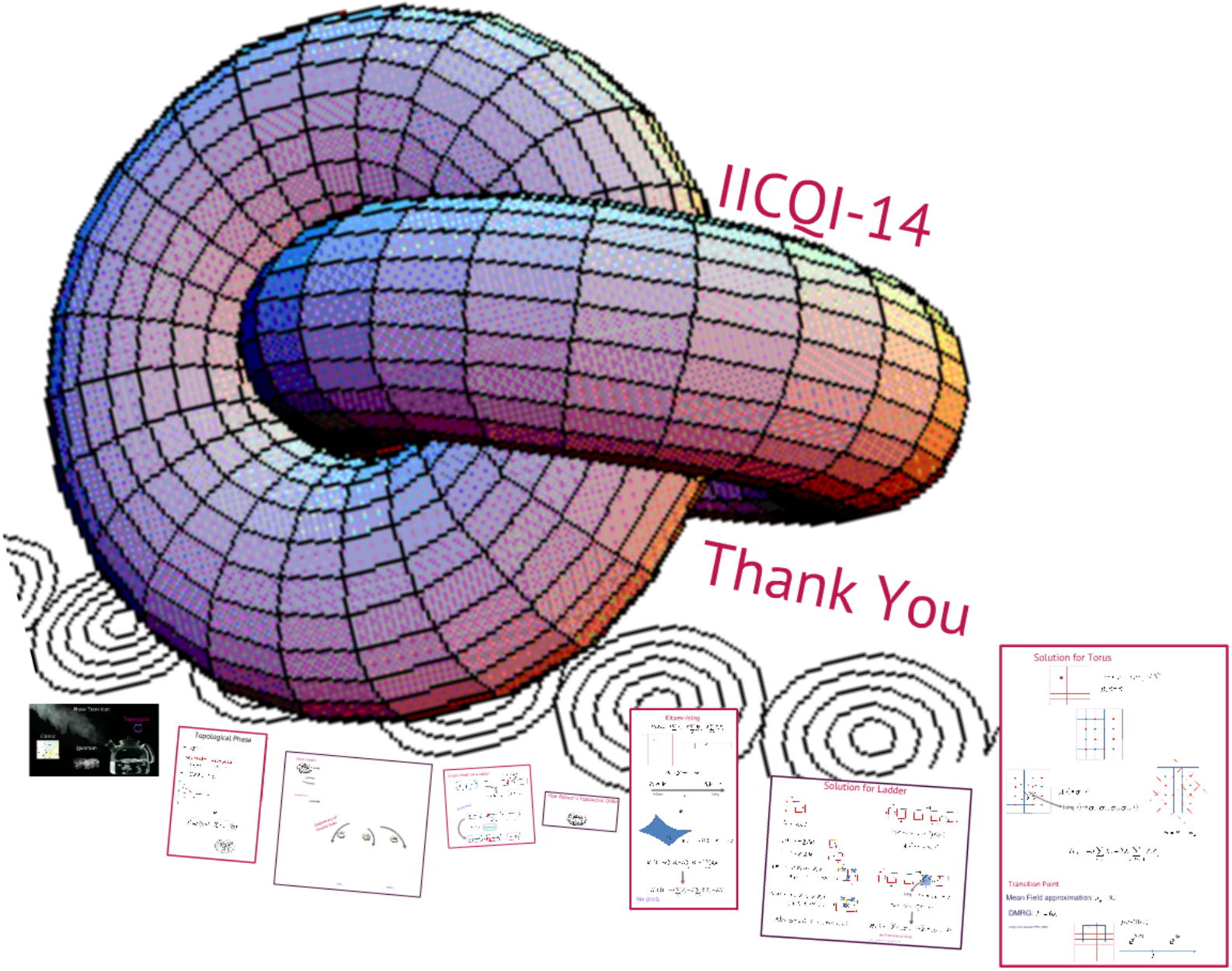

IICQI-14
Thank You
Phase Transition
Topological
CDW2
quantum
Topological Phase
Kitaev-Ising
Solution for Ladder
Solution for Torus
Transition Point
Mean Field approximation
DMRG

Transition between Topological and Ferromagnetic order in Kitaev-Ising model

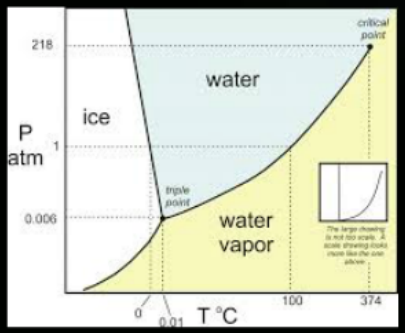
Laleh Memarzadeh
Sharif University of Technology

IICQI-14 Esfahan, Iran

Phase Transition

Topological

Classic



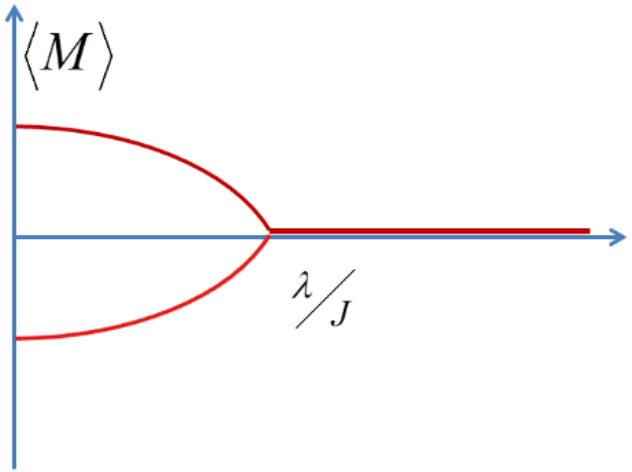
Quantum

Topological Phase

$$H = -J\sum_i Z_i Z_{i+1}$$

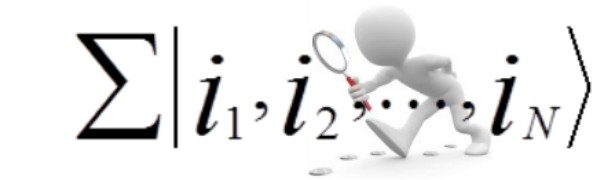
$$\langle M \rangle = 1 \qquad \langle M \rangle = -1$$

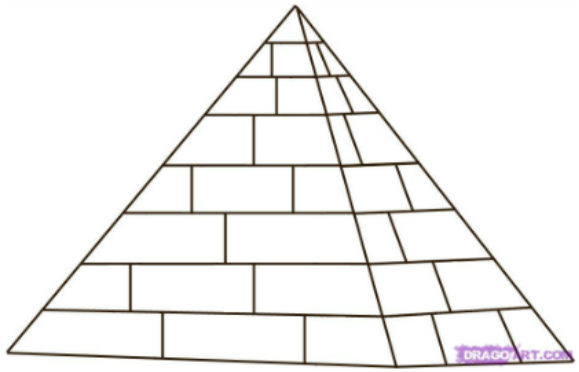
$$H = -J\sum_i Z_i Z_{i+1} - \lambda\sum_i X_i$$

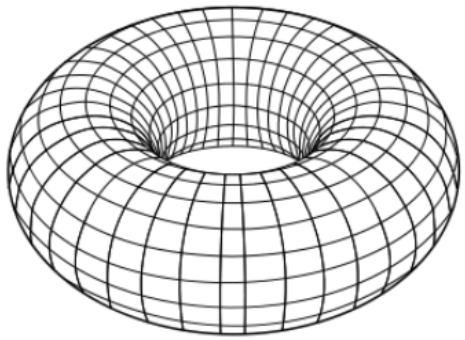


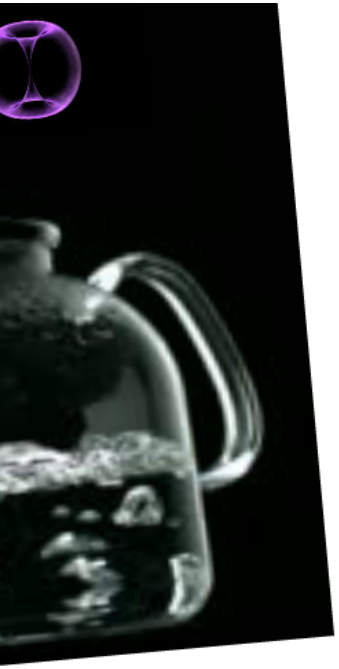
$$H$$

$$\sum \left| i_1, i_2, \cdots, i_N \right\rangle \qquad \sum \left| i_1, i_2, \cdots, i_N \right\rangle$$







