The quantum Random Access Memory

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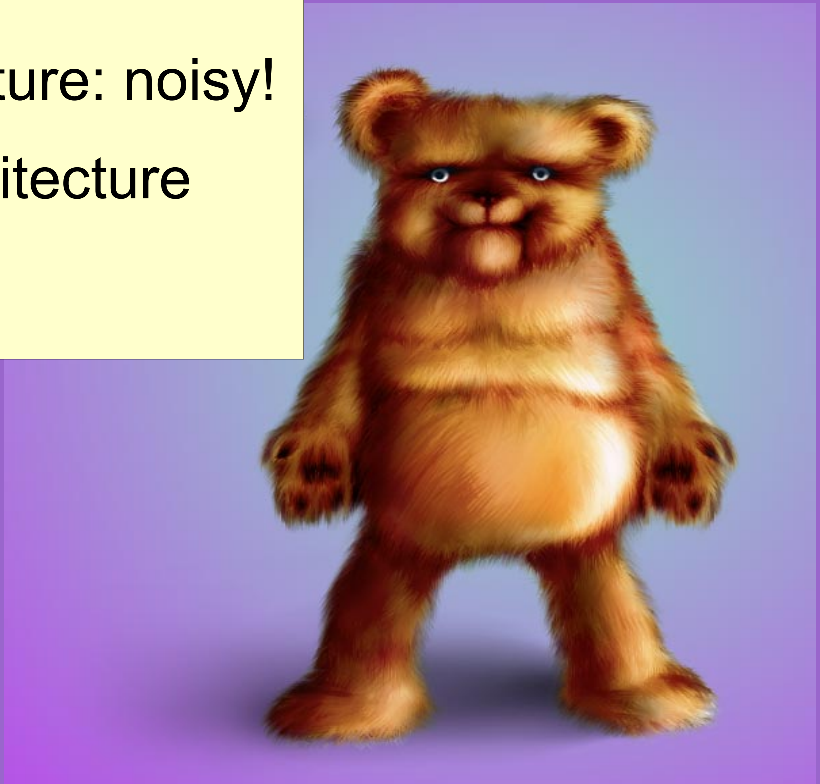
Outline: QRAM

Abstract:

We present a **protocol to address a memory array** using a **quantum register as index register**. This permits the adressing of **arbitrary quantum superpositions of memory cells**. If the memory array is classical, a small quantum computer suffices to implement the protocol: the **array can be exponentially larger than the required quantum resources**.

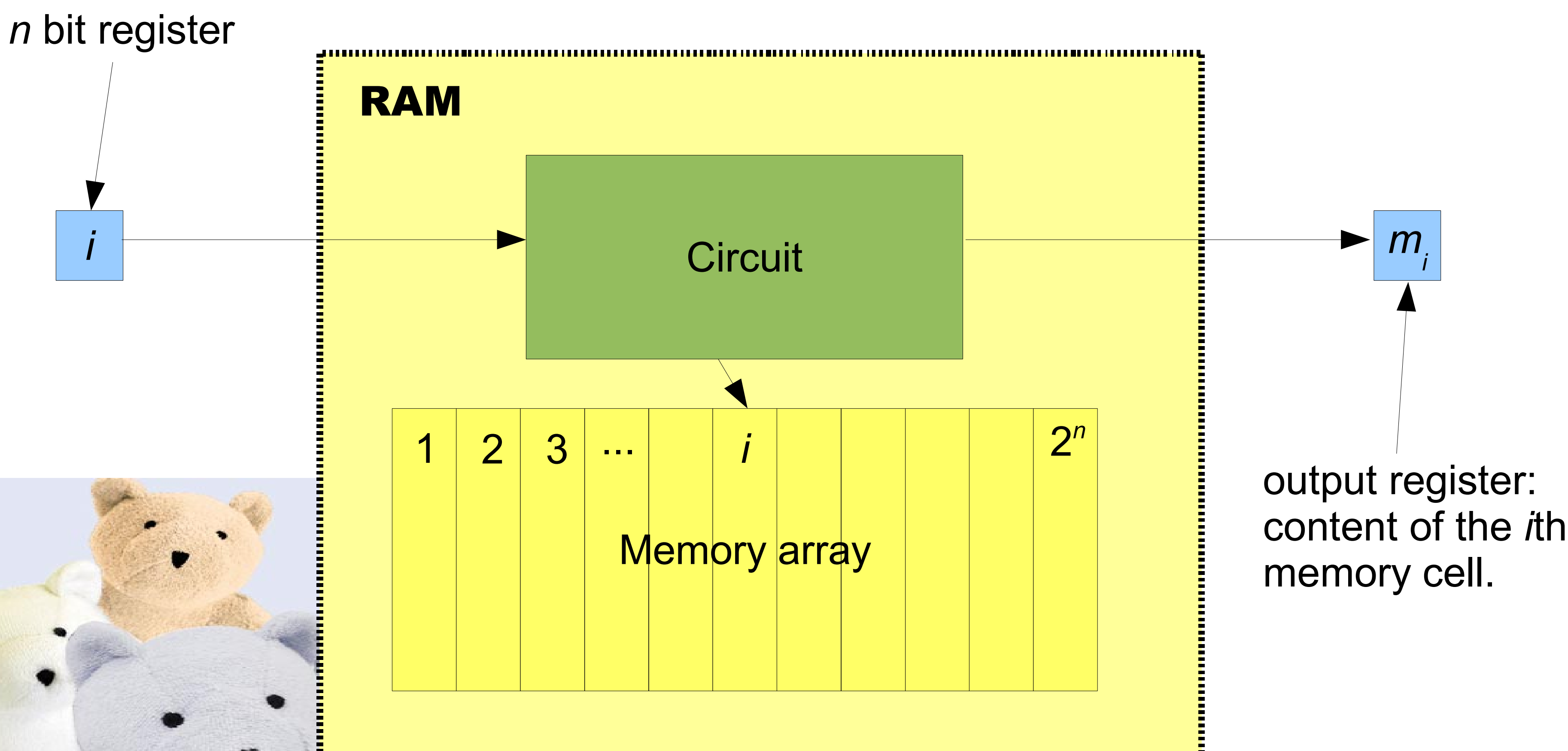
Outline:

- Ram
- quantum RAM
- conventional architecture: noisy!
- “Bucket brigade” architecture
- implementations



Random Access Memory (RAM)

