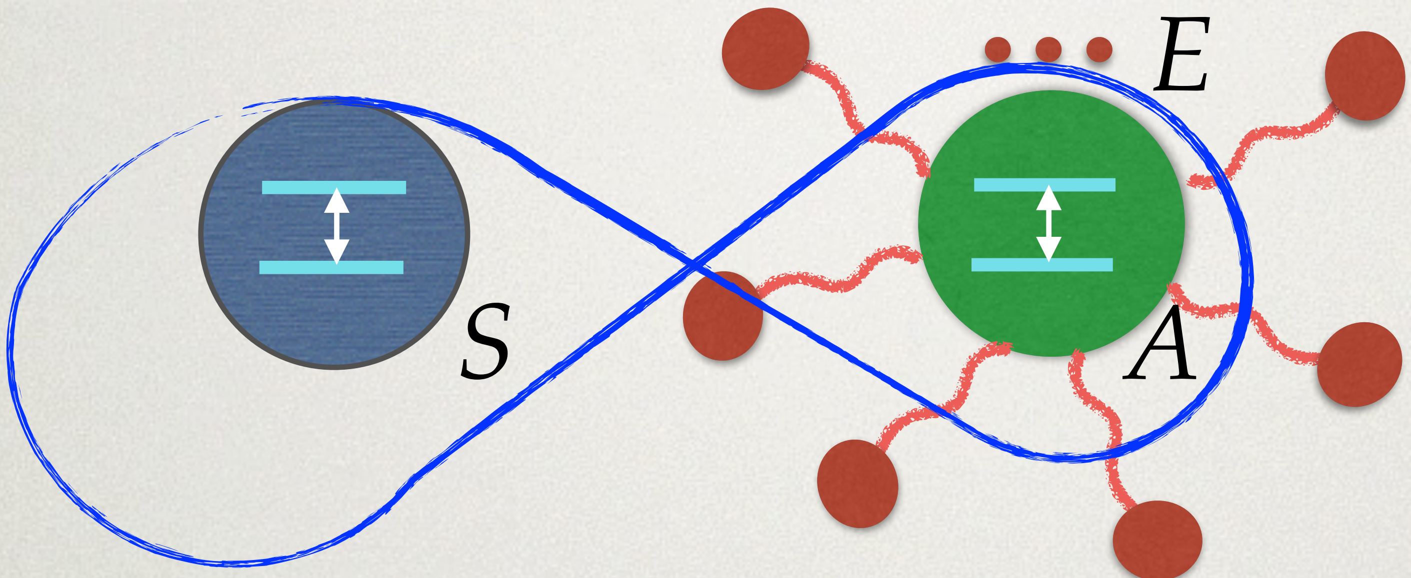

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Time-local master equation

$$\frac{\partial}{\partial t}\rho_{\mathcal{A}}(t) = \gamma(t)\left(\sigma_- \rho_{\mathcal{A}}(t)\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho_{\mathcal{A}}(t)\}\right)$$

$$\gamma(t) = \frac{2\gamma_0 \lambda \sinh(dt/2)}{d\cosh(dt/2) + \lambda \sinh(dt/2)}, \qquad d = \sqrt{\lambda^2 - 2\gamma_0 \lambda}$$

$$\rho(t) = \Lambda(\rho(0)) = \sum_{i=1}^{2} M_i(t)\rho(0)M_i^\dagger(t)$$ ← Solution

$$M_1(t) = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p(t)} \end{pmatrix}, M_2(t) = \begin{pmatrix} 0 & \sqrt{p(t)} \\ 0 & 0 \end{pmatrix},$$