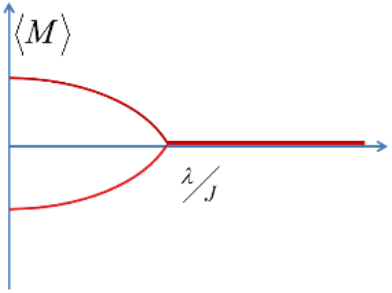


Topological Phase

$$H = -J\sum_i Z_i Z_{i+1}$$

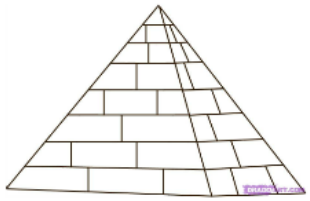
$$\langle M \rangle = 1 \qquad \langle M \rangle = -1$$

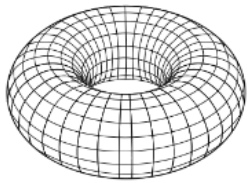
$$H = -J\sum_i Z_i Z_{i+1} - \lambda\sum_i X_i$$



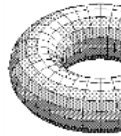
$$H$$

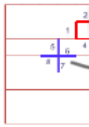
$$\sum |i_1, i_2, \cdots, i_N\rangle \qquad \sum |i_1, i_2, \cdots, i_N\rangle$$





Kitaev

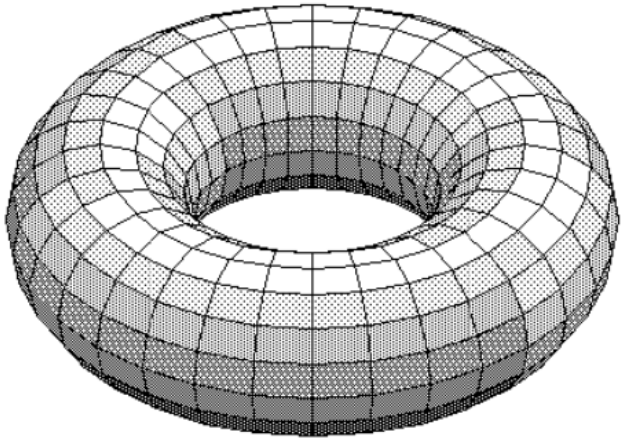


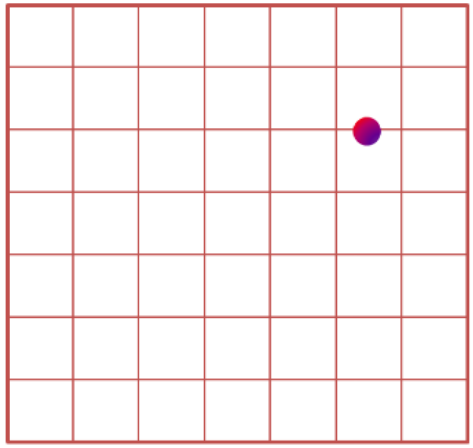


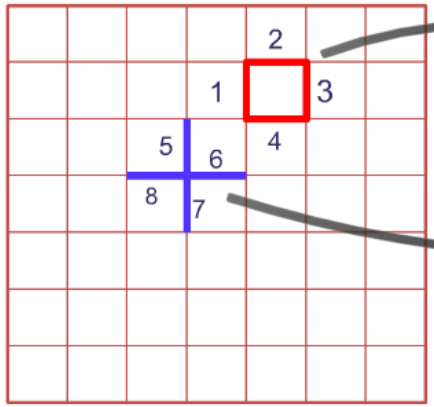
Ground S

$[A_v, B_p]$

Kitaev Model





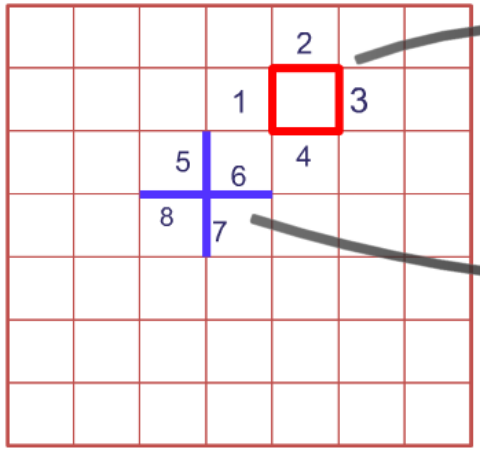


$$B_p = Z_1 Z_2 Z_3 Z_4$$

$$A_s = X_5 X_6 X_7 X_8$$

$$A_s^{\,2} = I \qquad B_p^{\,2} = I \qquad \left[A_s, B_p \right] = 0$$

$$H = -\sum_s A_s - \sum_p B_p$$



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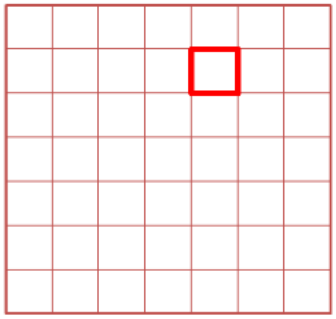
$$A_s^{\,2} = I \qquad B_p^{\,2} = I \qquad \left[A_s, B_p \right] = 0$$

$$H = -\sum_s A_s - \sum_p B_p$$

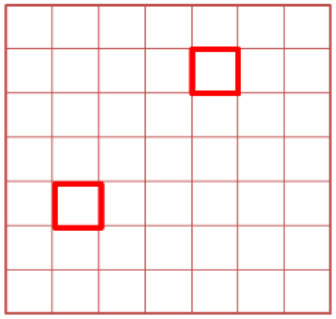
Ground State

$$\left[A_s, B_p \right] = 0 \longrightarrow A_s |\psi\rangle = |\psi\rangle \qquad B_p |\psi\rangle = |\psi\rangle$$

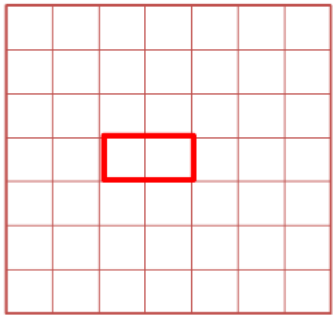
$$H|\psi\rangle = -(N_v + N_p)|\psi\rangle$$