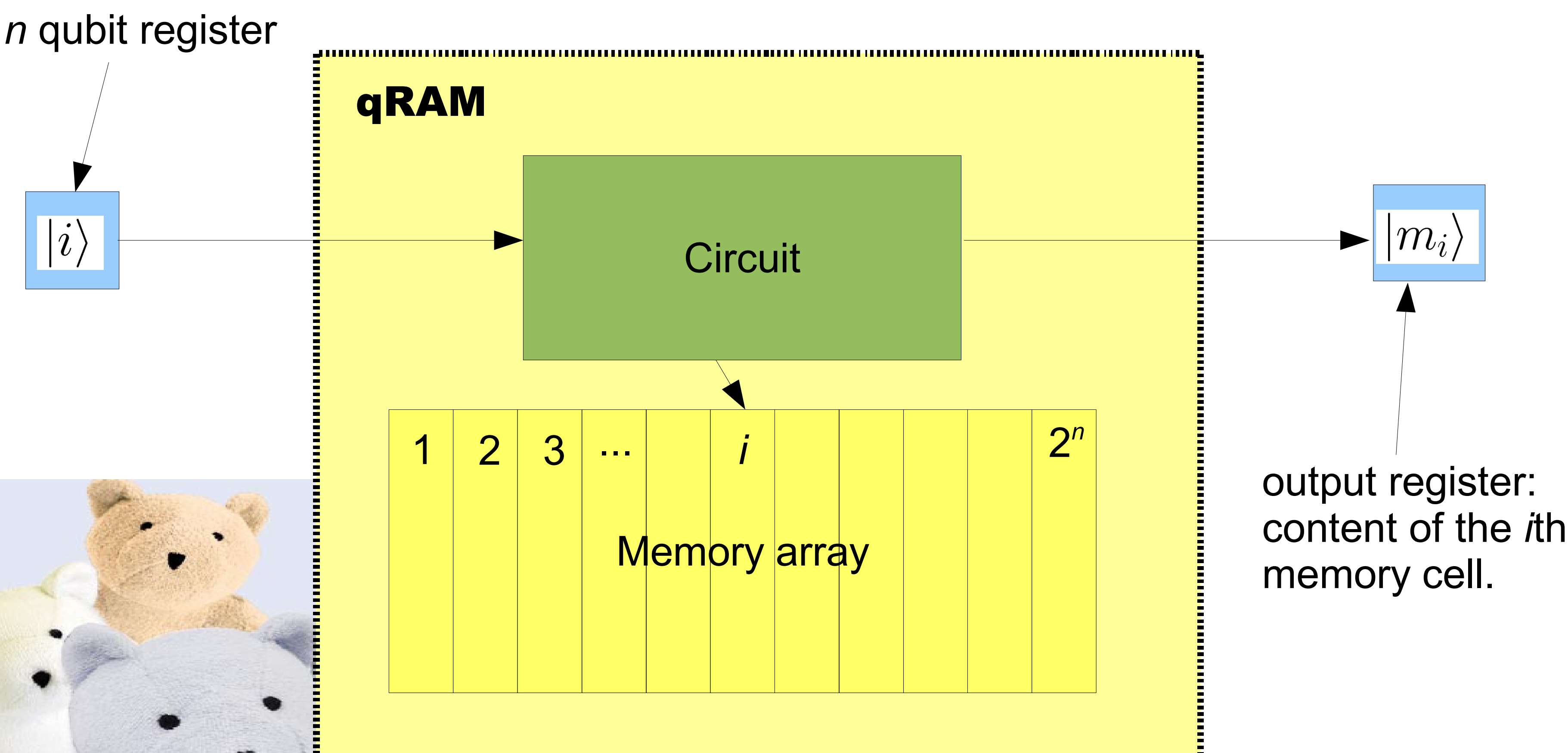
Each cell of a memory array can be univocally determined by its numerical address. An n-bit register can then address 2^n possible locations:



takes an n bit address as input,
provides the content of the memory cell as output

quantum Random Access Memory (qRAM)

Same as the previous, but the **index** and **output** registers are made of **qubits**!
(Eventually also is the memory array):



takes an n qubit address as input,
provides the content of the memory cell as output

...this has important consequences!

Suppose that the input register is in a superposition of querying for *N* different addresses:

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}|i\rangle \xrightarrow{\textbf{qRAM}} \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|i\rangle|m_i\rangle$$

Now the output register is **ENTANGLED** with the input register!

Is the qRAM useful?

YES!

It is explicitly or implicitly invoked in MANY known quantum algorithms:

•Quantum searching in a classical database

•Collision finding

•Element distinctness (in the classical and quantum settings)

•Pattern recognition

•New algorithms for evaluating general NAND trees

It's useful also for NEW algorithms we are developing:

•Quantum private queries (interrogate a database securely)

•Quantum routing (route signals in a quantum internet)

How can we build a qRAM?

1. Conventional architectures: modify the circuits for classical RAMs

not good! → **Difficult** to implement and **noisy**

2. Bucket brigade architecture

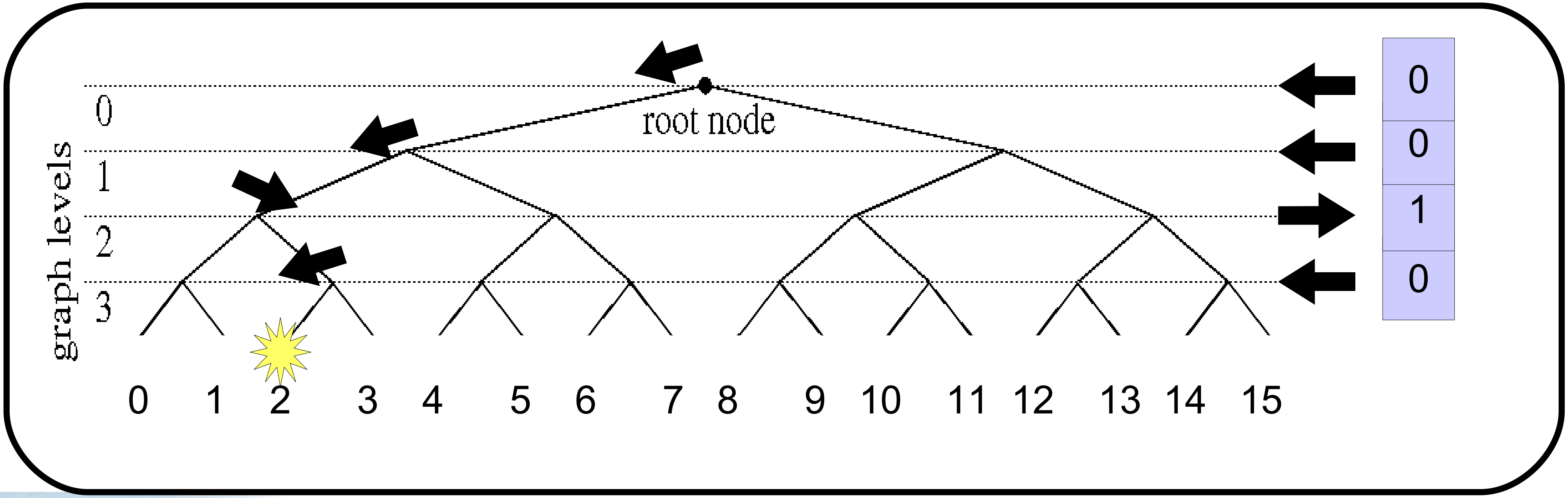
Solves the main problems of implementing a qRAM

Internal workings of conventional RAMs

E.g. address register ="2"=

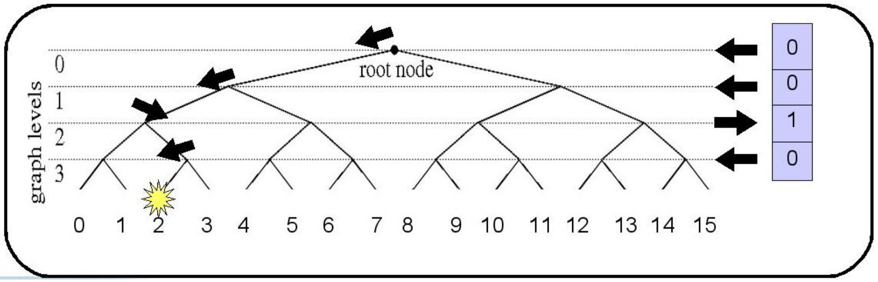
0	0	1	0

each register bit deviates the signal in one level of a bifurcation tree:

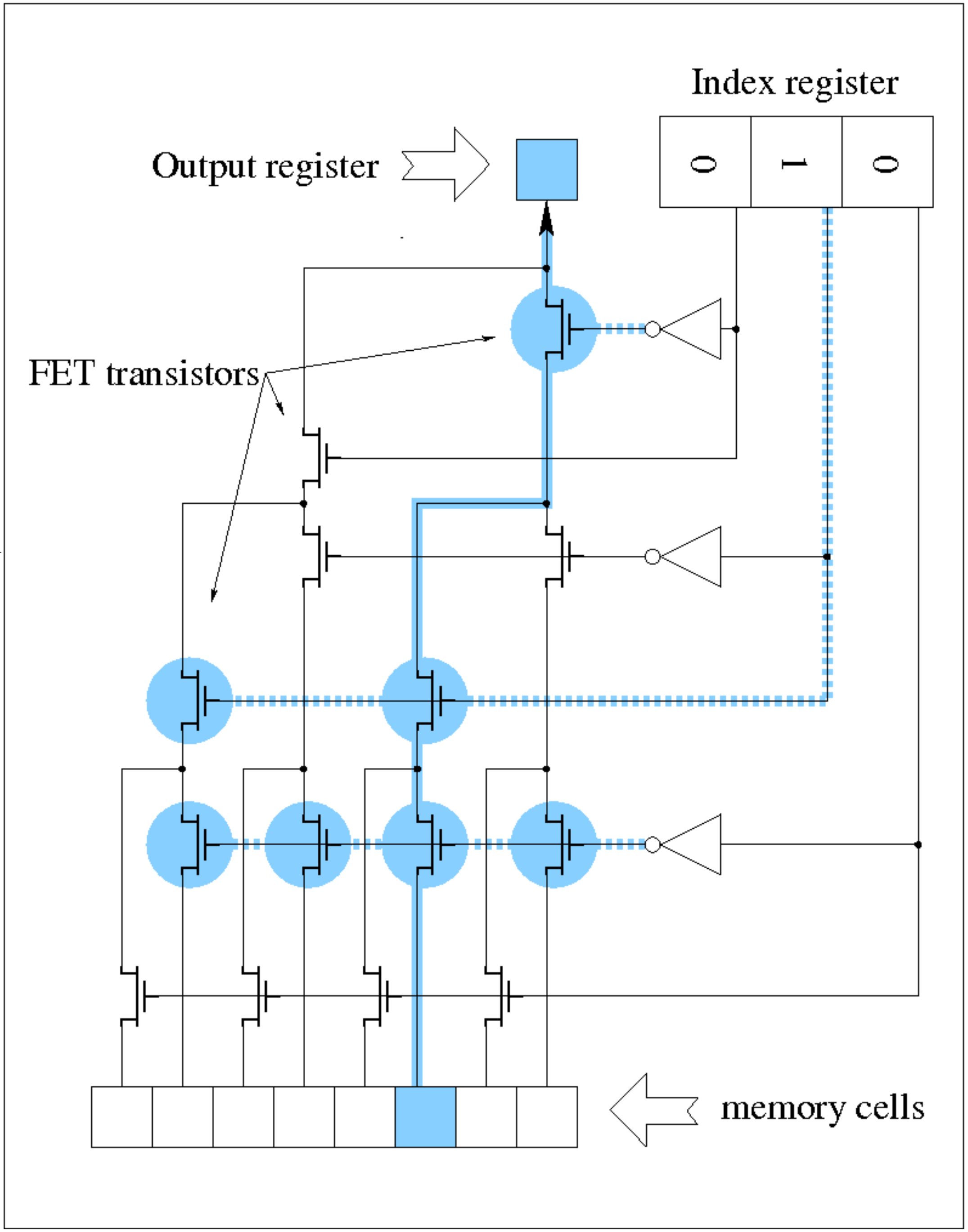


Internal workings of conventional RAMs 2

This ideal structure

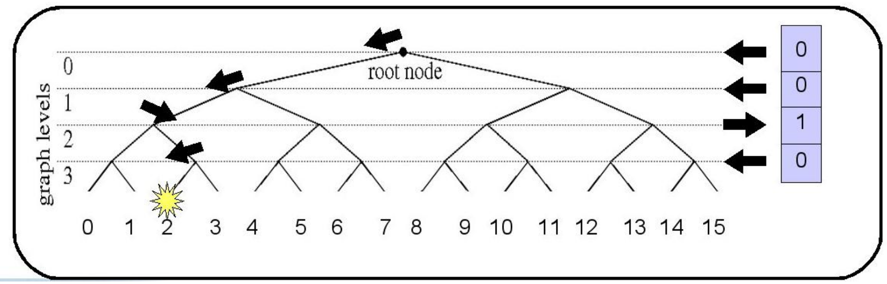


is almost faithfully copied in circuit diagrams of conventional RAMs

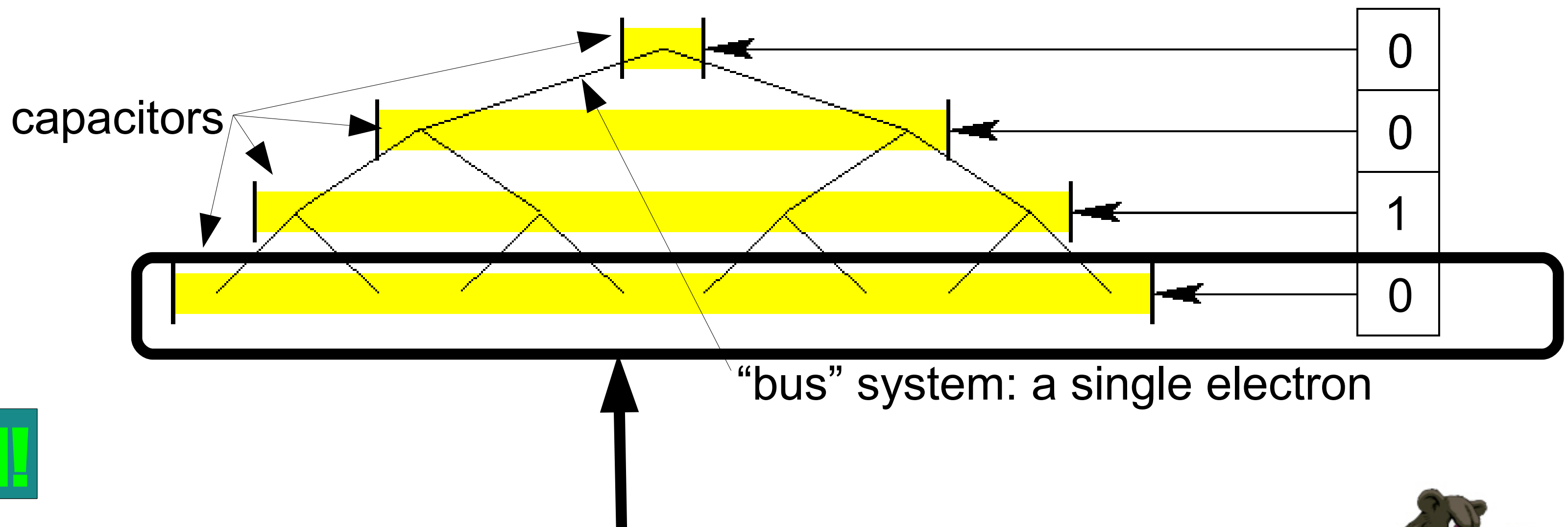


qRAMs proposals

...and was also proposed for building a qRAM [Nielsen and Chuang]



e.g.