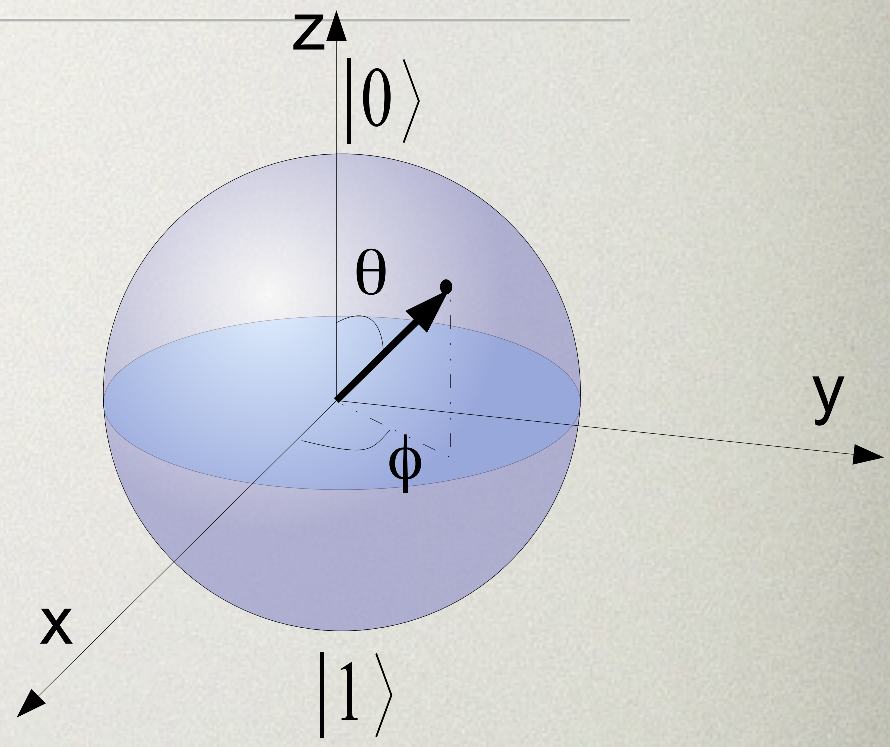
$$p(t) = 1 - e^{-\lambda t}\left[\cosh\left(\frac{dt}{2}\right) + \frac{\lambda}{d}\sinh\left(\frac{dt}{2}\right)\right]^2$$

Optimization

?

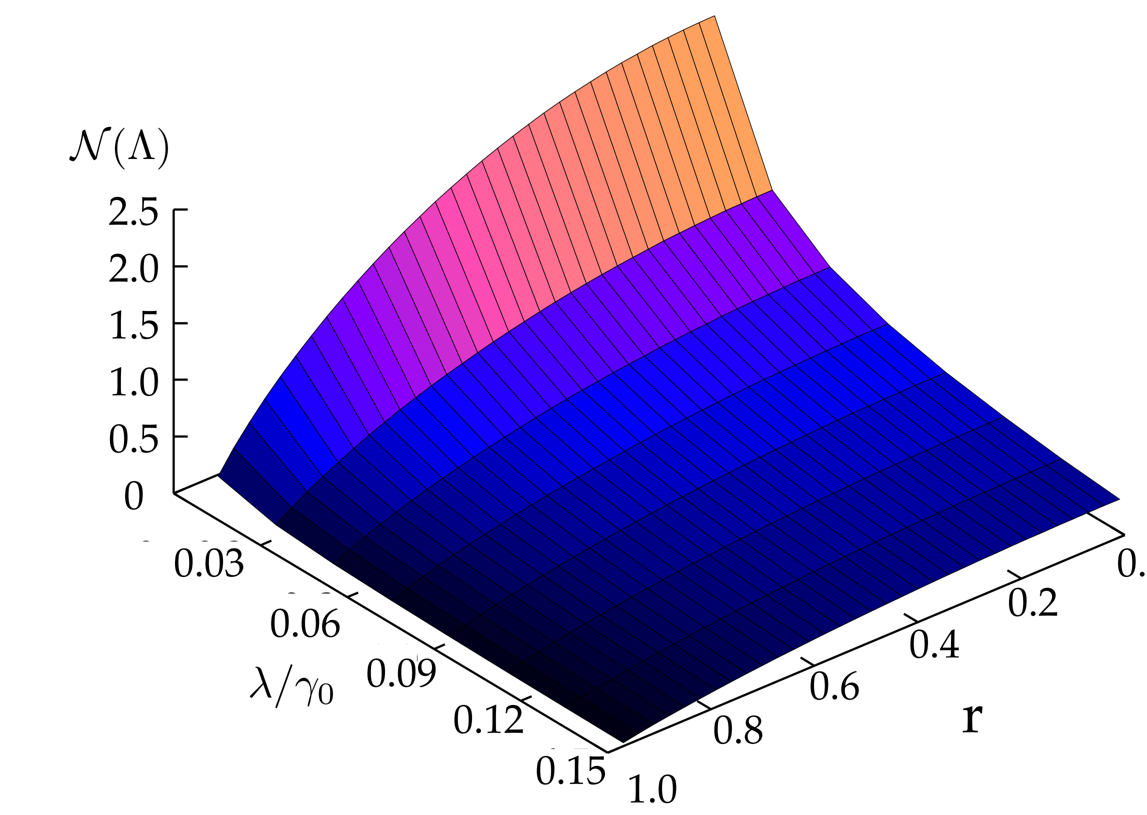
$$\rho_{\mathcal{A}} = \frac{1}{2}(I + \vec{r}.\vec{\sigma})$$

$$\vec{r} = (r\sin\theta\cos\phi, r\sin\theta\sin\phi, r\cos\theta)$$



$\mathcal{N}(\Lambda)$ is independent of θ and ϕ

The optimal state is the maximally mixed state



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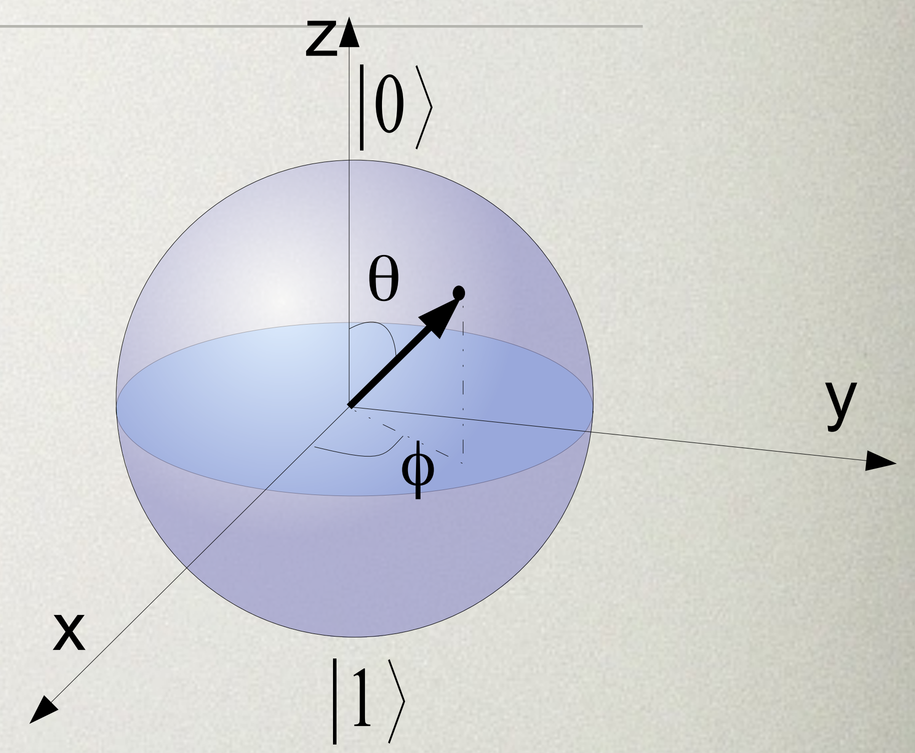

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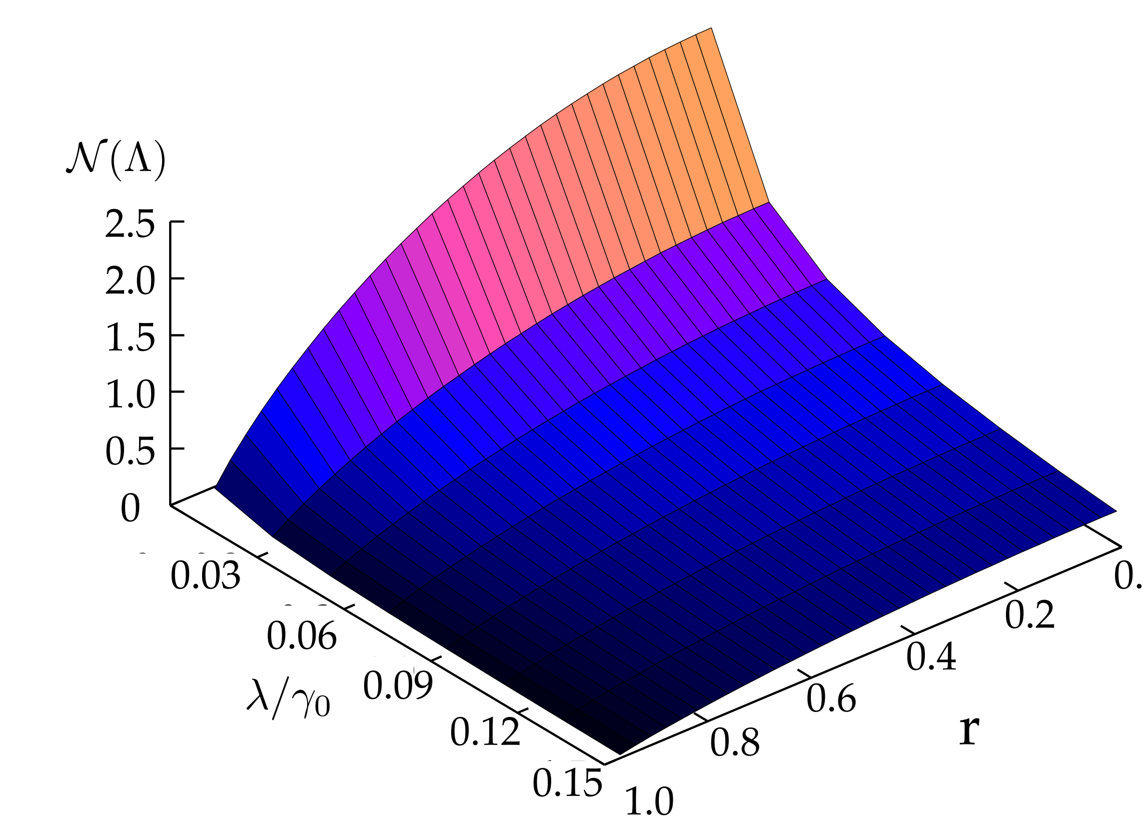
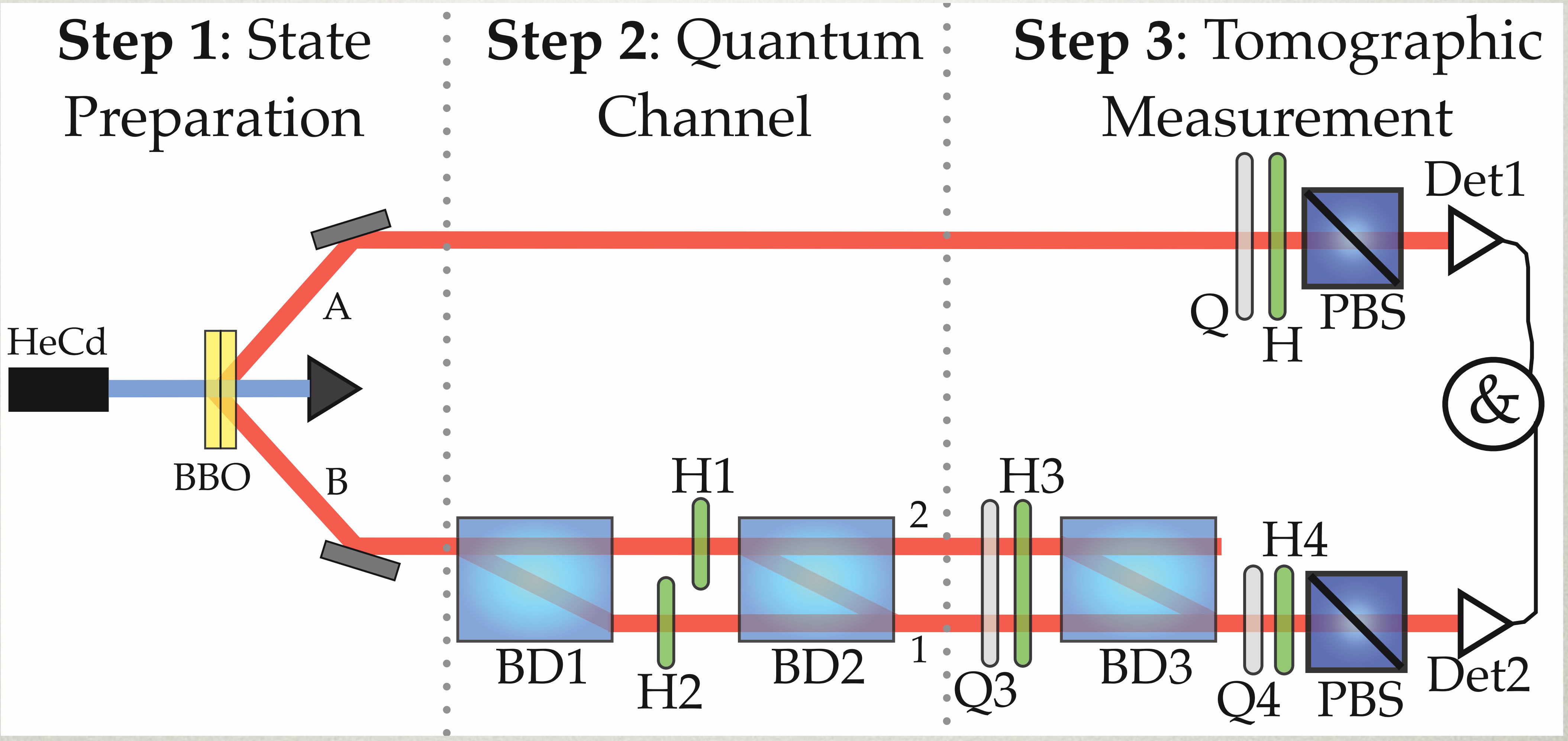


Experiment



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We assume the following triparite state :

$$|\Psi(0)\rangle_{\mathcal{SAE}} = \sqrt{\frac{1}{2}}\,[|10\rangle + |01\rangle]_{\mathcal{SA}}\,|0\rangle_{\mathcal{E}}$$

As a result of the interaction between *A* and *E*, the state *SA* evolves into

$$\rho_{\mathcal{SA}}(t) = \frac{1}{2}|\phi_{\mathcal{SA}}(t)\rangle\langle\phi_{\mathcal{SA}}(t)| + \frac{1}{2}p(t)|00\rangle\langle 00|$$

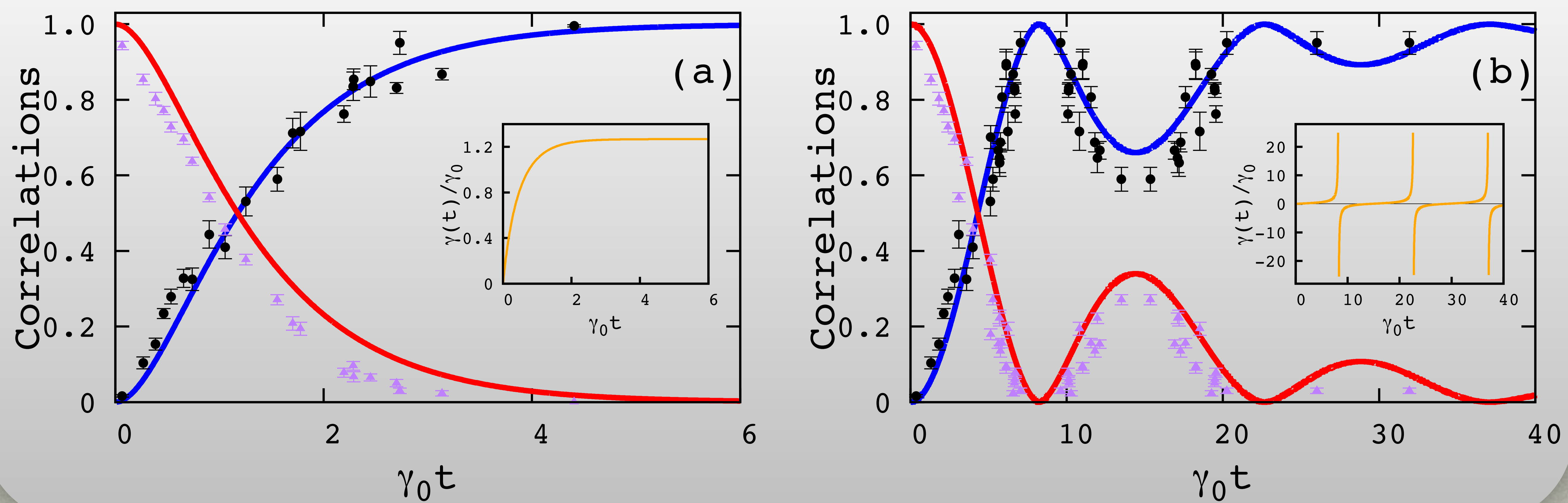
$$|\phi_{\mathcal{SA}}(t)\rangle = |10\rangle + \sqrt{1-p(t)}|01\rangle$$

and the state of *SE* evolves into

$$\rho_{\mathcal{SE}}(t) = \frac{1}{2}|\psi_{\mathcal{SE}}(t)\rangle\langle\psi_{\mathcal{SE}}(t)| + \frac{1}{2}[1-p(t)]|00\rangle\langle 00|$$

$$|\psi_{\mathcal{SE}}(t)\rangle = |10\rangle + p(t)|01\rangle$$

Results



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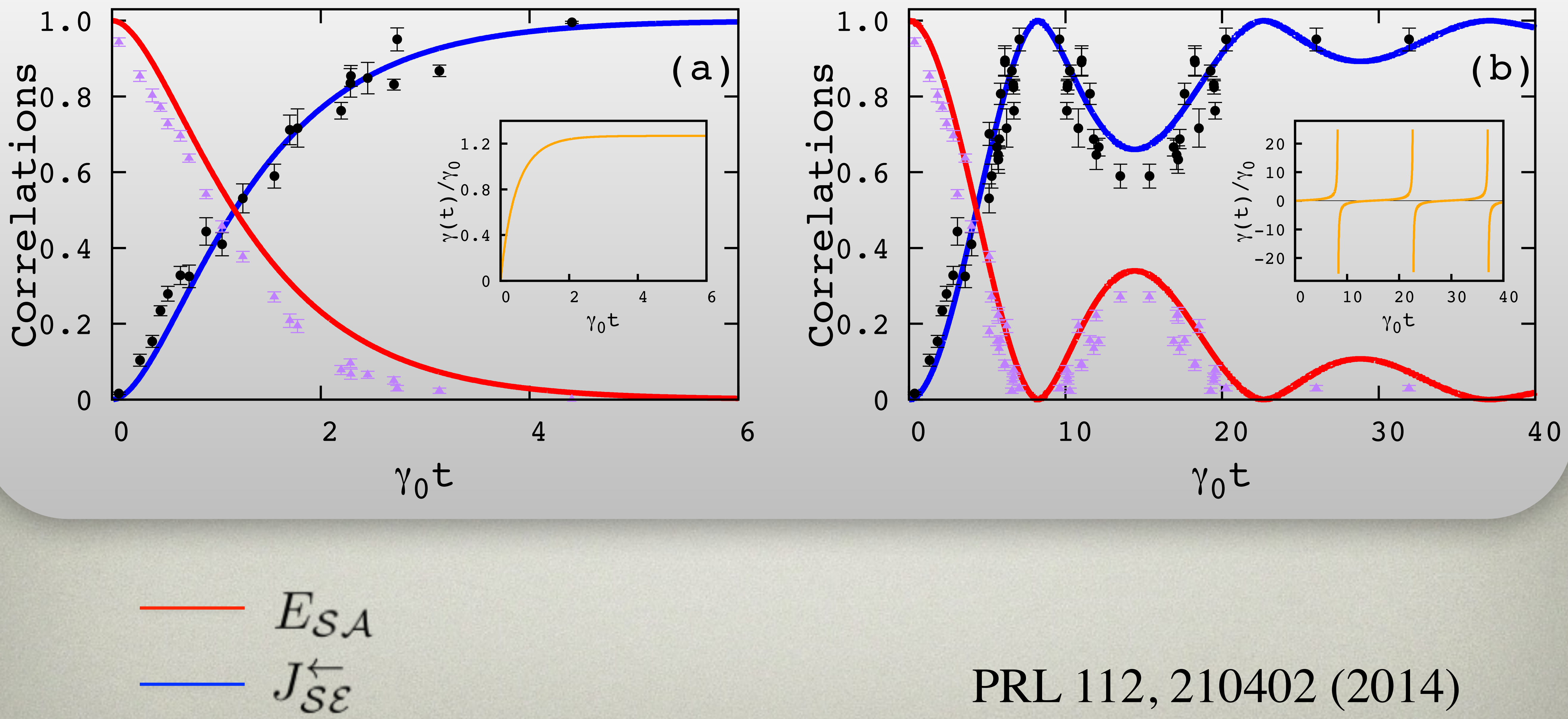

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Conclusions

- Simplified and computable measure of non-Markovianity

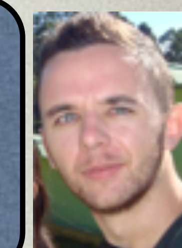
- Interpretation in terms of flow of information (measured by the classical correlation)

- Experimental demonstration using an optical setup

Quantum Information Theory

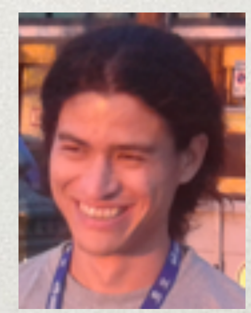
Measurement and control of quantum states

Non-classicality and complementarity



Quantum optomechanical and electromechanical resonators

Quantum phenomena in biological systems



Quantum correlations and entanglement theory

Control of dynamics in ultra cold atoms

Gaussian states and continuous variable entanglement

Implementation of quantum walks in opto- and electromechanical systems

INCT-IQ

Jalil Khatibi Moqadam