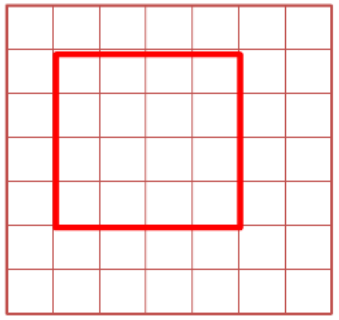
$B_p|+\rangle^{\otimes N}$



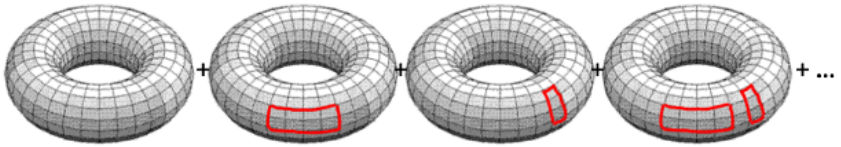
$B_p B_{p'}|+\rangle^{\otimes N}$

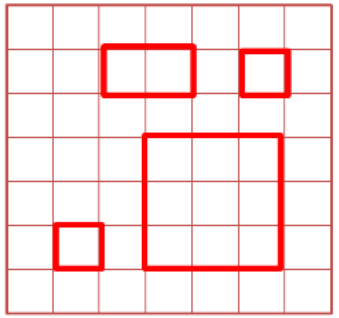


$B_p B_{p'}|+\rangle^{\otimes N}$

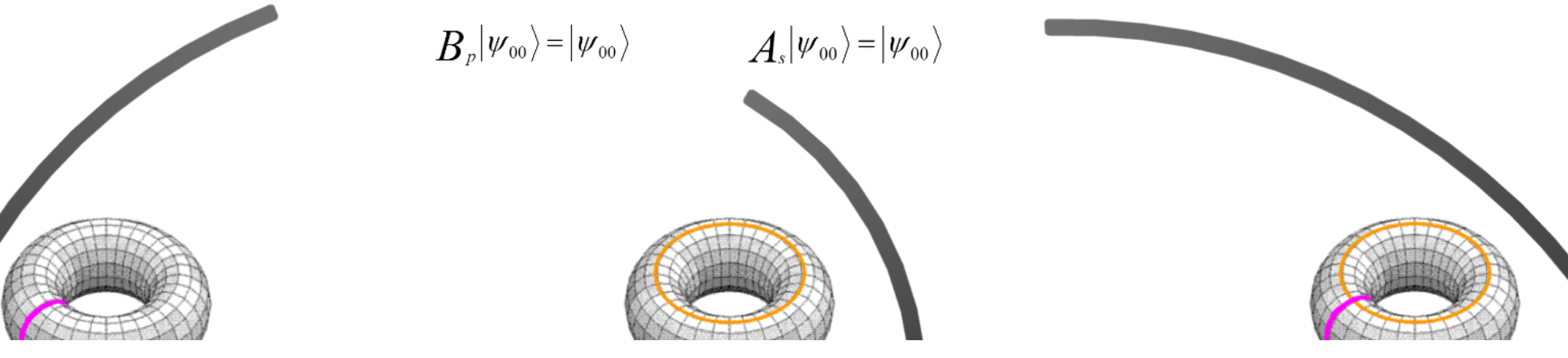


$$|\psi_{00}\rangle = \sum_{Loops} |\square\rangle$$





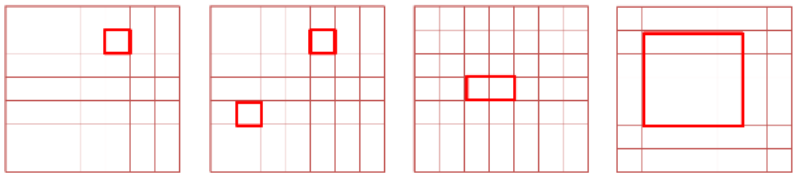
$B_p|\psi_{00}\rangle = |\psi_{00}\rangle$ $A_s|\psi_{00}\rangle = |\psi_{00}\rangle$



$-\sum_s A_s - \sum_p B_p$

$A_s|\psi\rangle = |\psi\rangle \qquad B_p|\psi\rangle = |\psi\rangle$

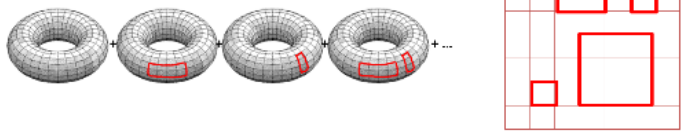
$-(N_v + N_p)|\psi\rangle$



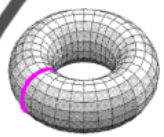
$B_p|\rangle^{\otimes N} \qquad B_p B_{p'}|+\rangle^{\otimes N} \qquad B_p B_{p'}|+\rangle^{\otimes N}$

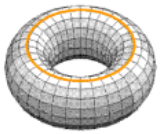
Degeneracy of Ground State

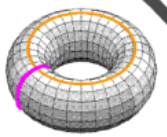
$|\psi_{00}\rangle = \sum_{Loops} |\,\rangle$

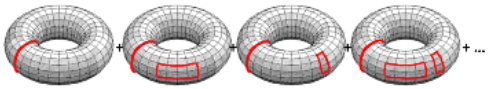


$B_p|\psi_{00}\rangle = |\psi_{00}\rangle \qquad A_s|\psi_{00}\rangle = |\psi_{00}\rangle$

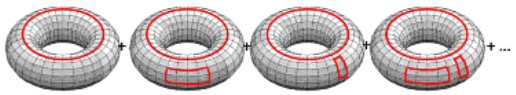




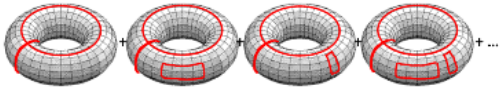




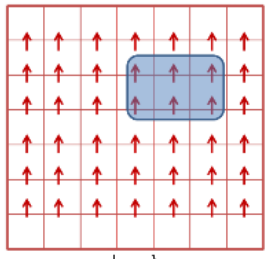
$|\psi_{01}\rangle = \sum_{Loops} |\,\rangle$



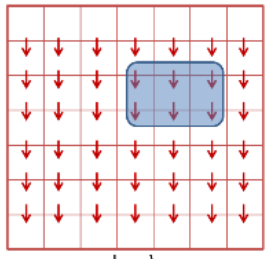
$|\psi_{10}\rangle = \sum_{Loops} |\,\rangle$



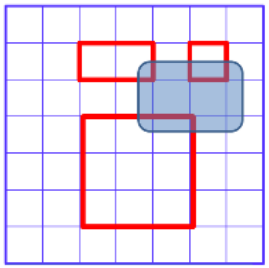
$|\psi_{11}\rangle = \sum_{Loops} |\,\rangle$



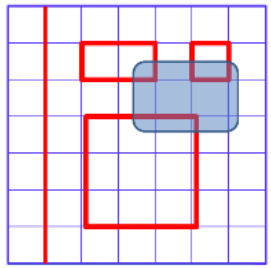
$|\varphi_0\rangle$



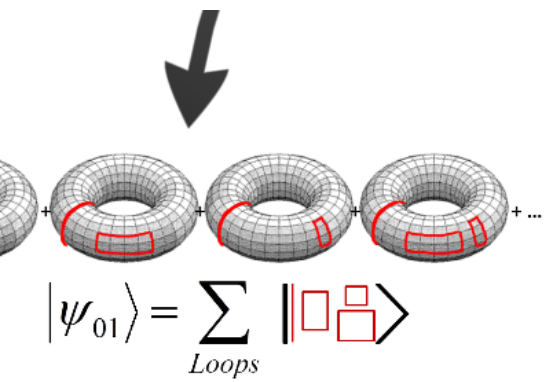
$|\varphi_1\rangle$



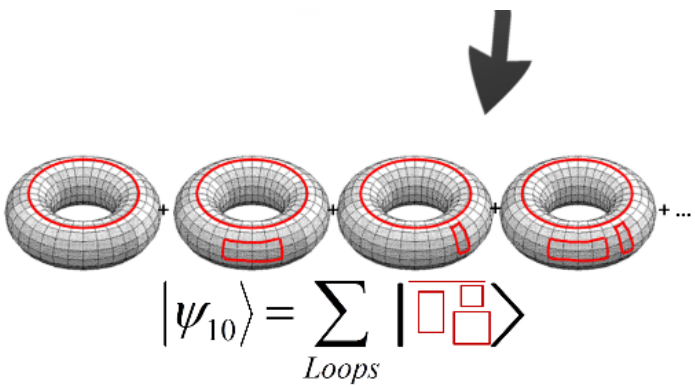
$|\psi_{00}\rangle$



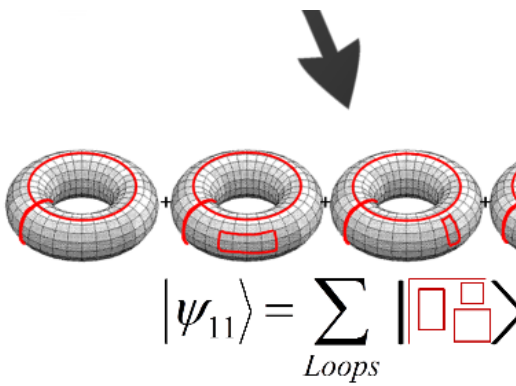
$|\psi_{01}\rangle$



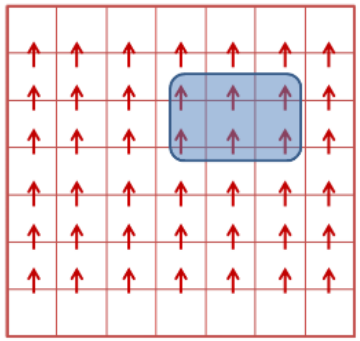
$$|\psi_{01}\rangle = \sum_{Loops} |$$ $$\rangle$$