PROBLEM!

The last few graph levels are BIG Schroedinger cats!!

It's difficult to maintain the necessary quantum coherence

$\Rightarrow$ **Impractical**

Bucket brigade

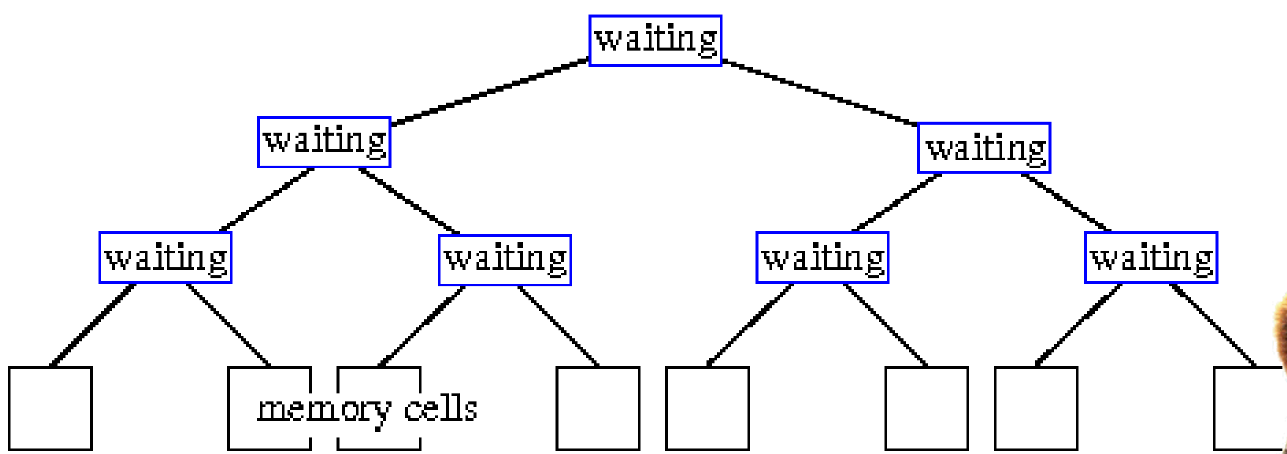
there's another way to implement the binary decision tree

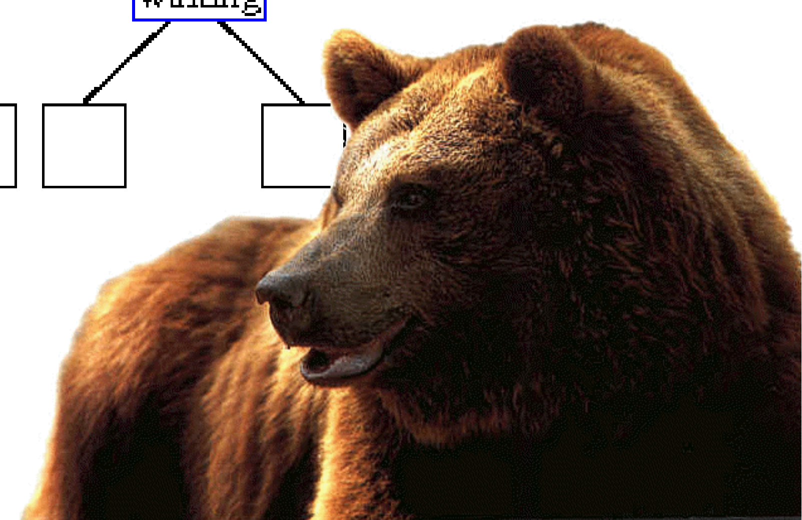
bucket brigade protocol

Put trit memory elements in the tree nodes.