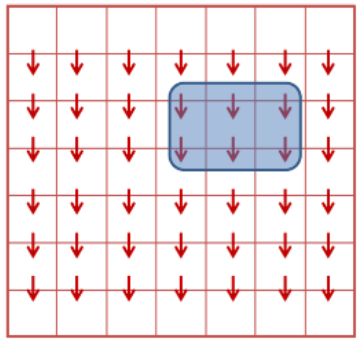
$$|\psi_{10}\rangle = \sum_{Loops} |$$ $$\rangle$$



$$|\psi_{11}\rangle = \sum_{Loops} |$$ $$\rangle$$

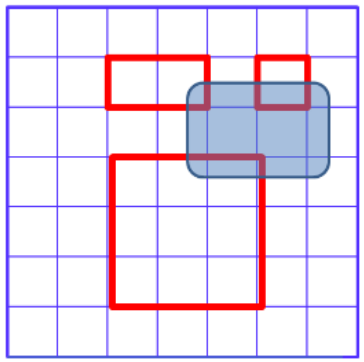


$|\varphi_0\rangle$

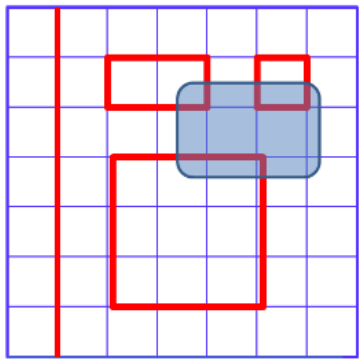


$|\varphi_1\rangle$

Ising



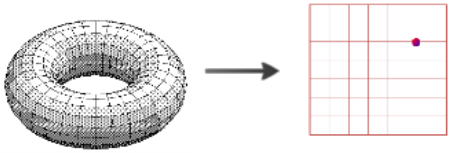
$|\psi_{00}\rangle$

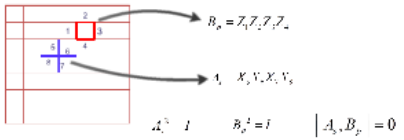


$|\psi_{01}\rangle$

Kitaev

Kitaev Model



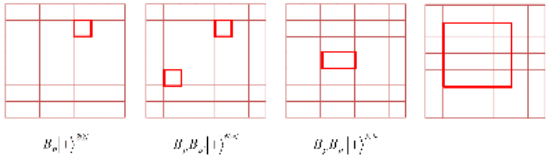


$$H = \sum_s A_s \sum_p B_p$$

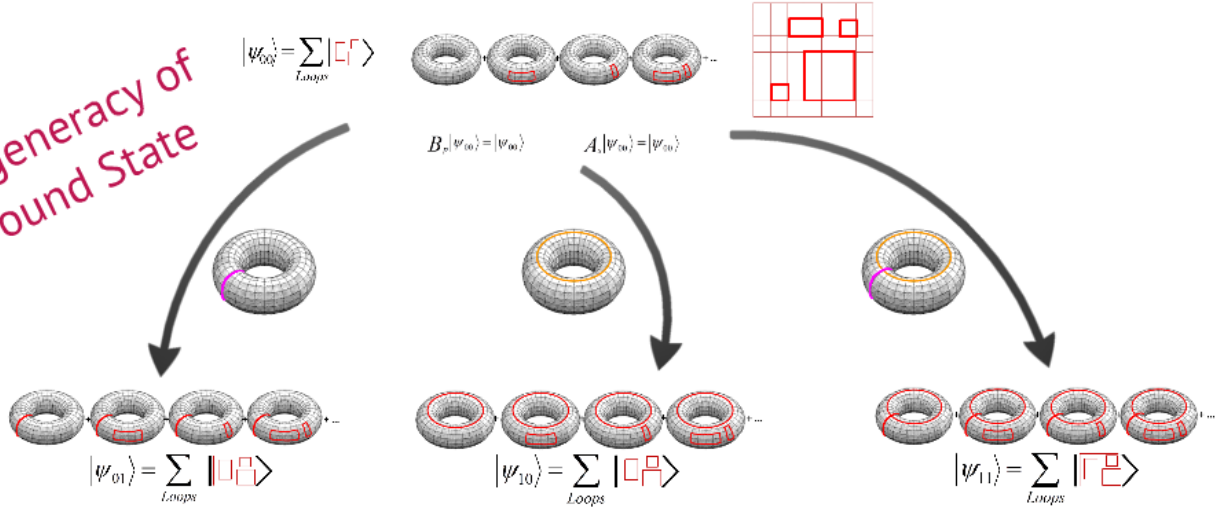
Ground State

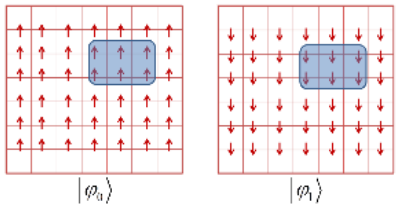
$$[A_s, B_p] = 0 \longrightarrow A_s|\psi\rangle = |\psi\rangle \qquad B_p|\psi\rangle = |\psi\rangle$$

$$H|\psi\rangle = -(N_s + N_p)|\psi\rangle$$

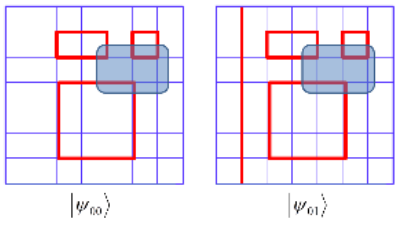


Degeneracy of Ground State



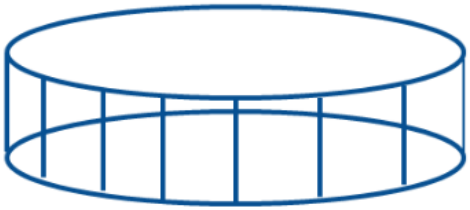


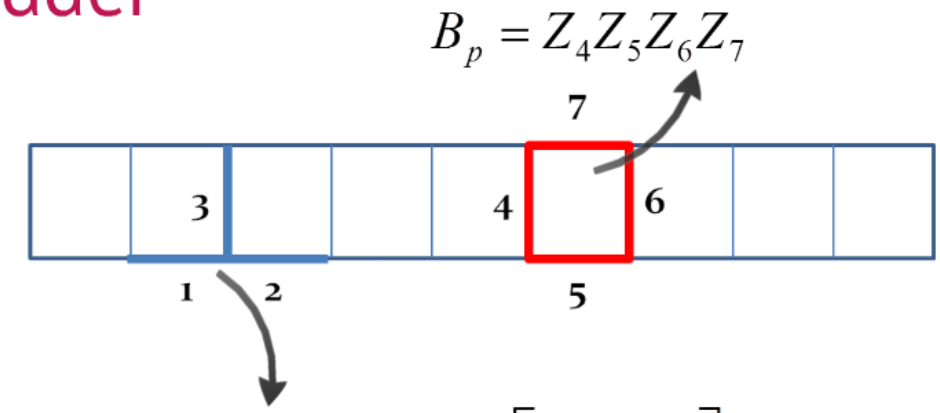
Ising



Kitaev

Kitaev Model on a Ladder



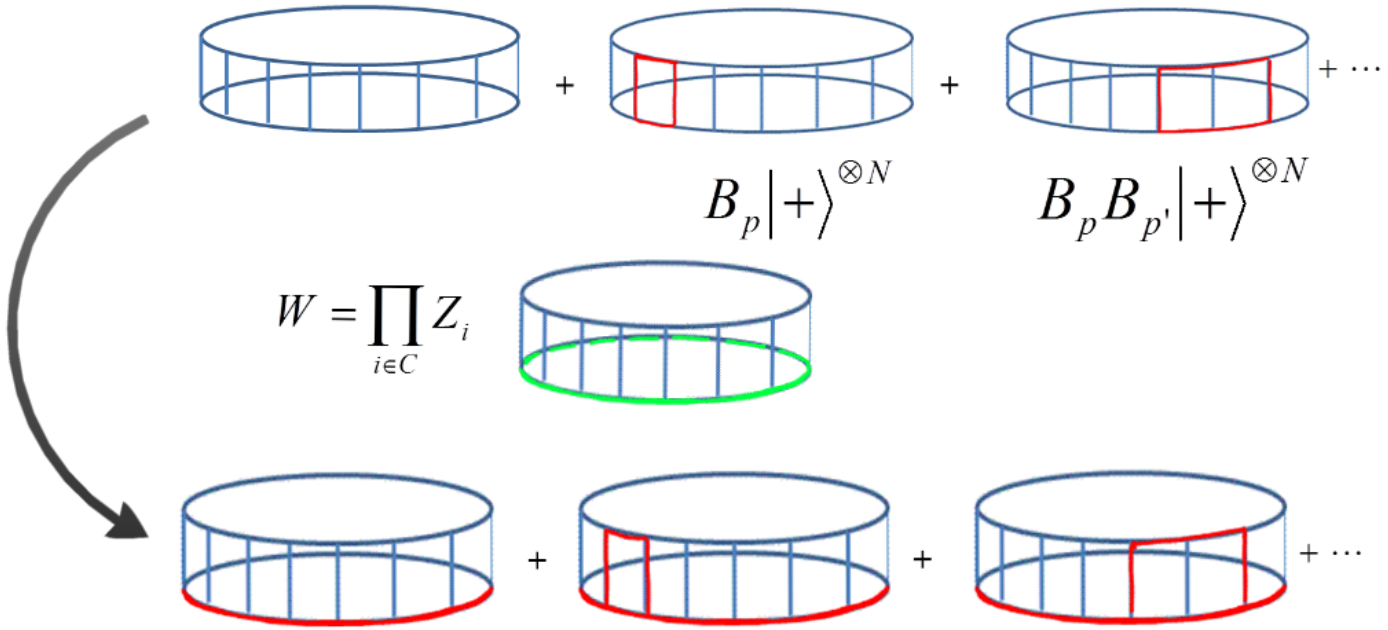


$$H = -\sum_s A_s - \sum_p B_p$$

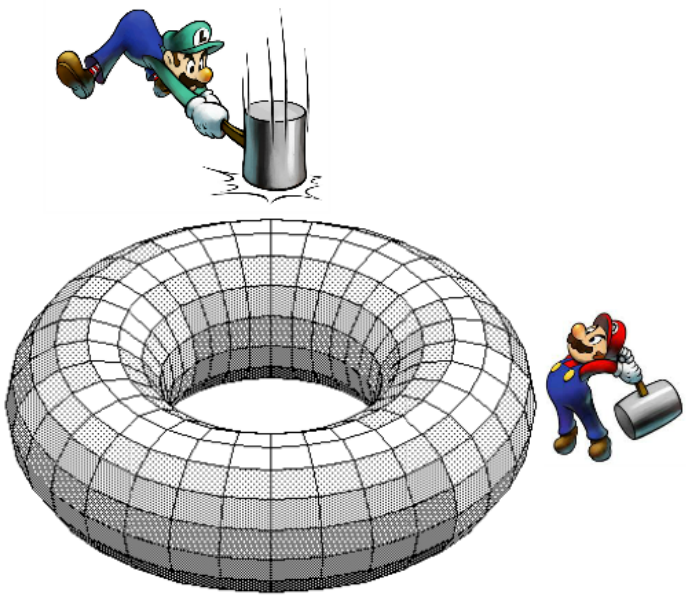
$$A_s = X_1 X_2 X_3$$

$$\left[A_s, B_p \right] = 0$$

Ground State

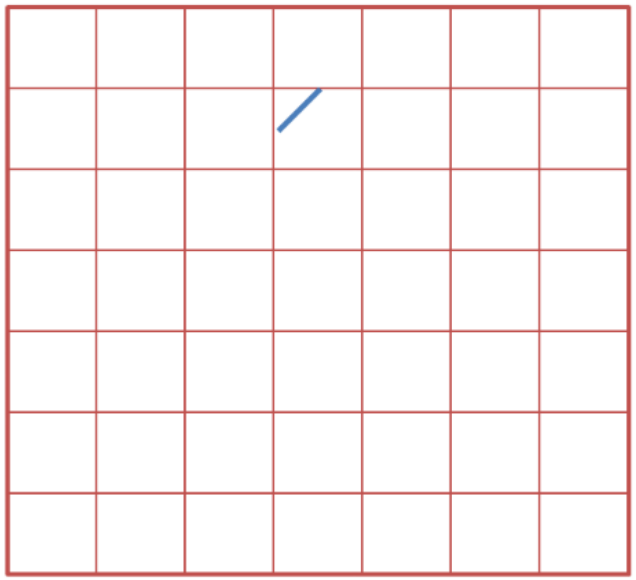


How Robust is Topological Order



Kitaev-Ising

$$H(\lambda) = -J\sum_{s} A_s - K\sum_{p} B_p - \lambda \sum_{<i,j>} Z_i Z_j$$



$$[H(\lambda), B_p] = 0 \qquad \forall p$$

$$B_p|\varphi\rangle = |\varphi\rangle \qquad\qquad B_p|\varphi\rangle = |\varphi\rangle$$

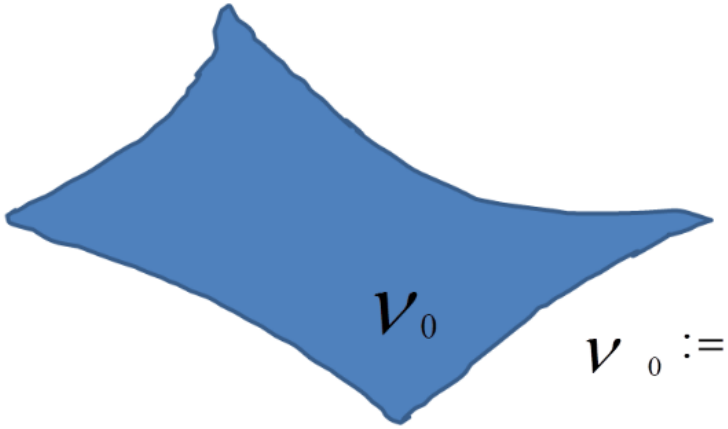
Kitaev λ Ising

rder

$B_p|\varphi\rangle = |\varphi\rangle$ $\qquad$ $B_p|\varphi\rangle = |\varphi\rangle$

Kitaev $\qquad$ λ $\qquad$ Ising

$$B_p = I$$



$$\nu_0 := \{|\phi\rangle \mid B_p|\phi\rangle = |\varphi\rangle\}$$

$$H(\lambda) = -J\sum_s A_s - K\sum_p B_p - \lambda\sum_{<i,j>} Z_i Z_j$$

$$H_0(\lambda) = -J\sum_s A_s - \lambda\sum_{<i,j>} Z_i Z_j - KN$$

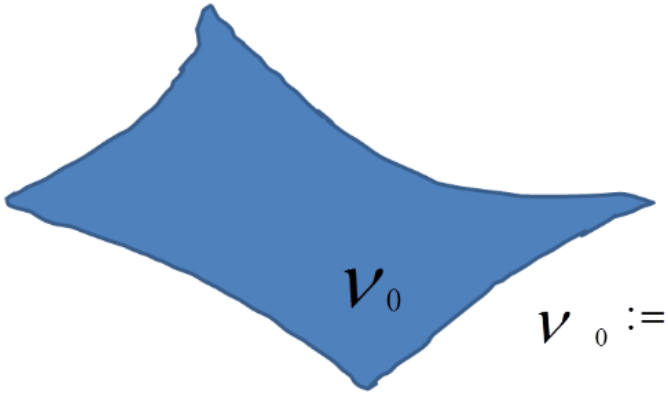
PRA (2013)

cal Order

$B_p|\Psi\rangle = |\Psi\rangle$ $B_p|\Psi\rangle = |\Psi\rangle$

Kitaev λ Ising

$$B_p = I$$



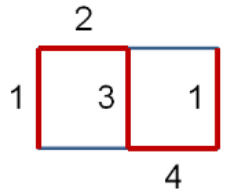
$$\nu_0 := \{|\phi\rangle \mid B_p|\phi\rangle = |\varphi\rangle\}$$

$$H(\lambda) = -J\sum_s A_s - K\sum_p B_p - \lambda\sum_{<i,j>} Z_i Z_j$$

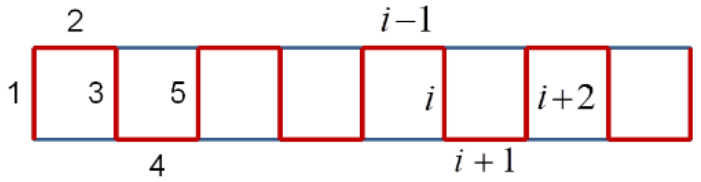
$$H_0(\lambda) = -J\sum_s A_s - \lambda\sum_{<i,j>} Z_i Z_j - KN$$