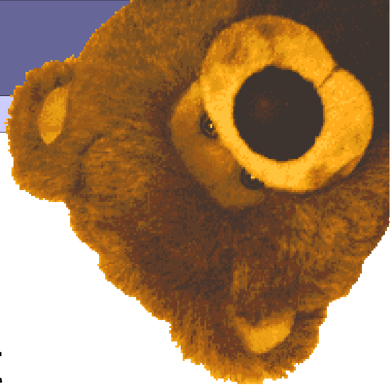
waiting | left | right

1. Initialize all of them in the “waiting” state:





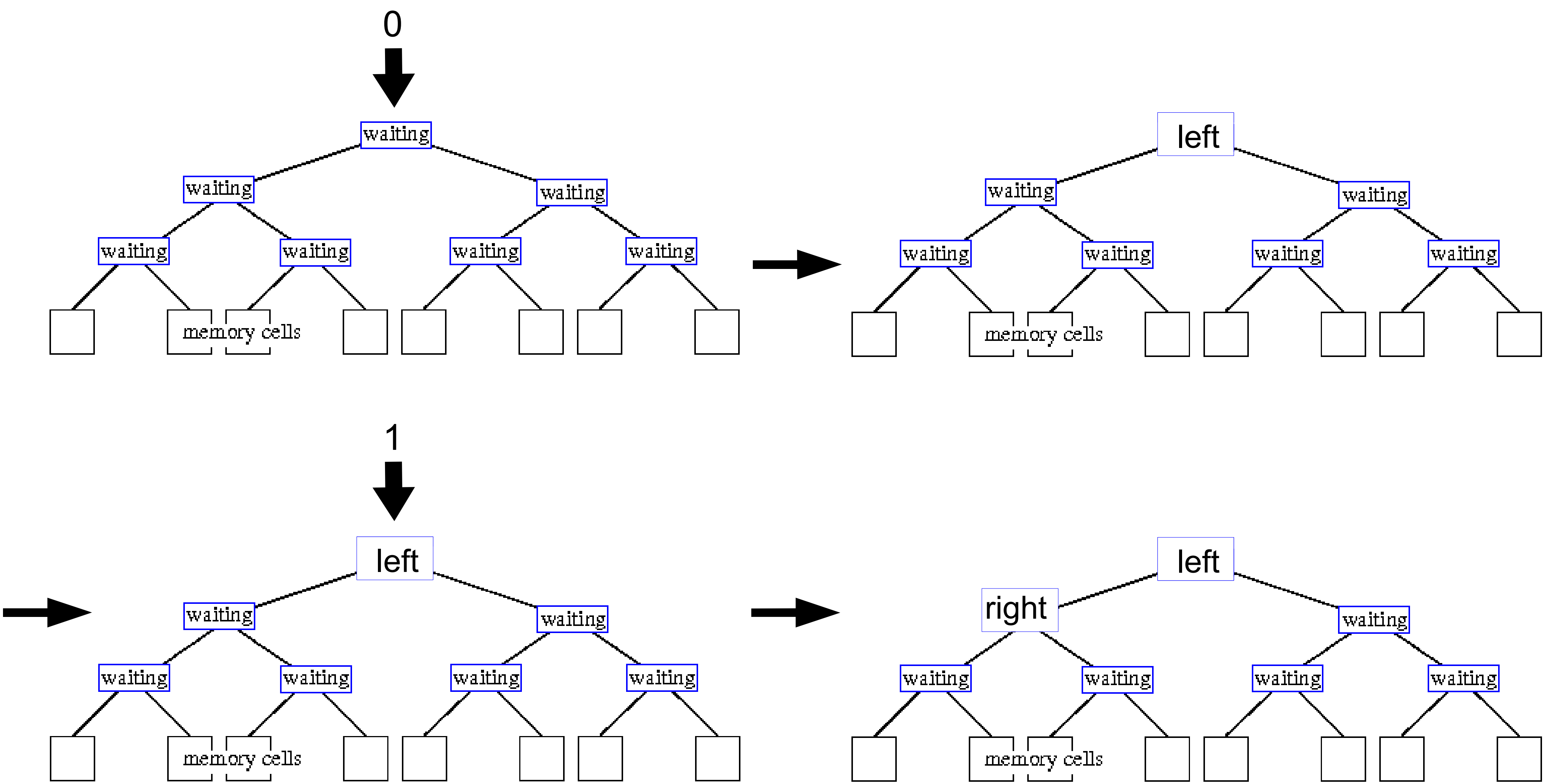
Bucket brigade 2



2. send the address bits in the network one by one

When it encounters a “waiting” trit a bit “0” becomes a “left” trit

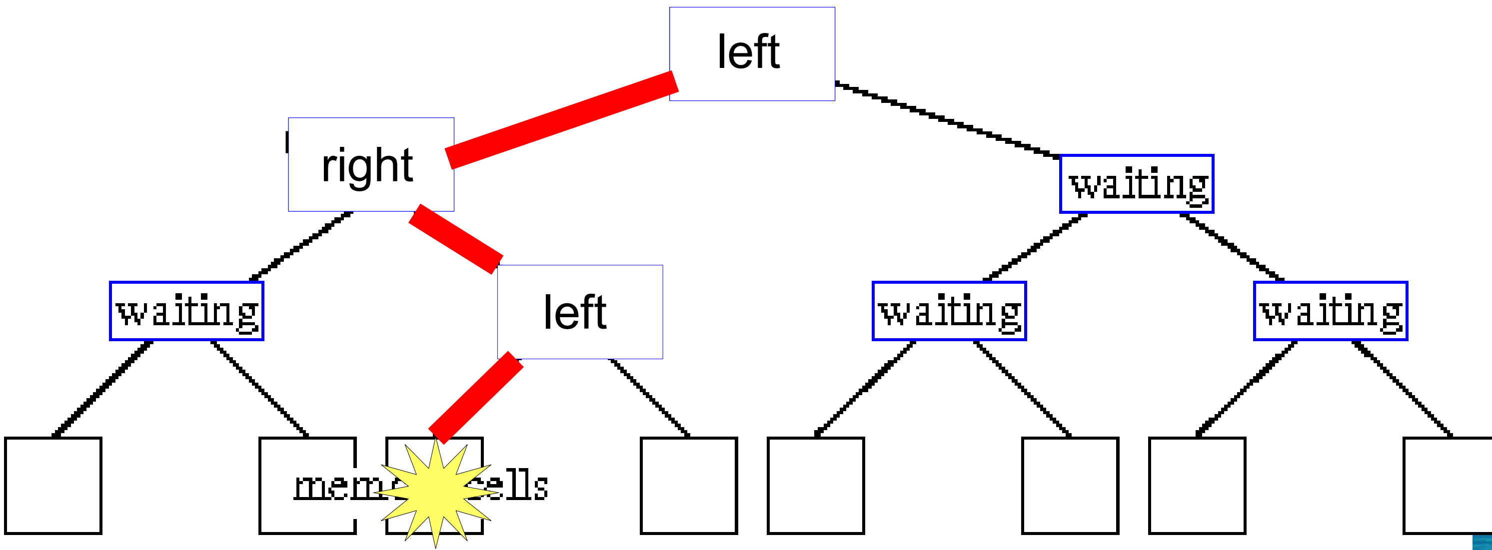
a bit “1” becomes a “right” trit



Bucket brigade 3

3. when all the address bits have been sent, there's a route carved in the tree:

e.g. for address register = 010:



4. extract the information through this route.

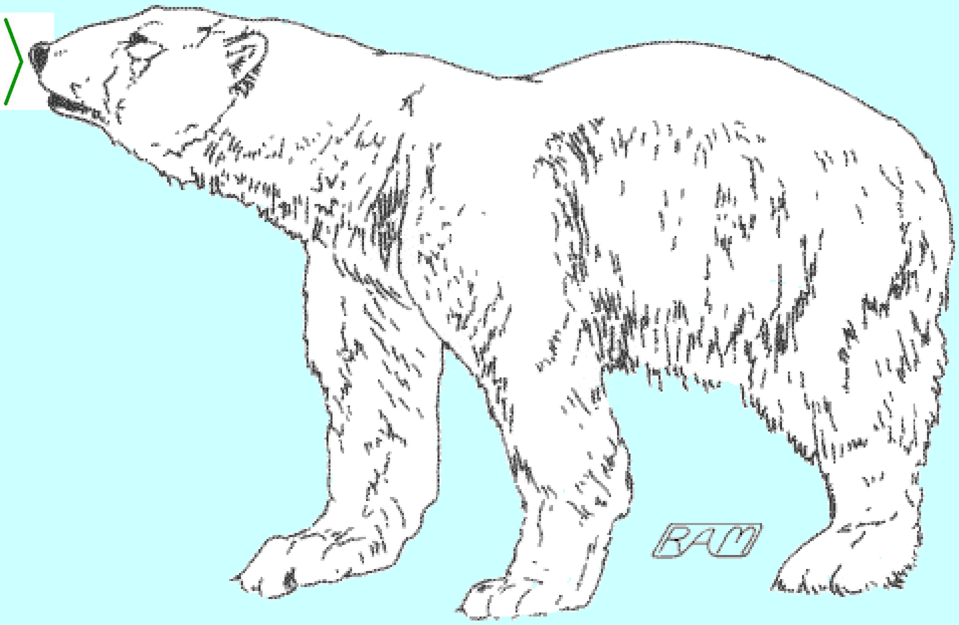
bucket brigade qRAM

It's very simple to quantize this RAM architecture:

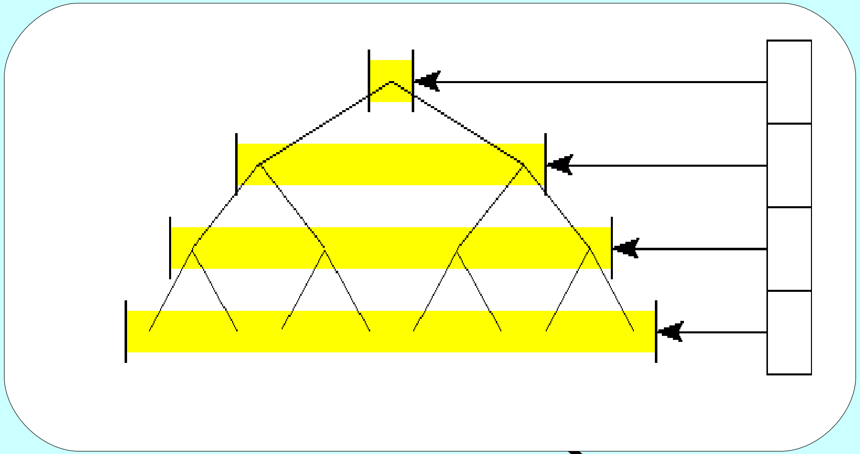
just **use qutrits** in place of trits (and little else)

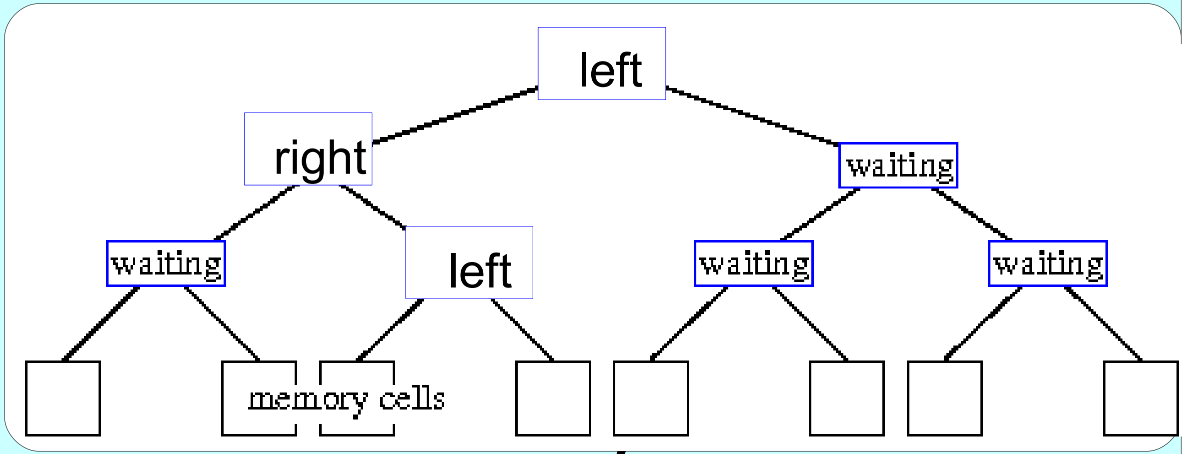
three-level quantum systems

$|waiting\rangle$ $|left\rangle$ $|right\rangle$



Resource accounting





	Conventional RAM arch.	Bucket brigade
coherent control	2^n	n
q. memories	$\boxed{n}$	$\boxed{2^n}$

index register

qutrits

The difficulty of maintaining coherence over many gates is traded for quantum memories.

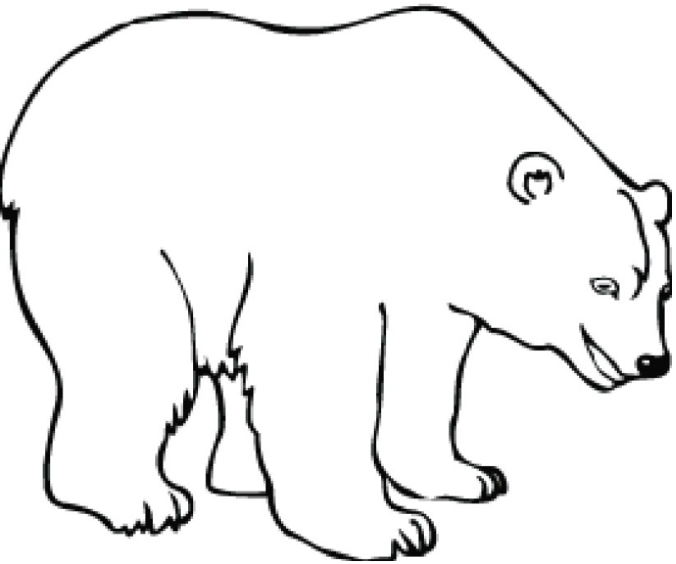
Resource accounting 2

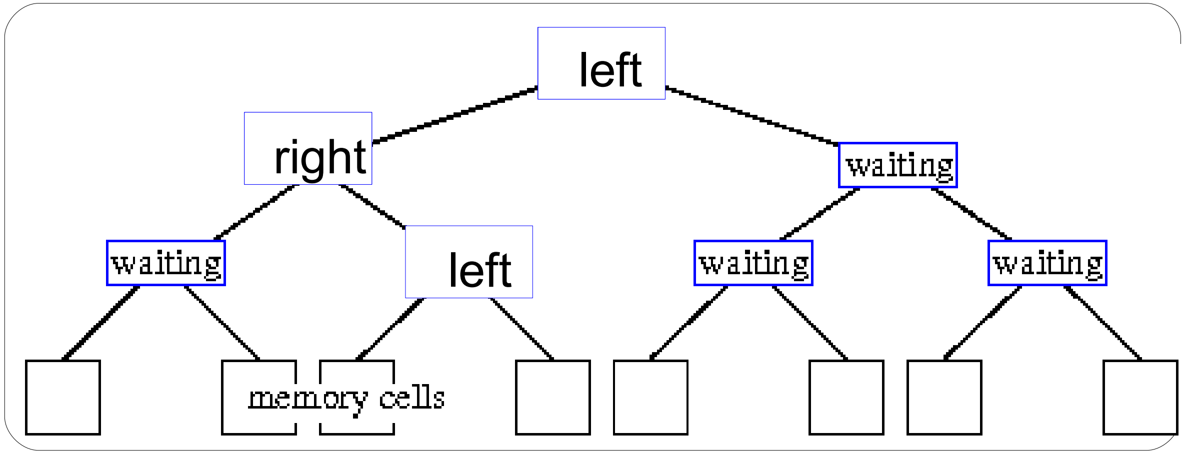
HOWEVER, note that:

1. O(2^n) memory elements are necessary **ANYHOW**, when the memory array is composed of quantum memories

2. most of the qutrits are always in the “waiting” state. If it is chosen appropriately, there is <u>**very little noise**</u> in the bucket brigade!

just choose the “waiting” state as a ground state.

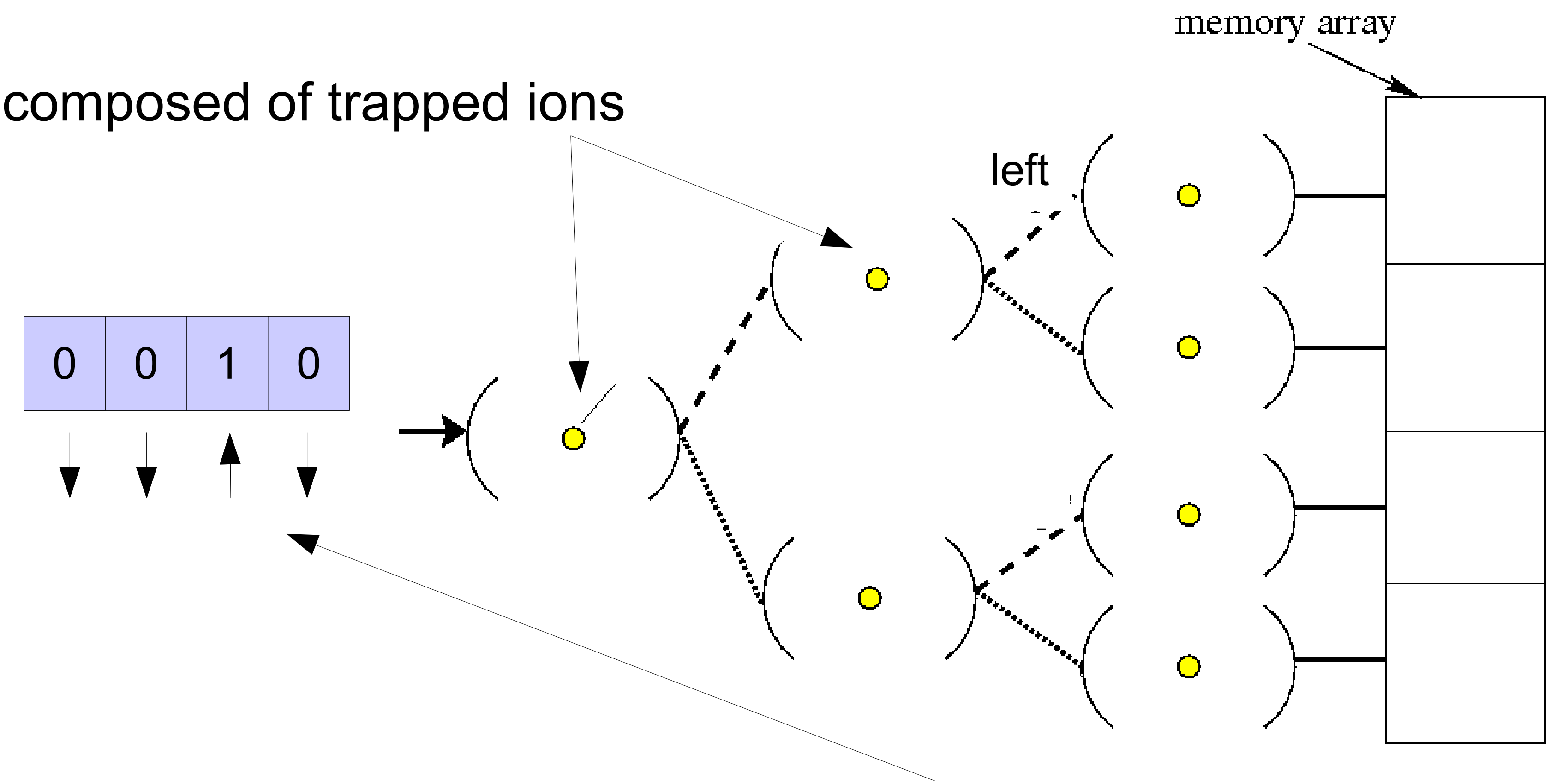




$\Rightarrow$ BB is more suited for qRAMs!

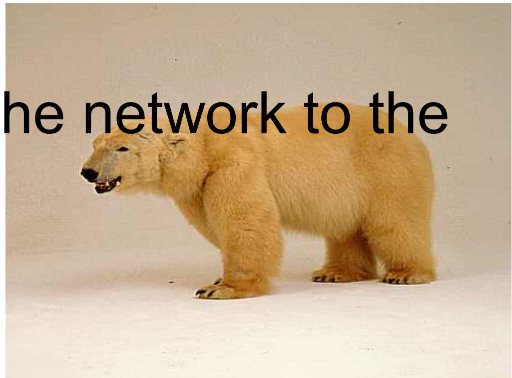
Implementation

• Qutrits composed of trapped ions

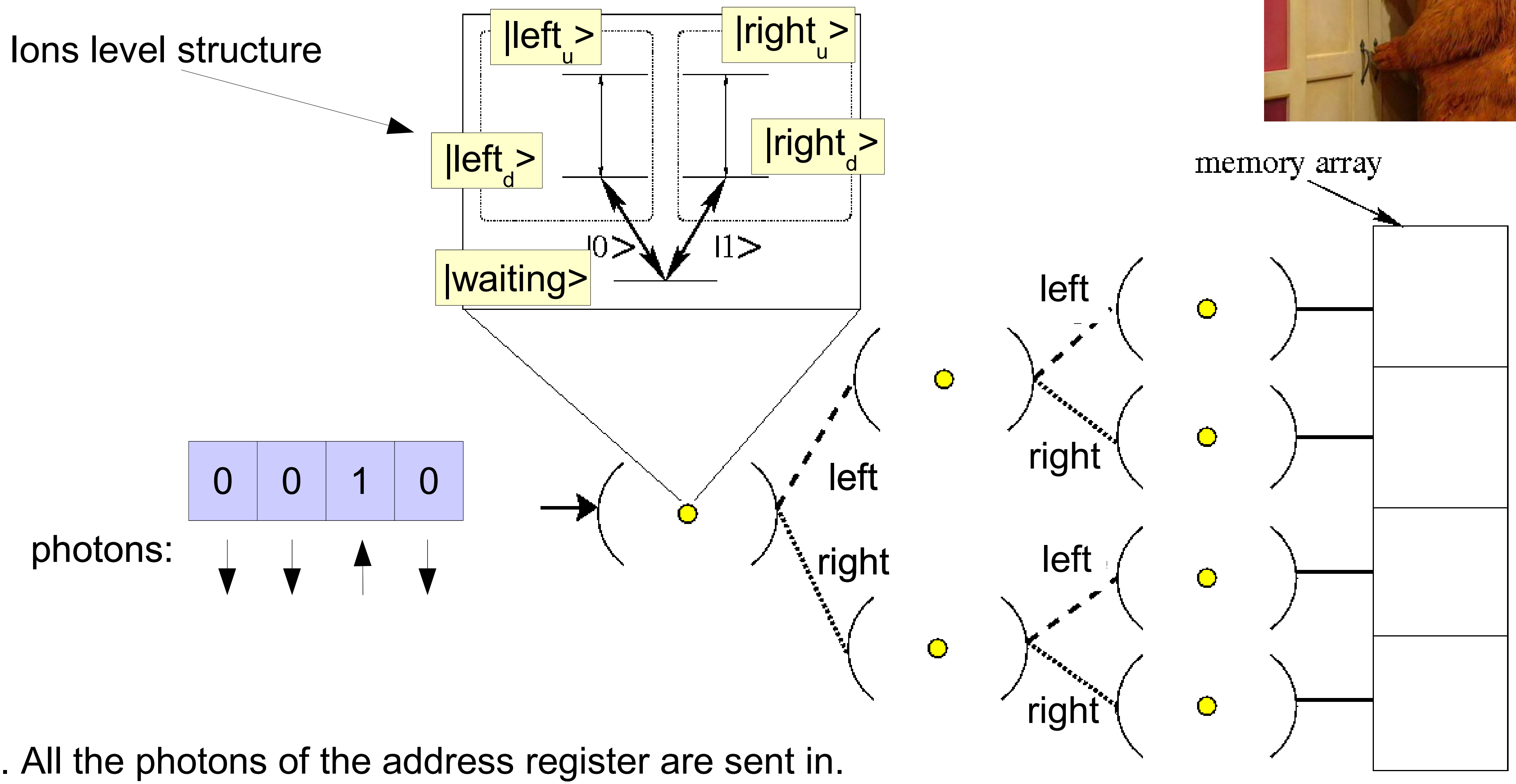


• Address register stored in the polarization of photon states

• “Bus” system composed of a photon, which flies through the network to the selected memory cell, stores its contents and flies back.



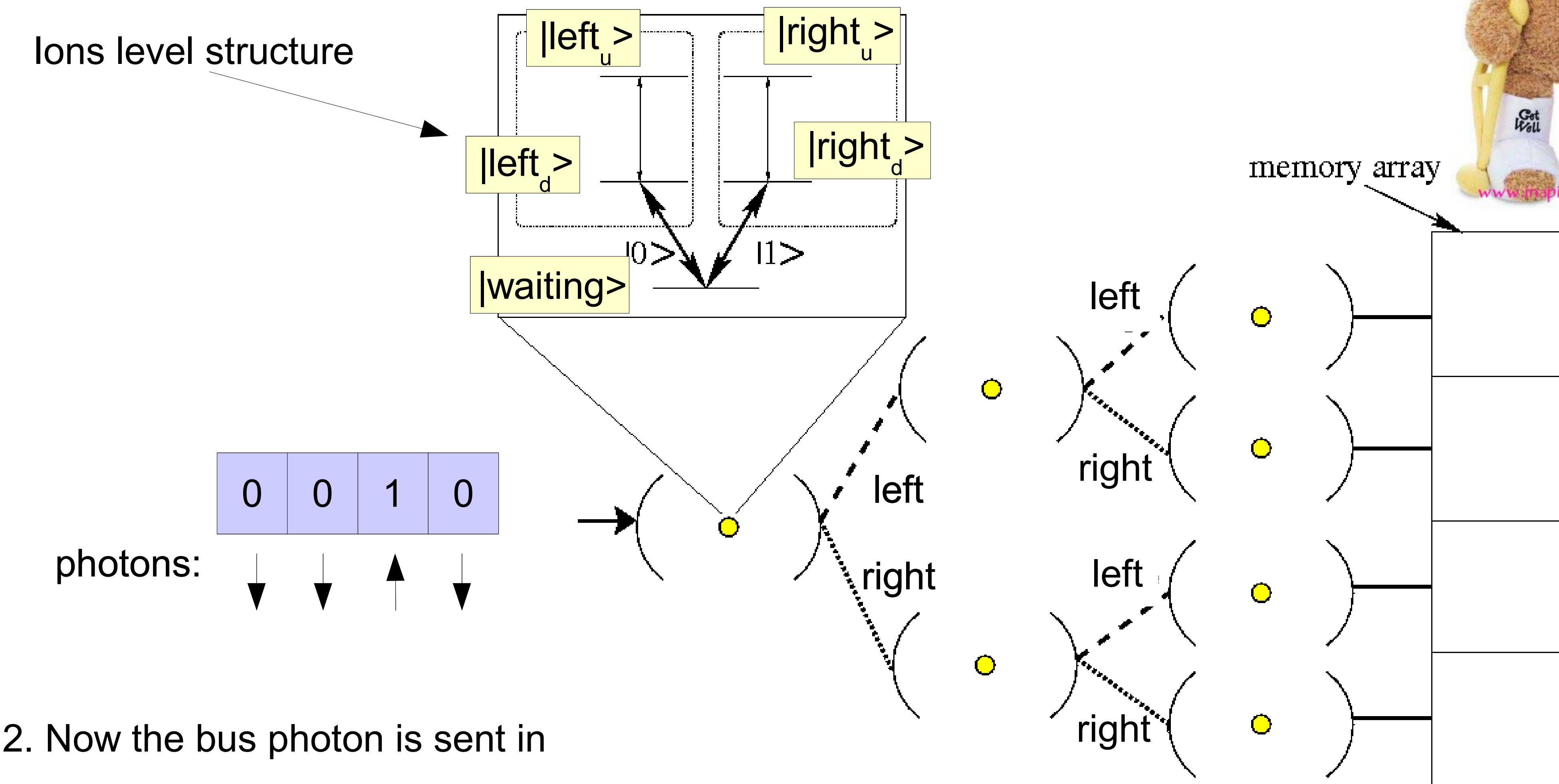
Implementation 2



1. All the photons of the address register are sent in.

- if they encounter a ion in |waiting>, they are stored either in level |left$_d$> or |right$_d$>, depending on their polarization.
- if they encounter a ion in |left$_d$> or |right$_d$>, they are re-emitted (using a Raman transition mediated through |left$_u$> and |right$_u$>) and continue along the tree, to the left or to the right, respectively (the |left$_x$> and |right$_x$> levels are spatially coupled only to the left and right outgoing modes).

Implementation 3



2. Now the bus photon is sent in

It follows the path carved by the address register photons (stored in the ions).

It reaches the memory cell, where it copies (or swaps) its content.

It is reflected back and exits at the tree root node with the memory cell content.

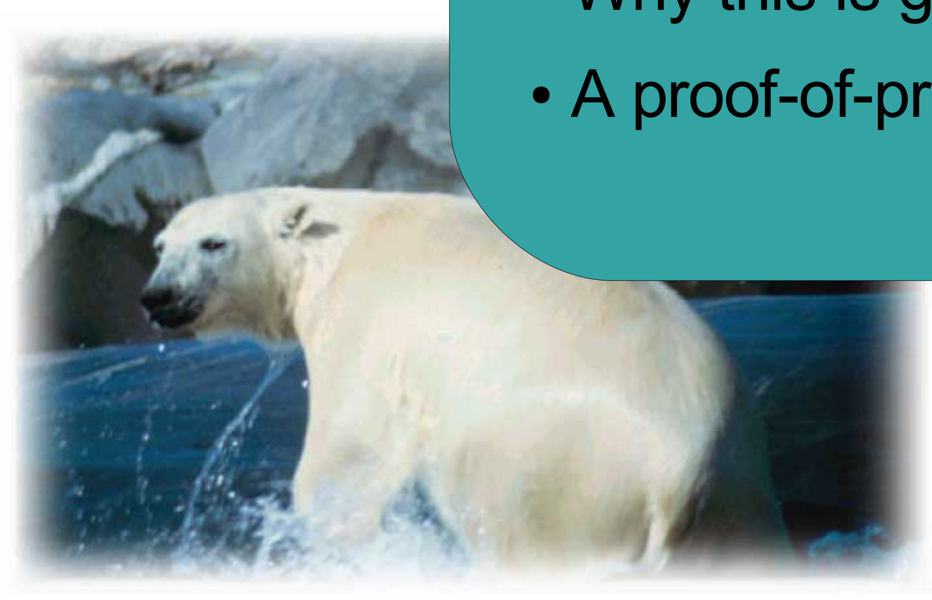
3. The address register photons are re-emitted one by one (starting from the last nodes in the tree

4. The memory has been accessed, the address register reobtained, and the network reset.

Conclusions

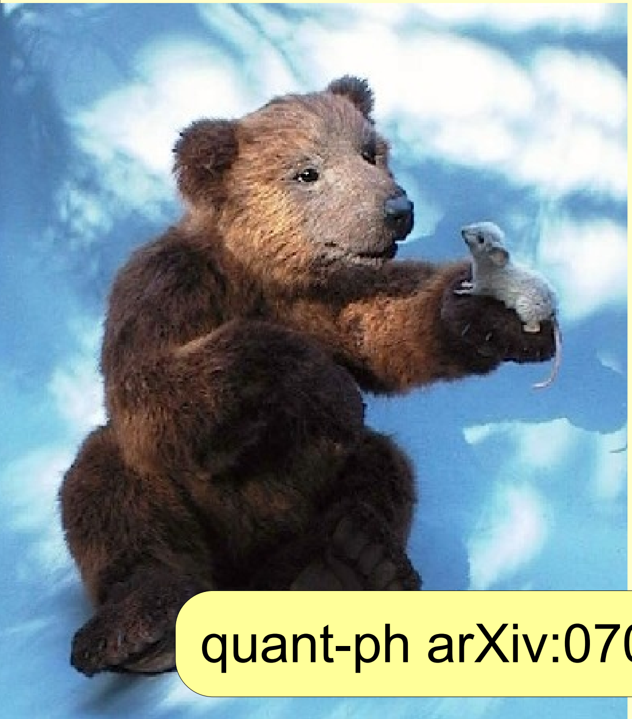
We have seen:

- What is a Random Access Memory
- What is a Quantum RAM
- How conventional RAMs work
- Why this isn't good for quantum computers
- The Bucket Brigade protocol
- Why this is good for quantum computers
- A proof-of-principle BB implementation



Take home message

A new architecture for random access memories, which is particularly suitable for quantum computers.



quant-ph arXiv:0708.1879

Comments and questions to

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