PRA (2013)

Solution for Ladder



$$|r_1, r_2, r_3, r_4\rangle$$



$$|r\rangle := |r_1, r_2, \cdots, r_{2N}\rangle = \prod_{i \in C} Z_i^{r_i} |\psi_0\rangle$$

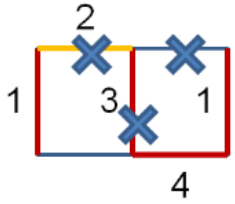
$$|0,1,0,0\rangle = Z_2 |\phi\rangle$$

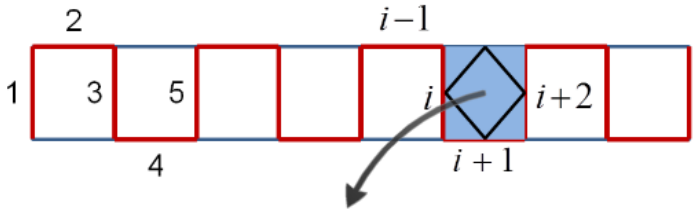
$$|0,1,1,0\rangle = Z_2 Z_3 |\phi\rangle$$

$$A_i |r\rangle = \sigma_{z,i-1} \sigma_{z,i} |r\rangle$$

$$A_3 |0,1,0,0\rangle = A_3 Z_2 |\phi\rangle = - Z_2 A_3 |\phi\rangle$$

$$A_3 |0,1,0,0\rangle = -|0,1,0,0\rangle$$

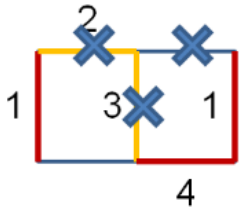




Ising $|r\rangle = 2(\sigma_{x,i}\sigma_{x,i+1} + \sigma_{x,i+1}\sigma_{x,i+2})$

$$A_3 |0,1,1,0\rangle = A_3 Z_2 Z_3 |\phi\rangle = Z_2 Z_3 |\phi\rangle$$

$$A_3 |0,1,1,0\rangle = |0,1,1,0\rangle$$



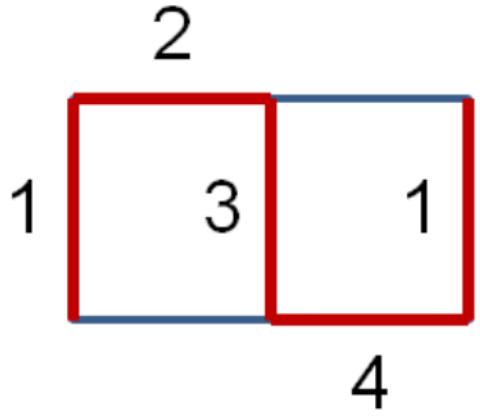
$$H_0(\lambda) = -J \sum_s A_s - \lambda \sum_{<i,j>} Z_i Z_j - KN$$

$$A_3 |r_1, r_2, r_3, r_4\rangle = \sigma_{z,2} \sigma_{z,3} |r_1, r_2, r_3, r_4\rangle$$

$$H_0(\lambda) = -J \sum_i \sigma_{z,i} \sigma_{z,i+1} - 2\lambda \sum_i \sigma_{x,i} \sigma_{x,i+1} - KN$$

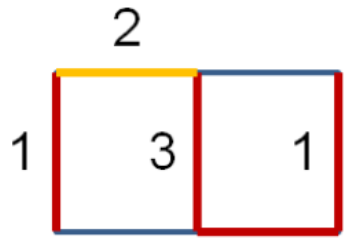
No Transition at finite λ

E. Leib, et. al. Ann. Phys (1961); E. Barouch, PRA (1970)

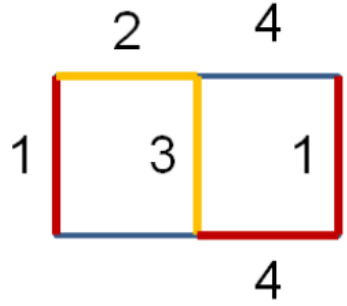


$$\left|r_1, r_2, r_3, r_4\right\rangle$$

$$\left|0,1,0,0\right\rangle = Z_2\left|\phi\right\rangle$$

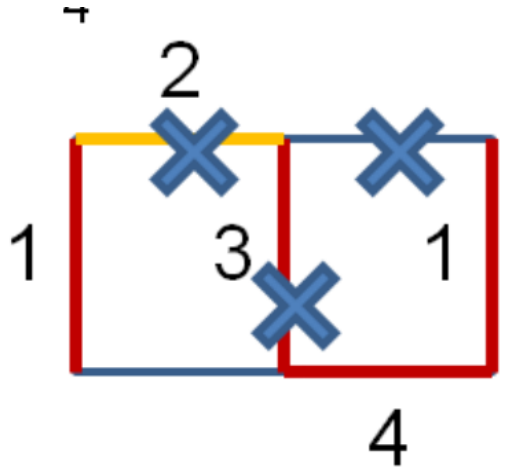


$$\left|0,1,1,0\right\rangle = Z_2 Z_3\left|\phi\right\rangle$$



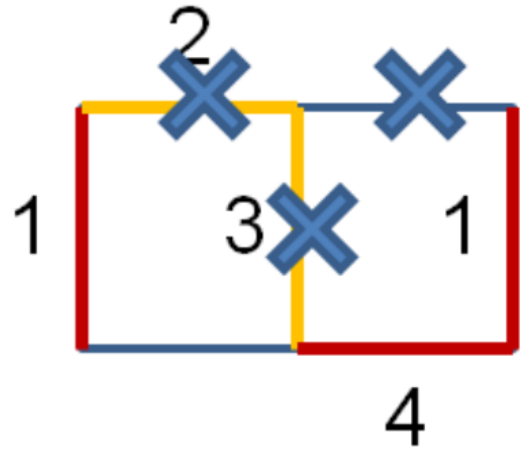
$$A_3|0,1,0,0\rangle = A_3 Z_2|\phi\rangle = -Z_2 A_3|\phi\rangle$$

$$A_3|0,1,0,0\rangle = -|0,1,0,0\rangle$$

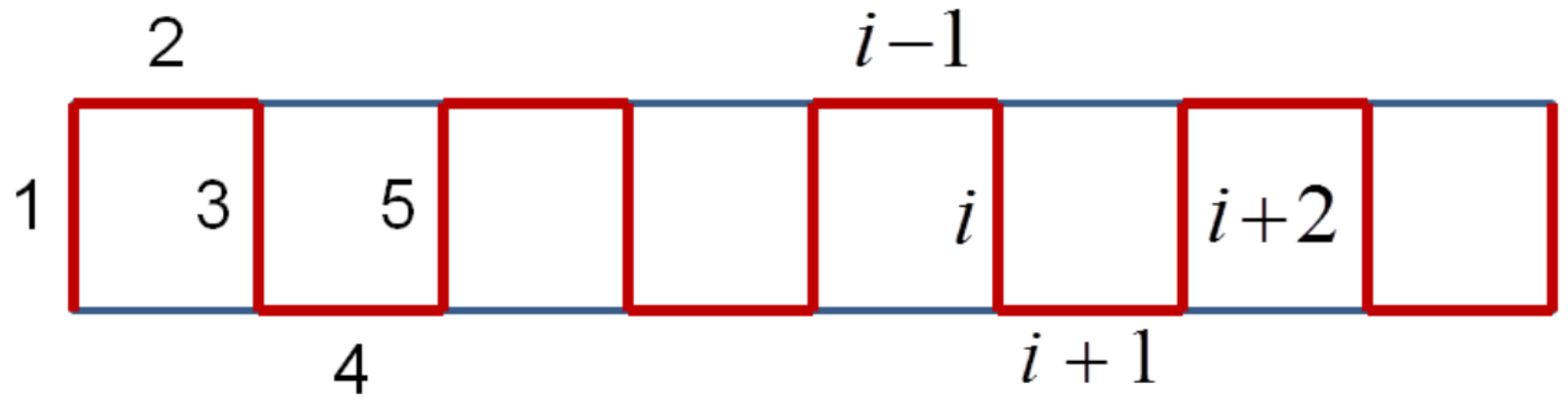


$$A_3|0,1,1,0\rangle = A_3 Z_2 Z_3|\phi\rangle = Z_2 Z_3|\phi\rangle$$

$$A_3|0,1,1,0\rangle = |0,1,1,0\rangle$$

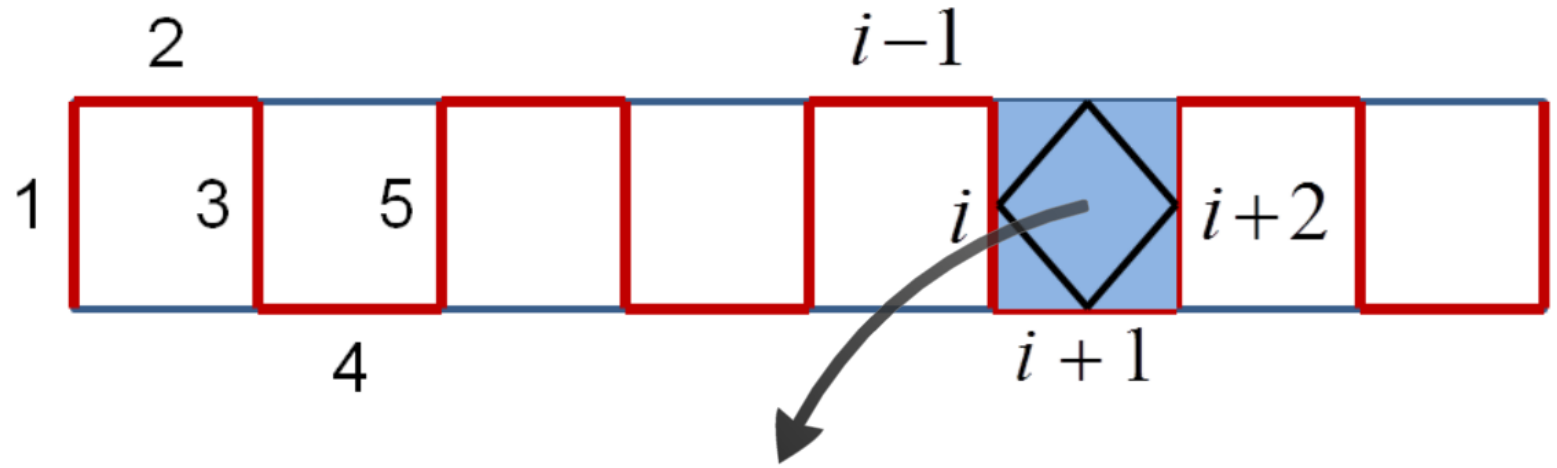


$$A_3|r_1,r_2,r_3,r_4\rangle = \sigma_{z,2}\sigma_{z,3}|r_1,r_2,r_3,r_4\rangle$$



$$|r\rangle := |r_1, r_2, \cdots, r_{2N}\rangle = \prod_{i \in C} Z_i^{r_i} |\psi_0\rangle$$

$$A_i |r\rangle = \sigma_{z,i-1} \sigma_{z,i} |r\rangle$$



$$\text{Ising}\,|r\rangle = 2(\sigma_{x,i}\sigma_{x,i+1} + \sigma_{x,i+1}\sigma_{x,i+2})$$

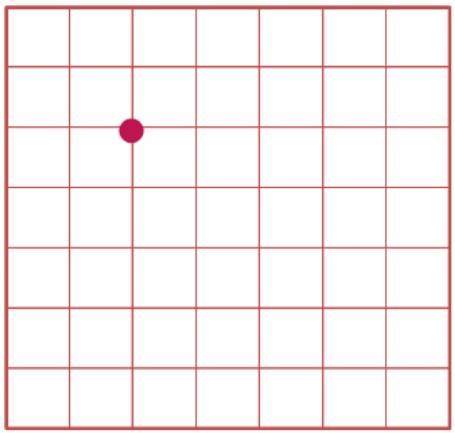
$$H_0(\lambda) = -J\sum_s A_s - \lambda\sum_{<i,j>} Z_i Z_j - KN$$

$$H_0(\lambda) = -J\sum_i \sigma_{z,i}\sigma_{z,i+1} - 2\lambda\sum_i \sigma_{x,i}\sigma_{x,i+1} - KN$$

No Transition at finite λ

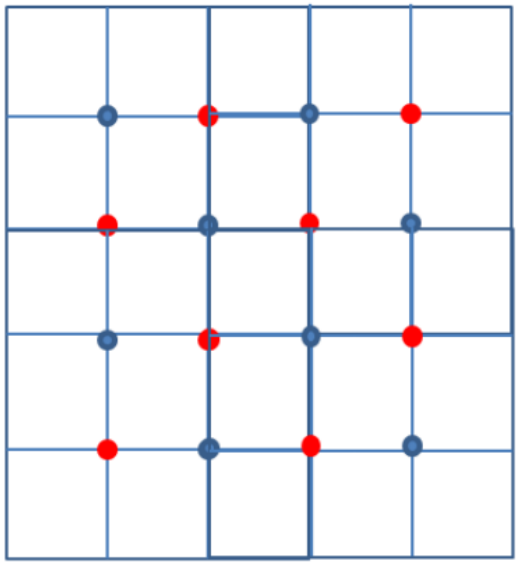
E. Leib, et. al. Ann. Phys (1961); E. Barouch, PRA (1970)

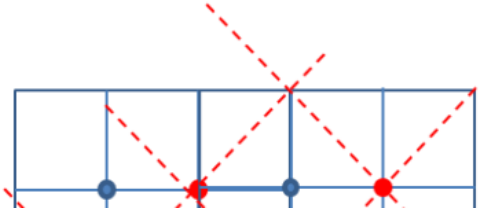
Solution for Torus

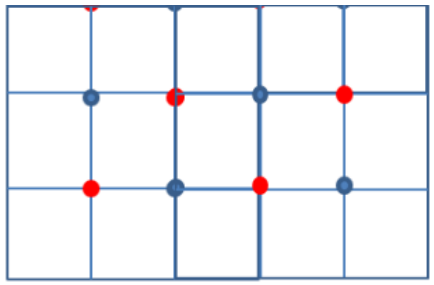


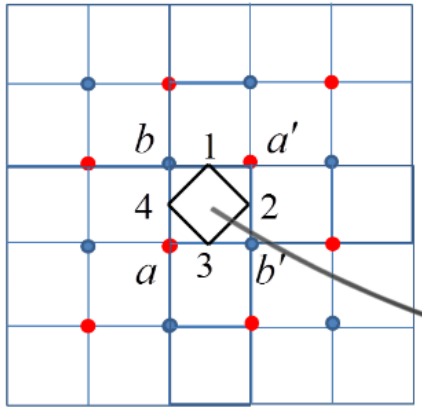
$$|r\rangle := |r_1, r_2, \cdots, r_N\rangle = \prod_{i \in C} A_i^{r_i} |0\rangle^{\otimes 2N}$$

$$B_p |r\rangle = |r\rangle$$



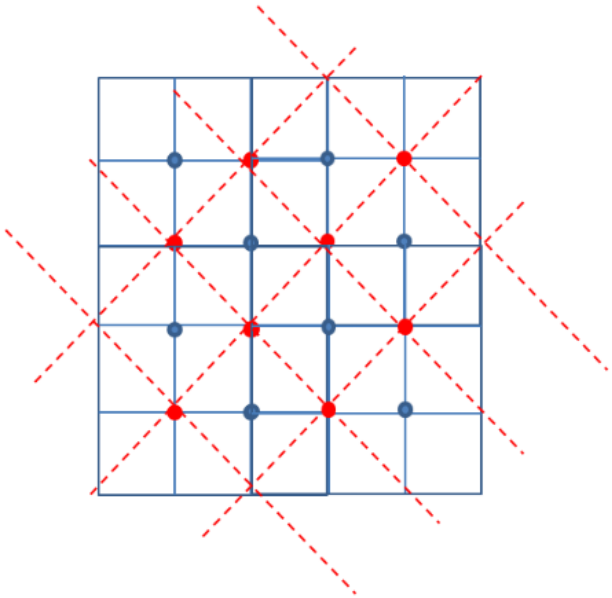






$$A_i|r\rangle = \sigma_{x,i}|r\rangle$$

$$\text{Ising}|r\rangle \rightarrow \sigma_{z,a}\sigma_{z,a'}\sigma_{z,b}\sigma_{z,b'}|r\rangle$$



$$H = H_A + H_B$$

$$H_A = -J\sum_{i\in A} X_i - 2\lambda \sum_{<i,j>\in A} Z_i Z_j$$

Transition Point

Mean Field approximation: $J_c = 8\lambda$

$$H = H_A + H_B$$

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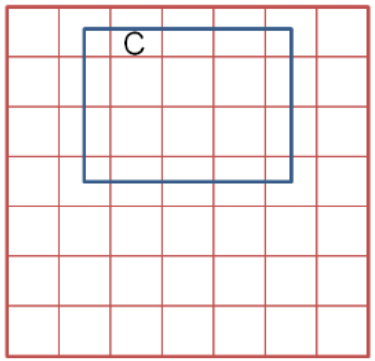
Transition Point

Mean Field approximation: $J_c = 8\lambda$

DMRG: $J_c = 6\lambda$

Jongh and Leeuwen PRB (1998)

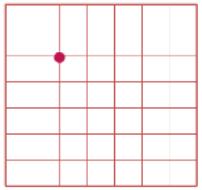


$$\langle W_C \rangle := \left\langle \prod_{i\in C} \sigma_{x,i} \right\rangle$$

$$e^{\beta|\partial S|} \qquad e^{\gamma|S|}$$

$$J_c$$

Solution for Torus



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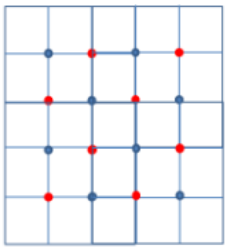
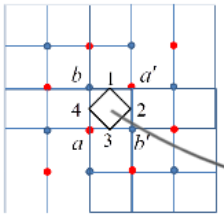
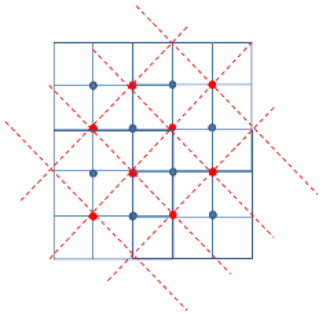




$$A_i |r\rangle = \sigma_{x,i} |r\rangle$$

$$\text{Ising} |r\rangle \rightarrow \sigma_{z,a} \sigma_{z,a'} \sigma_{z,b} \sigma_{z,b'} |r\rangle$$



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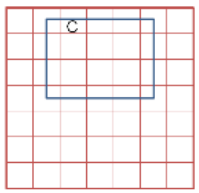
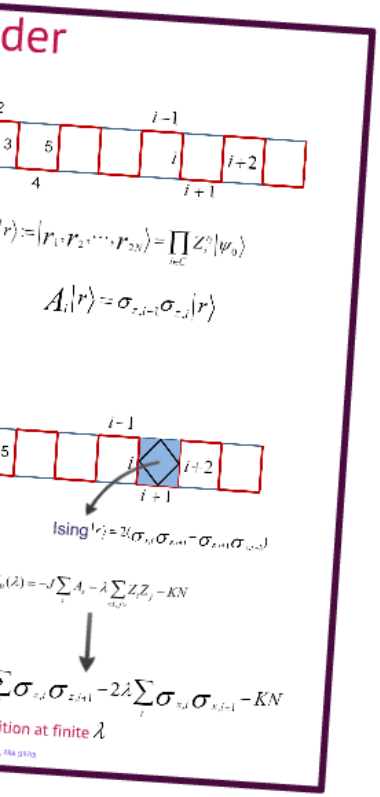
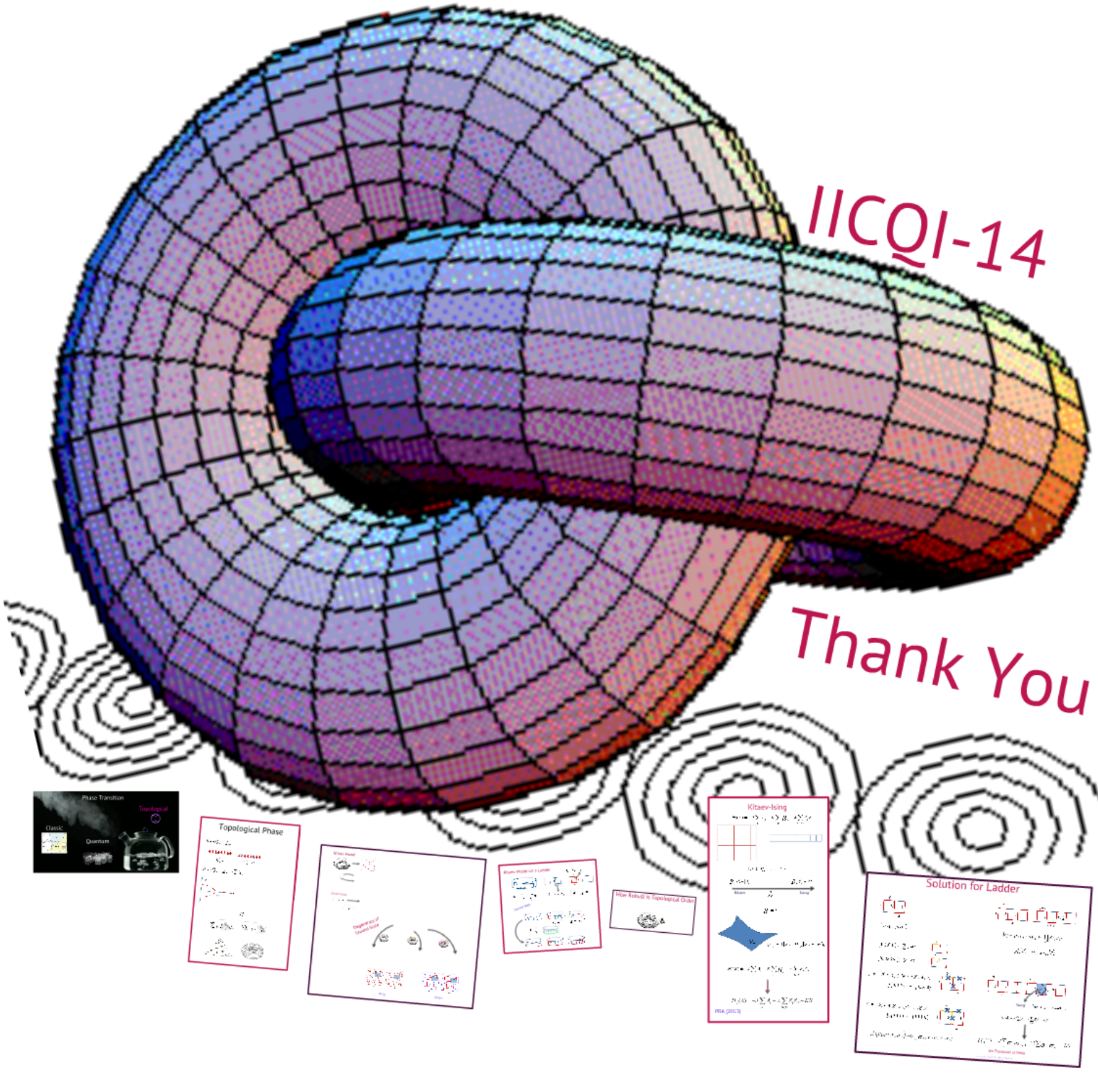


$$\langle W_C \rangle = \left\langle \prod_{x,i \in C} \sigma_{x,i} \right\rangle$$

$$e^{\beta |\partial S|} \qquad e^{\gamma |S|}$$

$$J_c$$




IICQI-14
Thank You
Phase Transition
Classic
Quantum
Topological
Topological Phase
How Robust is Topological Order?
Kitaev-Ising
Solution for Ladder
Solution for Torus
Transition Point
Mean Field approximation:
DMRG: