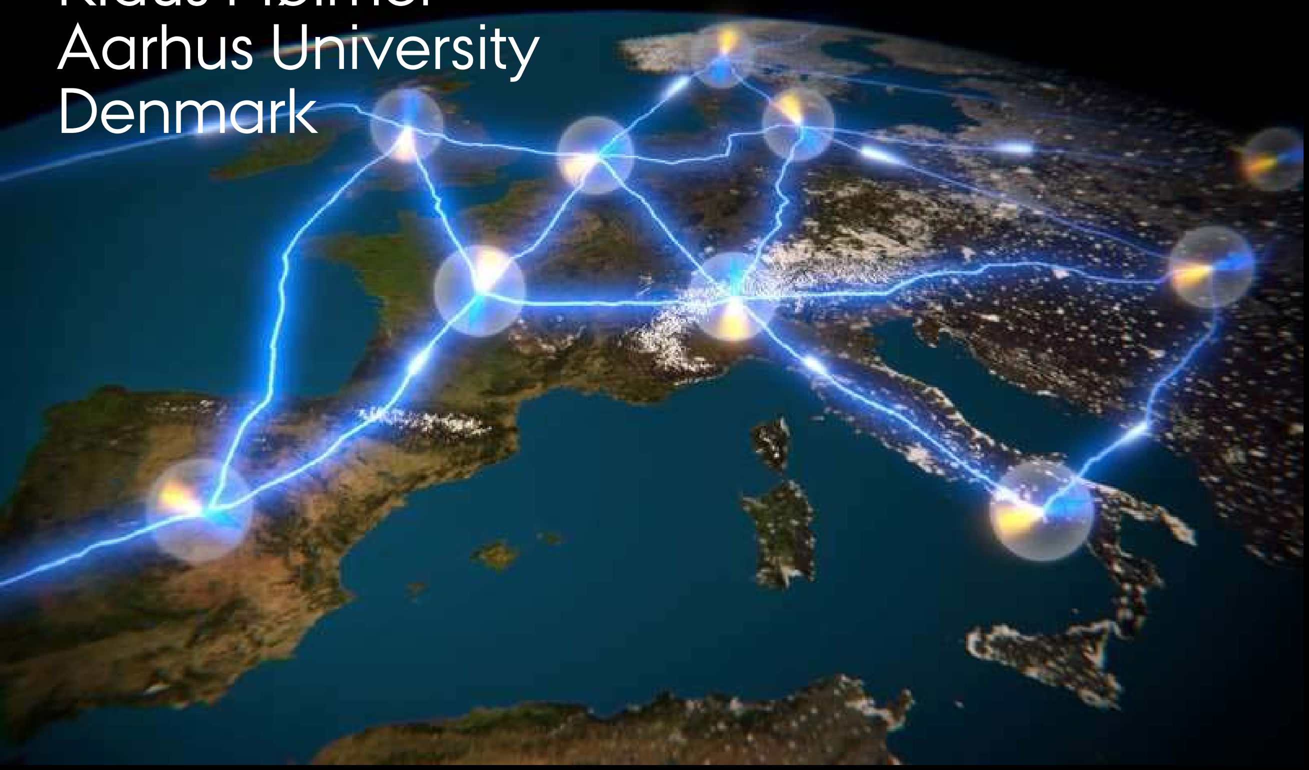
How do flying and stationary qubits interact in a quantum network ?

Klaus Mølmer
Aarhus University
Denmark



QUANTUM FLAGSHIP

SQUARE

SCIENCE TO SHAPE THE FUTURE OF THE ARMY
ARMY RESEARCH OFFICE
1951

Carlsberg
CARLSBERG FOUNDATION

VILLUM FONDEN

Iran International Conference on Quantum Information
IICQI-20, July 16, 2020.

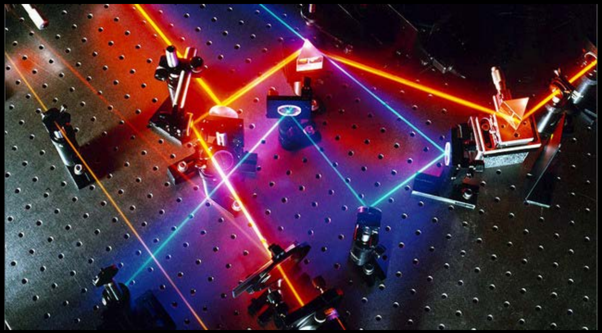


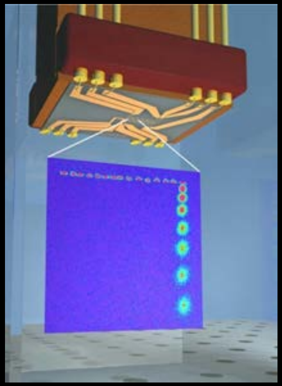


Isfahan
IICQI 2014 ;=)

Tehran
IICQI 2018 ;=)

The Internet
IICQI 2020

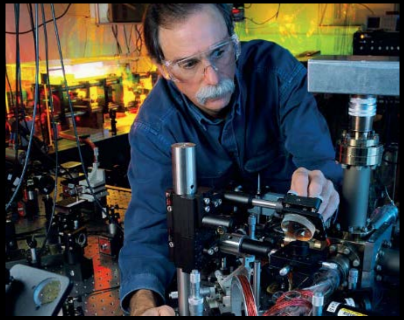


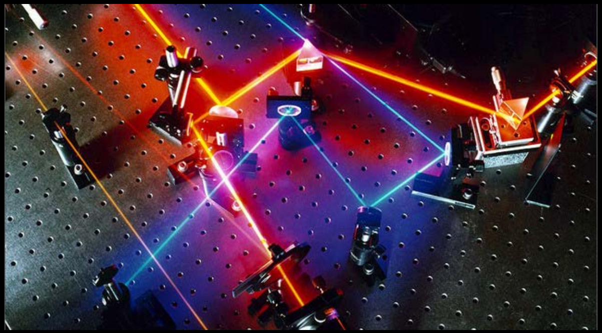


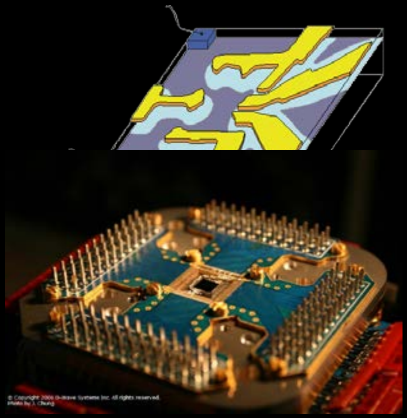
Quantum Optics

Atoms
Ions
Photons
    Cavities
    Travelling fields

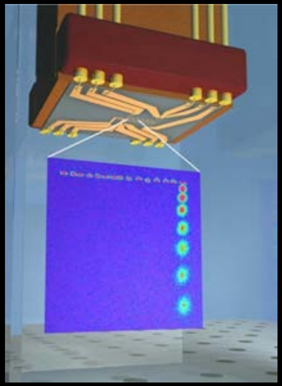
…







Quantum Optics

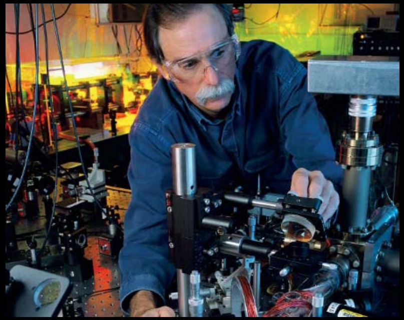


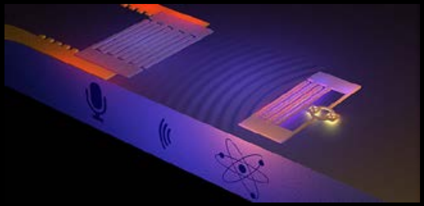
Atoms
Ions
Photons
- *Cavities*
- *Travelling fields*

…

Bits and Pieces

Quantum dots
Superconductors
Magnons
Cantilevers
Microwaves
Bulk and surface waves
…





The quantum theory of light

Maxwell’s equations

$$\begin{aligned}\nabla \cdot \vec{B} &= 0\\ \nabla \times \vec{E} + \partial \vec{B}/\partial t &= 0\\ \nabla \cdot \vec{D} &= \rho\\ \nabla \times \vec{H} - \partial \vec{D}/\partial t &= \vec{J}\end{aligned}$$

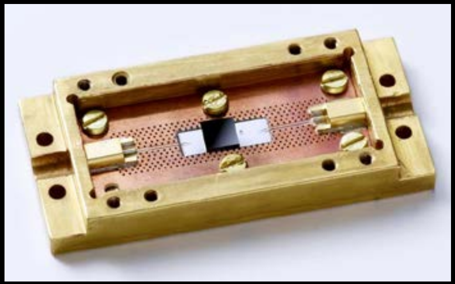
electron coordinates are quantum observables

Quantum mechanics is a ”virus”

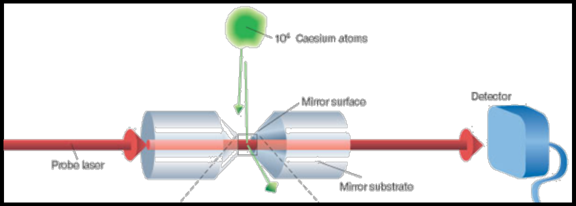
Fields → quantum observables

Maxwell’s Eqs → Heisenberg Eqs of motion for field *operators*

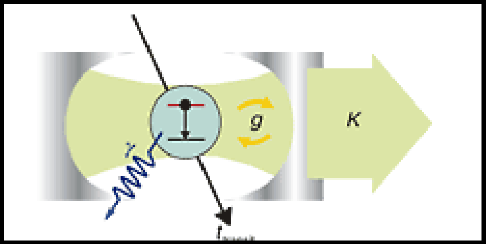
Quantum states of light



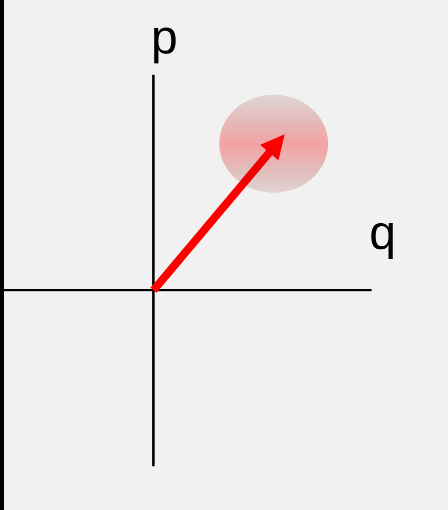
Annihilation and creation a, a^+
number operator $n=a^+a$,



Fock or number states, $|n>$



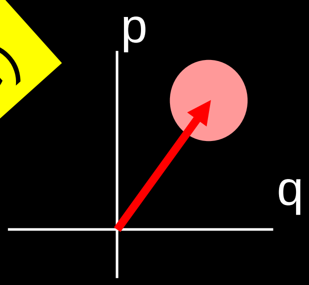
Coherent state, $|\alpha>$

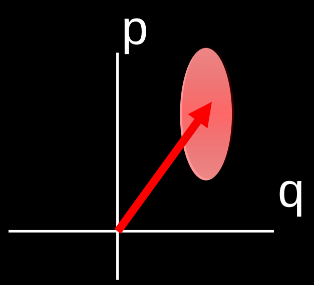


Interaction
$g(a^+\sigma_- + a\sigma_+)$

The small print in quantum optics textboks

Single mode
(standing wave)





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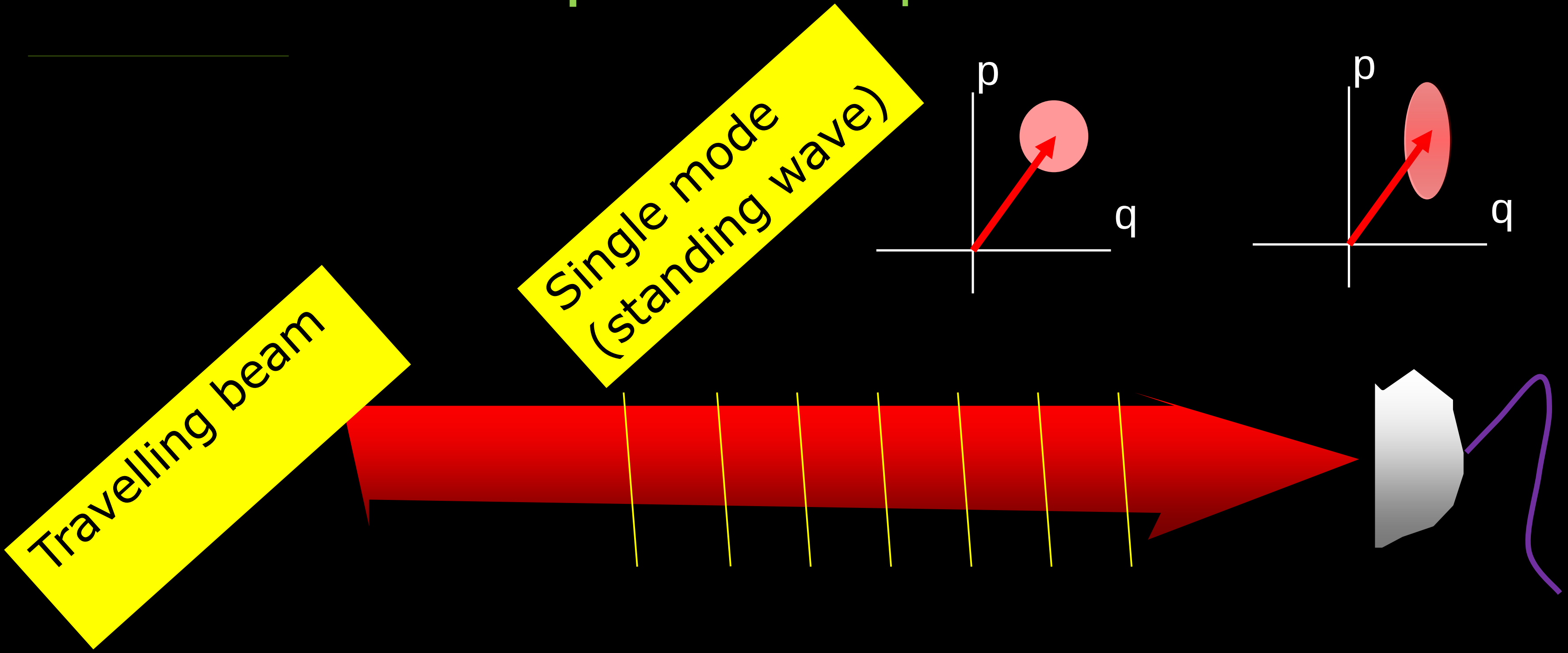

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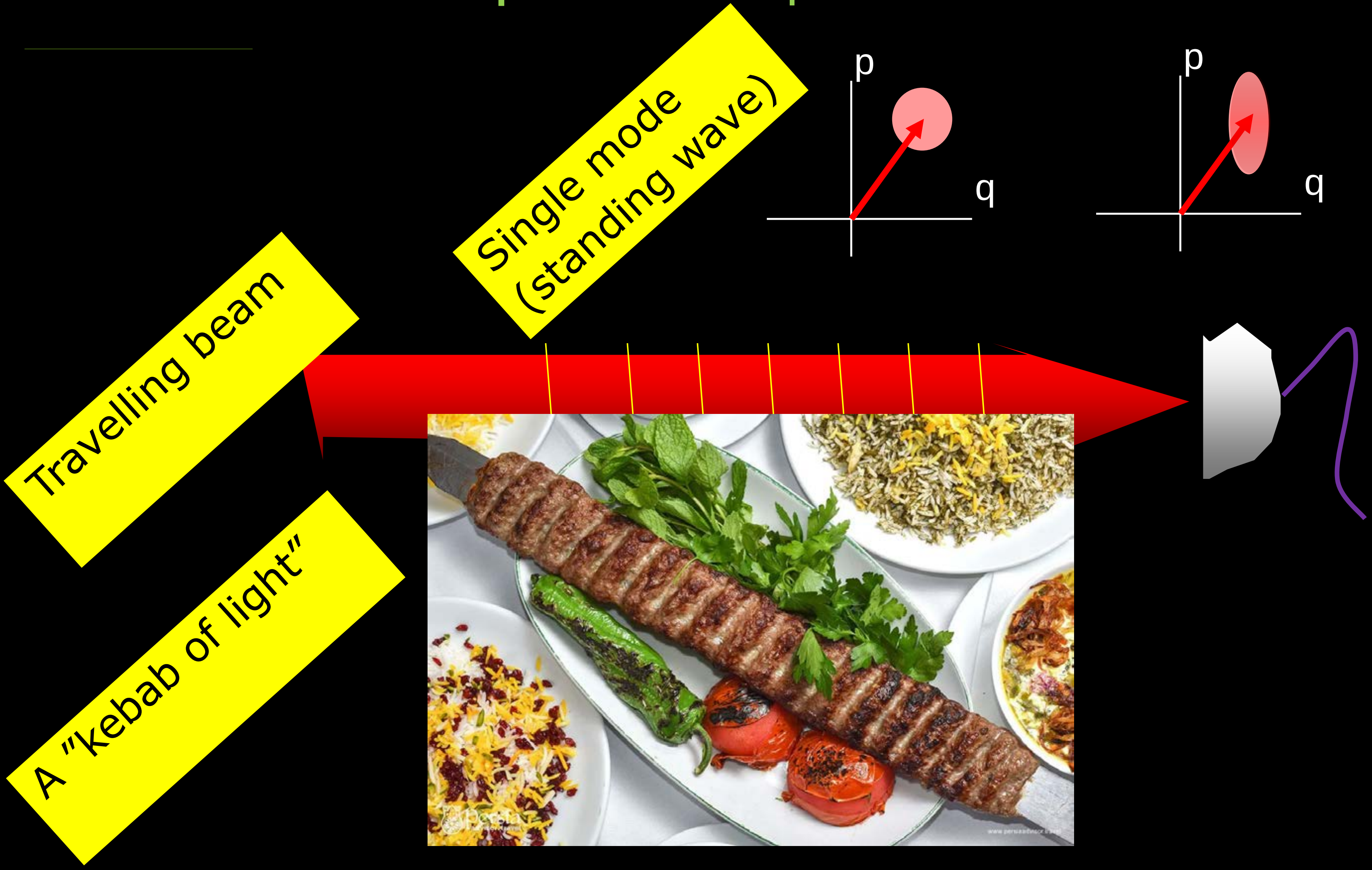

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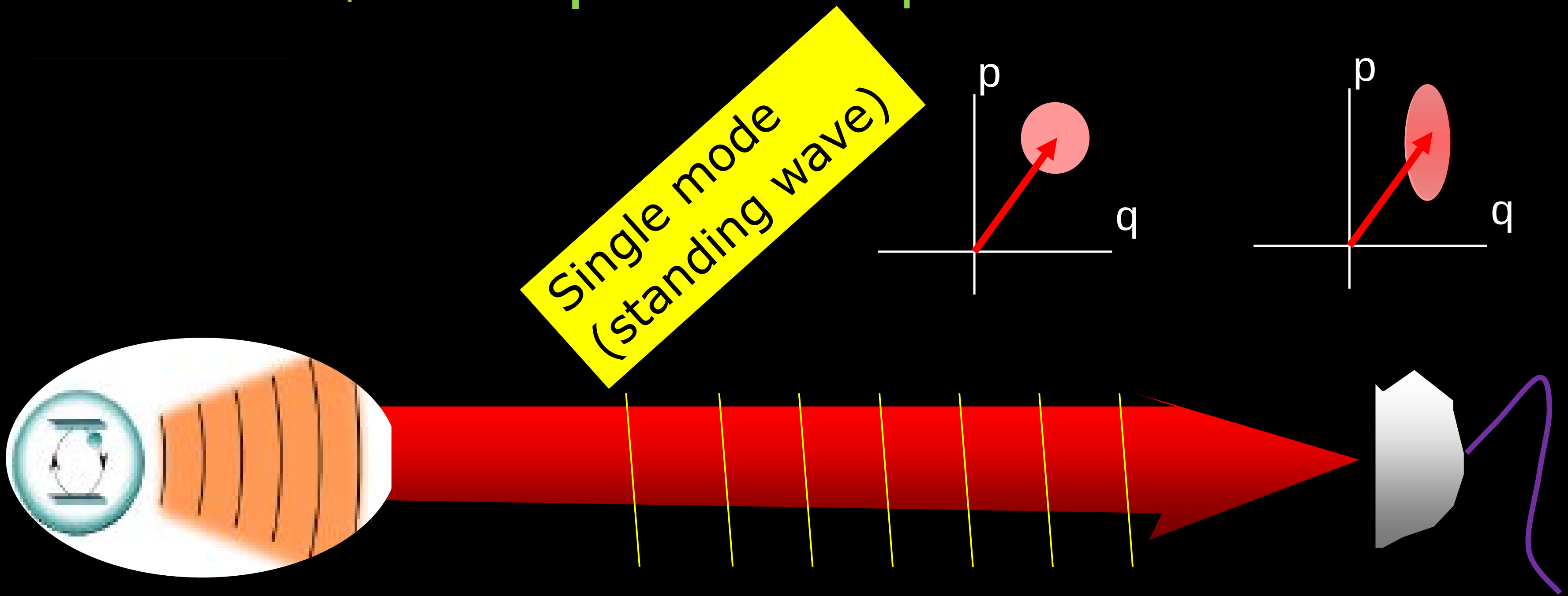
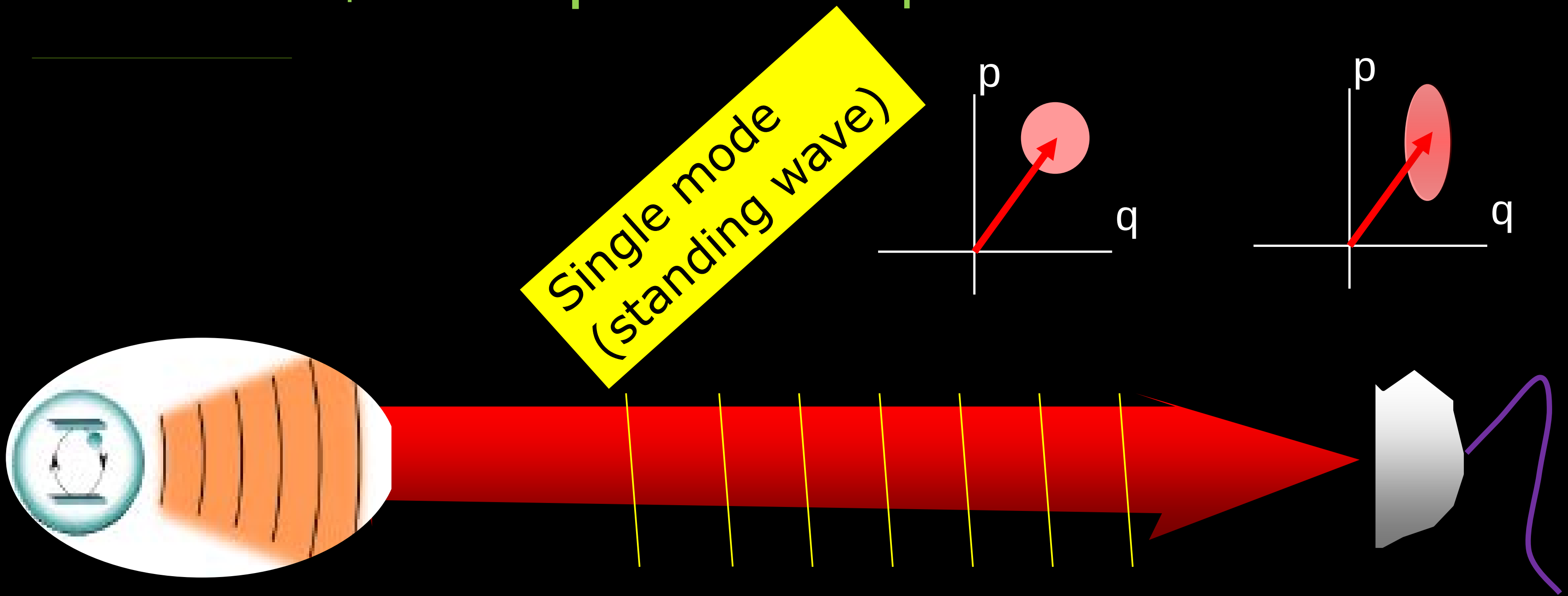


Schrödinger picture (expansion on number states)
is practically impossible.

Heisenberg picture (field observables)
yields mean values, correlation functions.

The small print in quantum optics textboks



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Heisenberg picture (field observables) yields mean values, correlation functions.

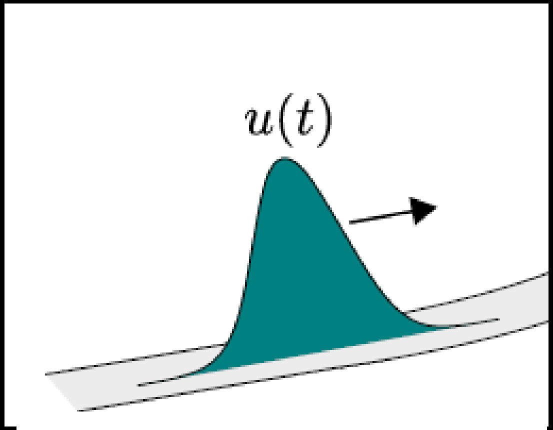
(Source) master equation: $\frac{d}{dt}\rho = \frac{1}{i\hbar}[H,\rho] - \frac{1}{2}(L^+L\,\rho + \rho\,L^+L) + L\rho L^+$

$Emitted\ field \propto L = \sqrt{\gamma}\,\sigma$

The state of a *pulse* of light (microwave, SAW, ...)

Wave packet:
solution of wave equation

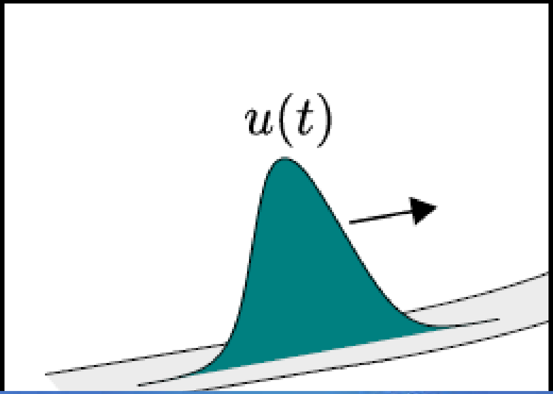
Second quantization:
|*n*> Fock state
or superposition state $\Sigma_n c_n |n>$



$$\hat{b}_u^\dagger = \int dt\, u(t)\hat{b}^\dagger(t)$$

Such pulses may drive quantum systems,
may work as flying qubits,may probe quantum systems,
may transport pure or mixed states, transport energy …

The state of a *pulse* of light (microwave, SAW, …)



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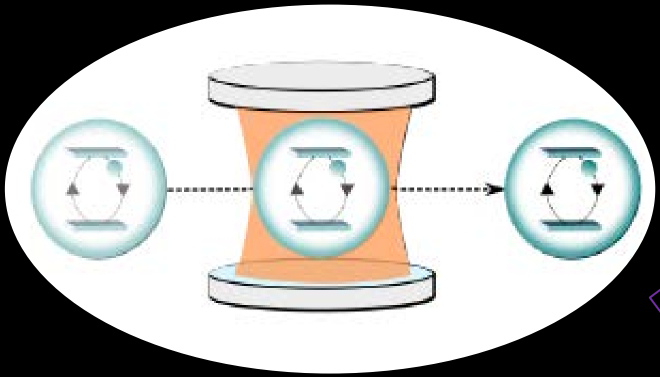
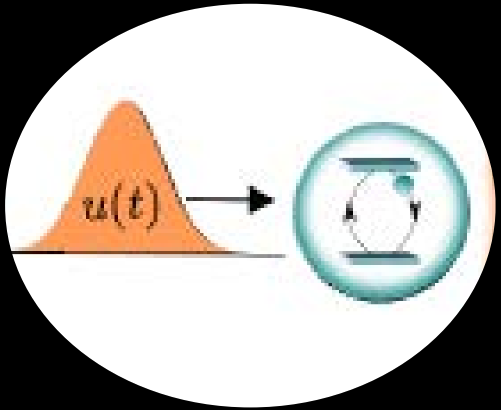
How does a quantum pulse interact with a qubit ?





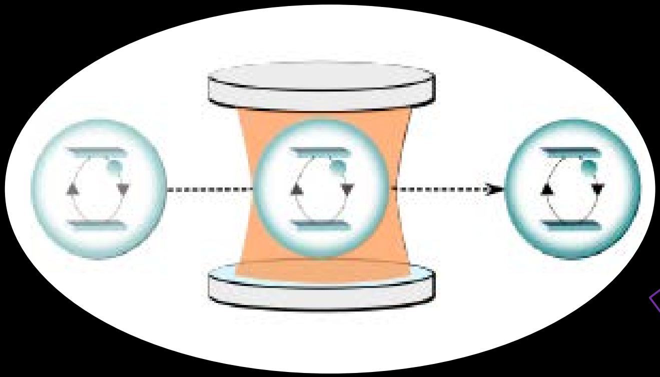
The same ?

NO !

Flying atom, fixed mode:
coupling g → g(t)

Flying mode, fixed atom:
coupling g → u(t) ?

How does a quantum pulse interact with a qubit ?



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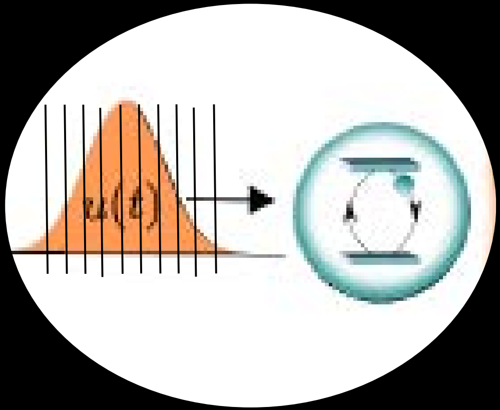

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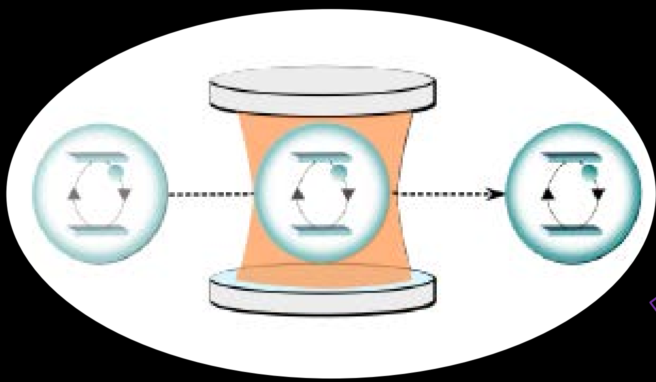

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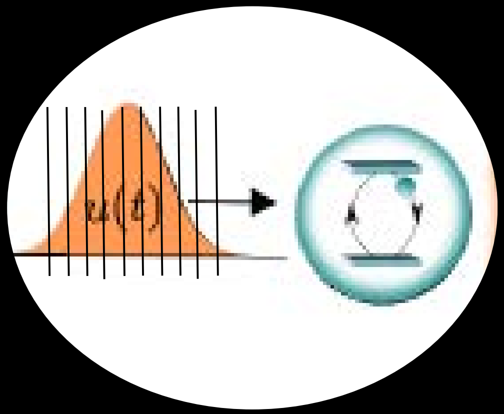


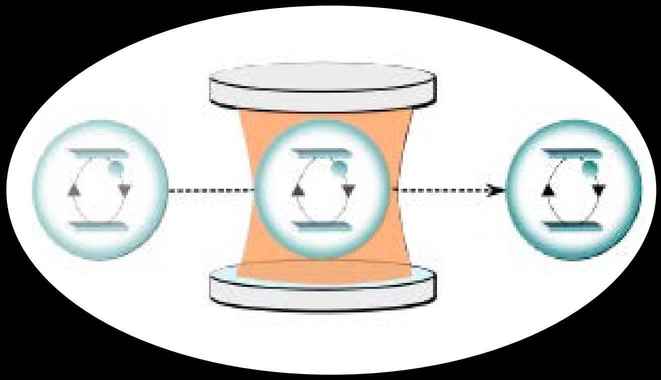
Flying atom, fixed mode:
coupling g → g(t)

NO !

Flying mode, fixed atom:
coupling g → u(t) ?

→ Exchange of quanta between emitter and field
→ Distortion of the pulse (mode continuum)
mix of the two: genuine multi-mode theory

How does a quantum pulse interact with a qubit ?



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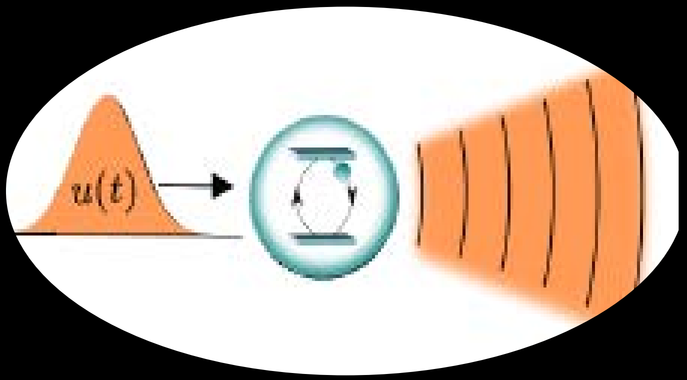


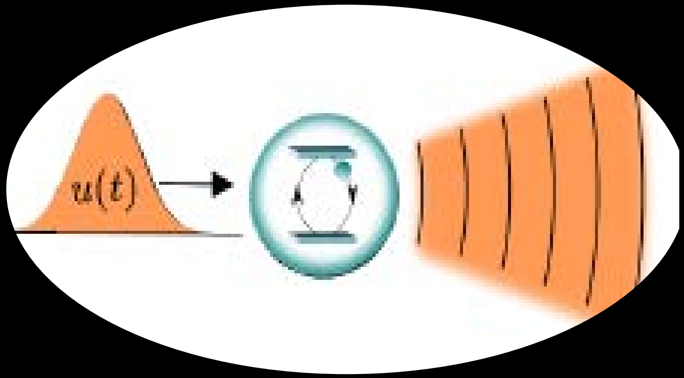
Flying atom, fixed mode:
coupling g → g(t)

One atom & mode **continuum**
Open quantum system

Cascaded system master equation
(Gardiner 1993, Carmichael 1993)

See also:
B. Q. Baragiola, et al (J. Combes), “n-photon wave packets interacting with an arbitrary quantum system,” Phys. Rev. A 86, 013811 (2012).

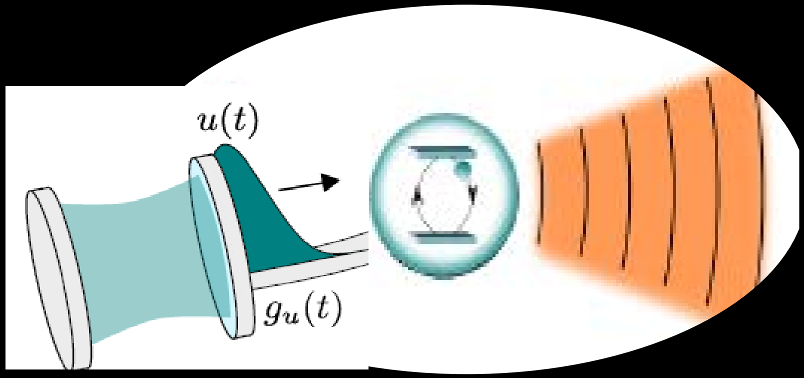
How does a quantum pulse interact with a qubit ?



Alexander Holm Kiilerich and Klaus Mølmer
Input-Output Theory with Quantum Pulses
Phys. Rev. Lett. **123**, 123604 (2019).

How does a quantum pulse interact with a qubit ?

A trick !



$$g_u(t) = \frac{u^*(t)}{\sqrt{1 - \int_0^t dt' \, |u(t')|^2}}$$

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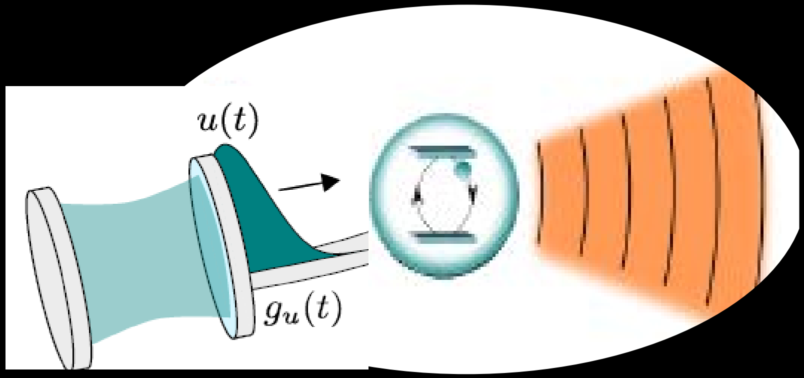

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Single-mode cavity and an atom

"Jaynes-Cummings" Hamiltonian:

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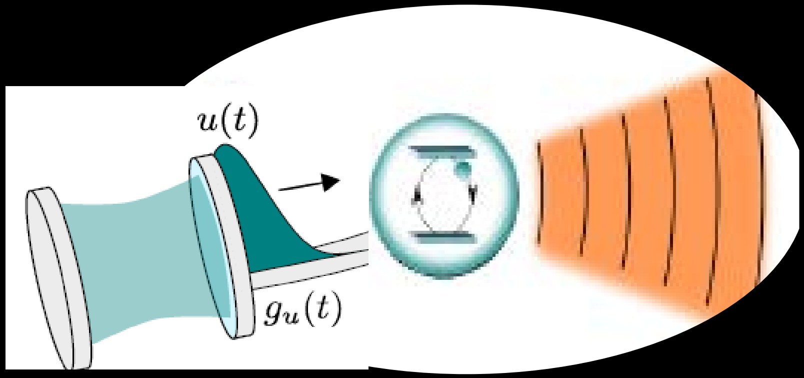

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Single-mode cavity and an atom

"Jaynes-Cummings" Hamiltonian:

$$H = \frac{i\sqrt{\gamma}}{2} \left(g_u(t) a_u^+ \sigma - g_u^*(t) a_u \sigma^+ \right)$$

Damping (Lindblad) operator:

$$L = g_u^*(t) a_u + \sqrt{\gamma}\, \sigma$$

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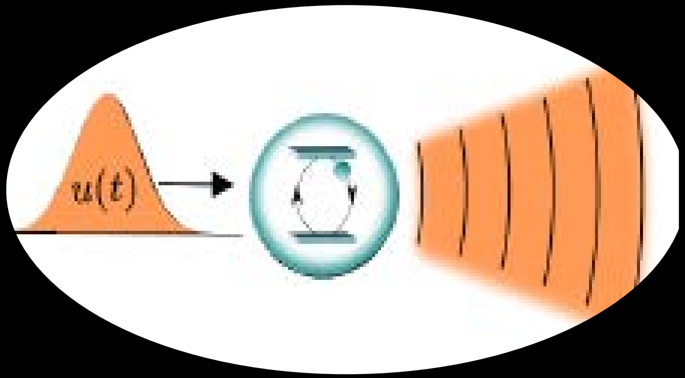

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$$L = g_u^*(t) a_u + \sqrt{\gamma}\,\sigma$$

Master equation:

$$\frac{d}{dt}\rho = \frac{1}{i\hbar}[H,\rho] - \frac{1}{2}(L^+L\,\rho + \rho\, L^+L) + L\rho L^+$$

$$= \sqrt{\gamma}\,\{ g_u^*(t)\,(a_u\,\rho\,\sigma^+ - a_u\,\sigma^+\rho) + g_u(t)(\sigma\,\rho\, a_u^+ - \rho\, a_u^+\,\sigma)\}$$

$$+ D[\sqrt{\gamma}\,\sigma]\rho + D[g_u^*(t) a_u]\rho$$

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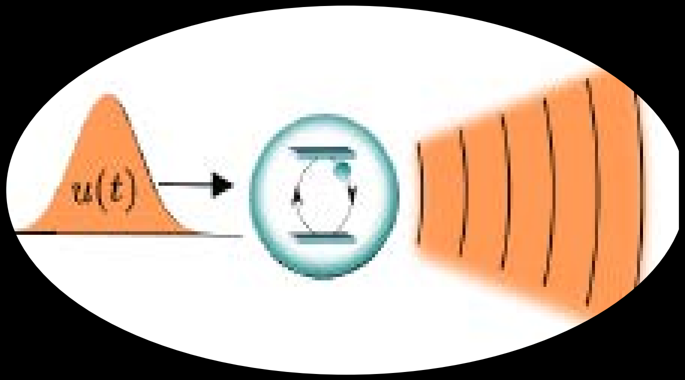

$$H = \frac{i\sqrt{\gamma}}{2}\left(g_u(t)a_u^{+}\,\sigma - g_u^{*}(t)a_u\sigma^{+}\right)$$

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$$+\; D[\sqrt{\gamma}\,\sigma]\rho + D[g_u^{*}(t)a_u]\rho$$

Chiral "Hamiltonian"
Excitations: "→"

$\rho_F \rightarrow$ *vacuum state*

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How does a quantum pulse interact with a qubit ?

Field + Atom Master Equation:

$$\frac{d}{dt}\rho = \sqrt{\gamma}\,\{\, g_u^*(t)\,(a_u\,\rho\,\sigma^+ - a_u\,\sigma^+\rho) + g_u(t)(\sigma\,\rho\,a_u^+ - \rho\,a_u^+\,\sigma)\}$$

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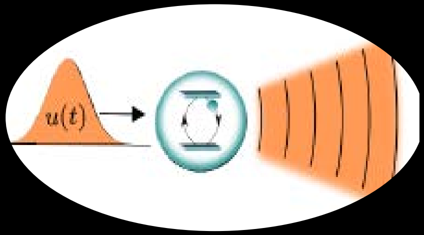

If input "cavity" field is in a *coherent state*: $|\,\alpha >< \alpha| \;\rightarrow\; |\alpha(t) >< \alpha(t)|$

$\rightarrow |\,0 >< 0|$

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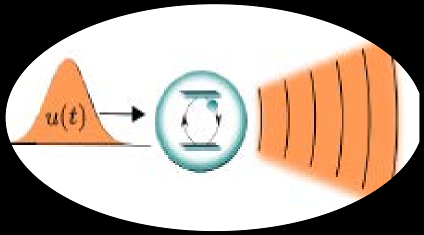

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Atom Master Equation :

$$\frac{d}{dt}\rho = \sqrt{\gamma}\,[u(t)\alpha^*(0)\,\sigma \;- u^*(t)\alpha(0)\sigma^+, \rho_A] \;+\; D[\sqrt{\gamma}\,\sigma]\rho_A$$

classical drive *atomic decay*

How does a quantum pulse interact with a qubit ?

Field + Atom Master Equation:

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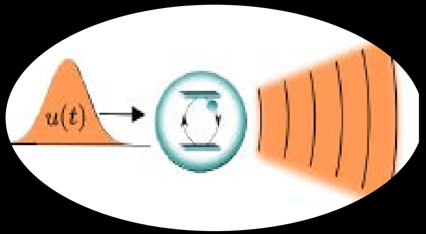

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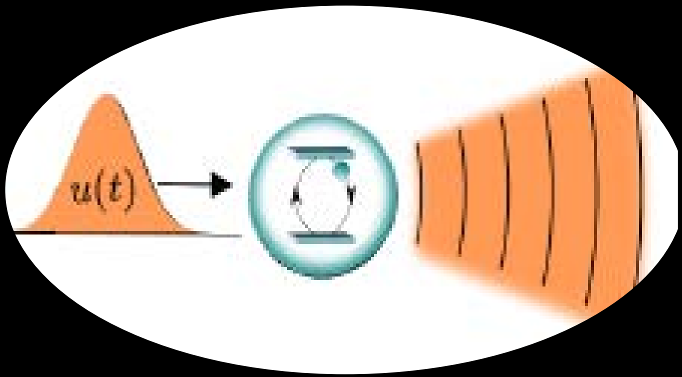
classical drive *atomic decay*

Opposite to "real" Jaynes-Cummings model: Fock state easy.

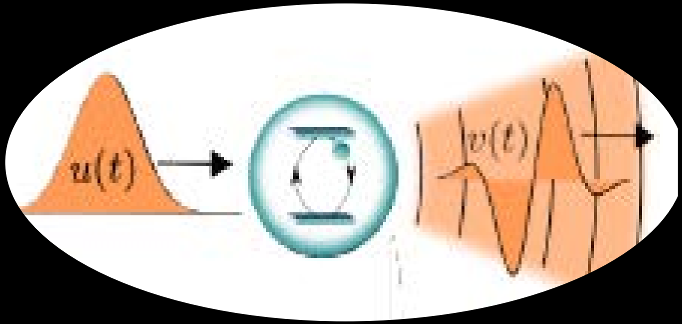
Input Fock state is more difficult: solve $\rho_{FA}(t)$

How does a quantum pulse interact with a qubit ?



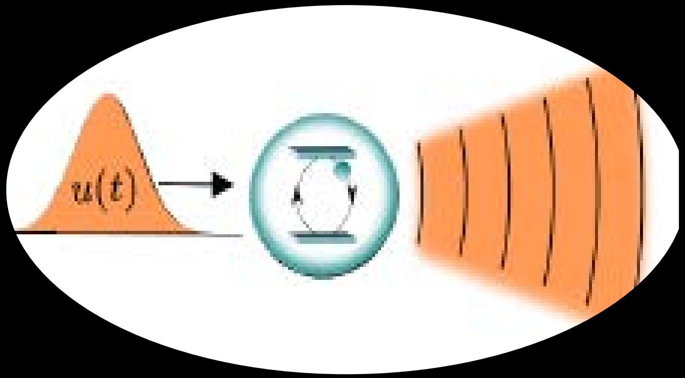
What about the state of the pulse after the interaction ?

$\rho_F \rightarrow vacuum\ state$



State contents of pulse $v(t)$ after the interaction ?

How does a quantum pulse interact with a qubit ?



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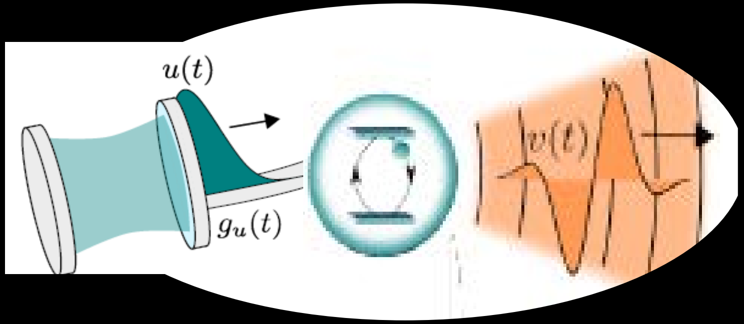

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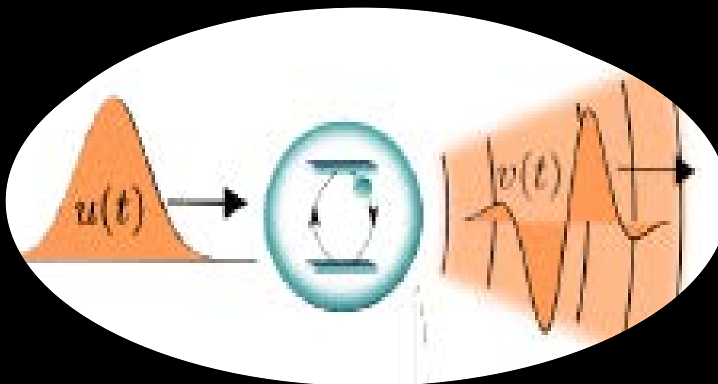

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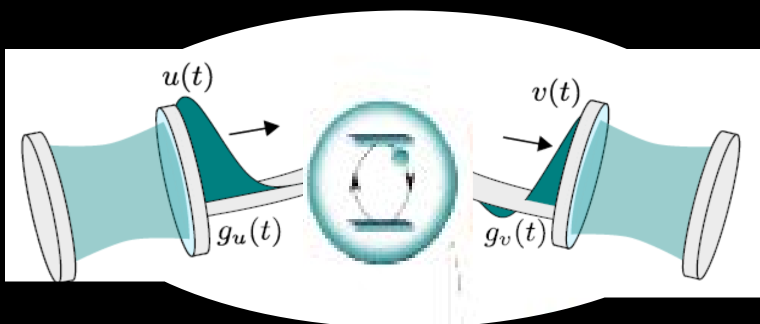

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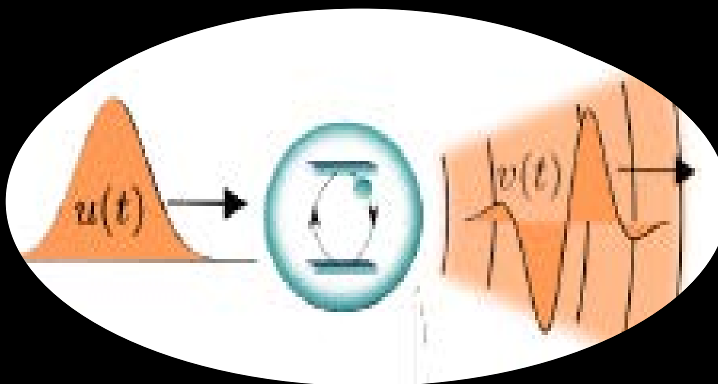

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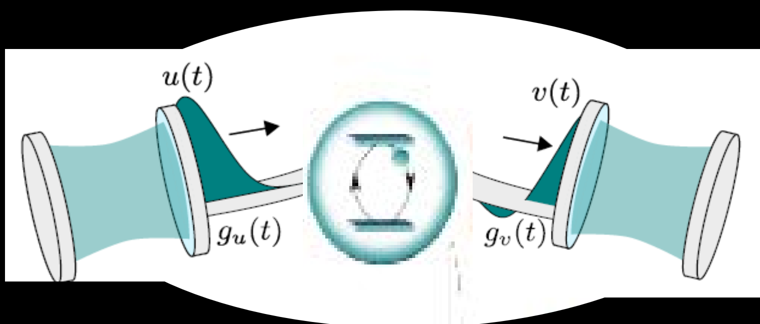


$$g_v(t) = -\frac{v^*(t)}{\sqrt{\int_0^t dt' \, |v(t')|^2}}$$

State contents of pulse $v(t)$ after the interaction ?

Cascaded Master Equation for $u(t)$-cavity + qubit + $v(t)$-cavity

More general "scatterer": H_s $\{L_i\}$

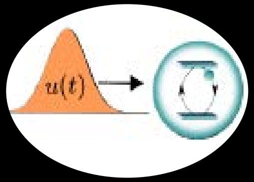
How does a quantum pulse interact with a qubit ?

More general ”scatterer”: H_s $\{L_i\}$

How does a quantum pulse interact with a qubit ?

More general "scatterer": H_s $\{L_i\}$

Quantum input pulse (u)	Quantum scatterer (s)	Quantum output pulse (v)	
X	X	0	ρ_{us}
0	X	X	ρ_{sv}
X	X	X	ρ_{usv}



How does a quantum pulse interact with a qubit ?

More general ”scatterer”: H_s $\{L_i\}$

	Quantum input pulse (u)	Quantum scatterer (s)	Quantum output pulse (v)	
➡	X	X	0	ρ_{us}
➡	0	X	X	ρ_{sv}
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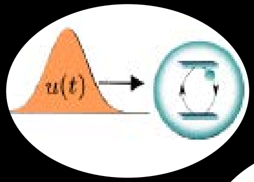

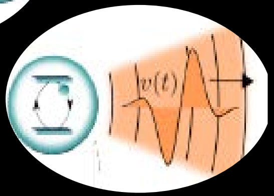

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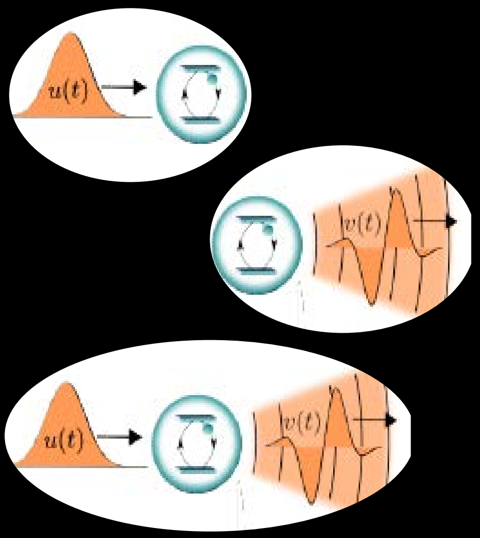

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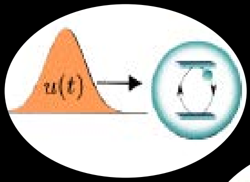
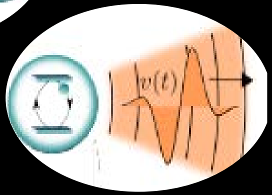
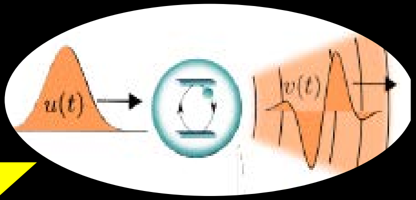






$$H = -\frac{i}{2}\left(g_u(t)\sqrt{\gamma}\, a_u\, \sigma^+ + g_u(t) g_v^*(t) a_u\, a_v^+ + \sqrt{\gamma} g_v(t) \sigma a_v^+ - h.c.\right)$$

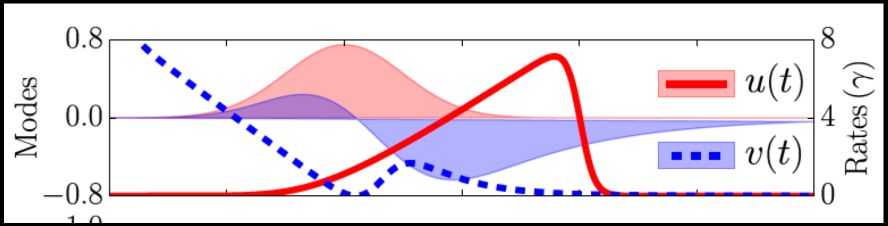
$$L = g_u(t) a_u + \sqrt{\gamma}\, \sigma + g_v(t) a_v$$

Chiral "Hamiltonian"
Excitations: "→"

Examples

Single photon scattering on an empty cavity ($\sigma \rightarrow c$).

Input wave packet $u(t)$, $g_u(t)$ → output wave packet $v(t)$, $g_v(t)$



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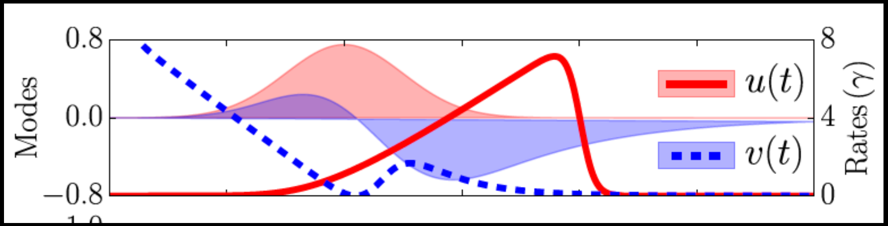
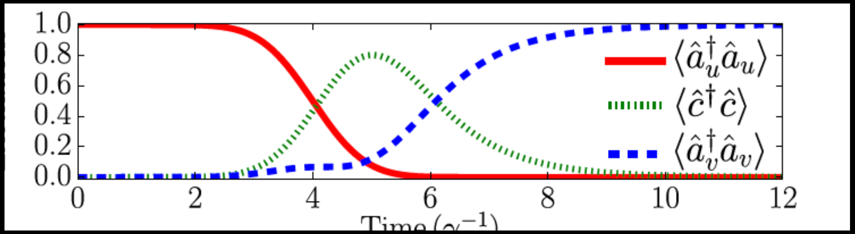


Result:
Occupation of input, cavity and output:



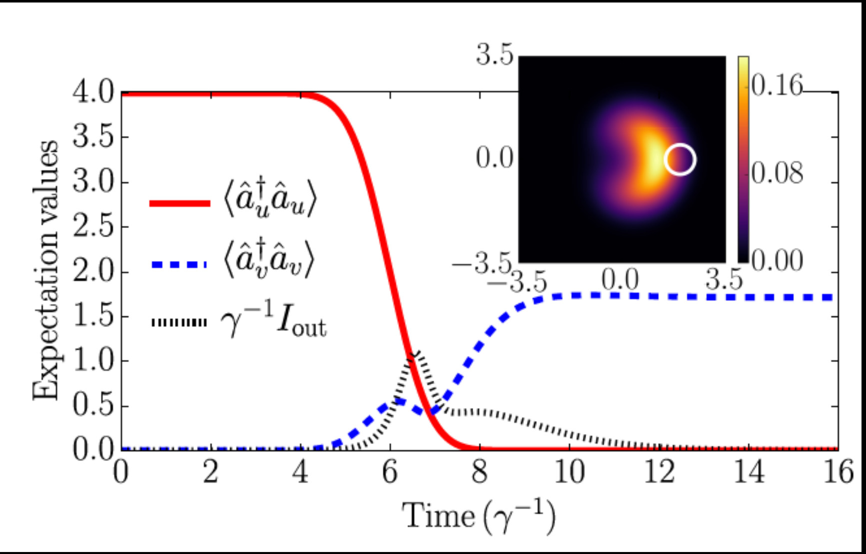
Examples

Scattering of a coherent state scattering on an empty cavity ($\sigma \rightarrow c$).

Cavity with phase noise (shaking mirror)

$|\alpha=2\rangle$ coherent input state ($\langle n\rangle=4$)

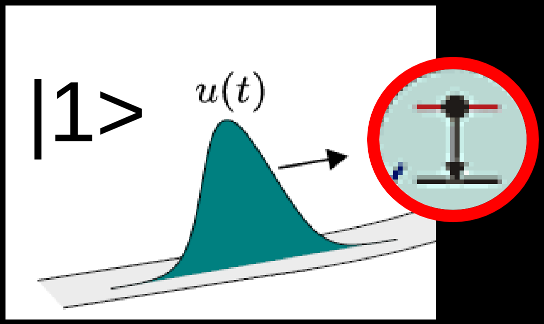
Output mode is damped and dephased, see *W(q,p)*



→ Output is multi-mode
We consider only the dominant output mode here.

Examples

Stimulated emission (same mode)



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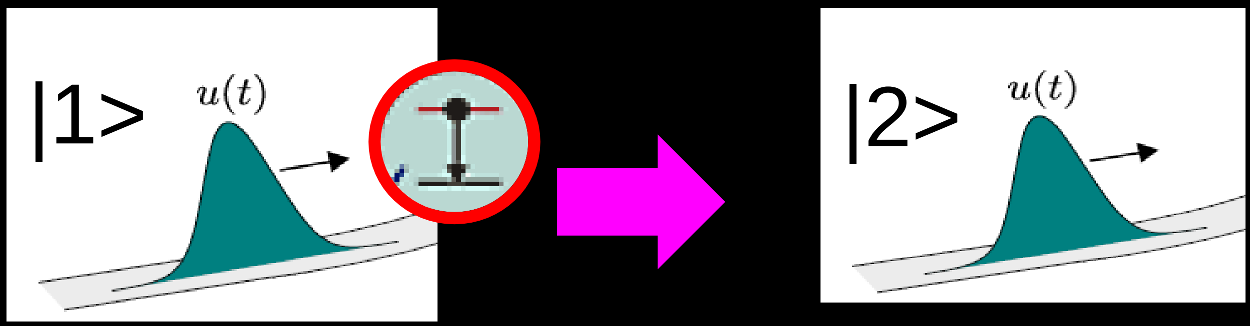

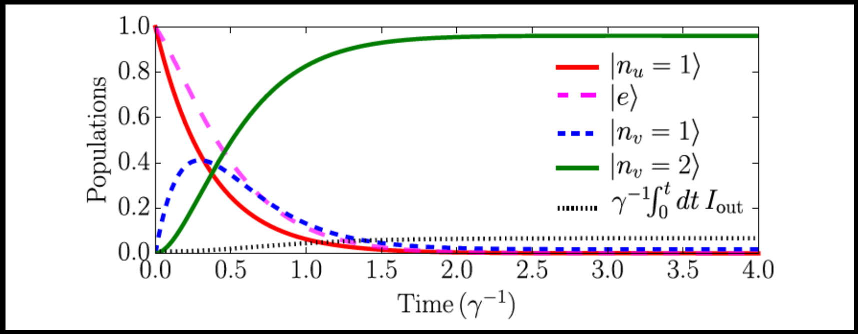

Examples

Schrödinger’s cat

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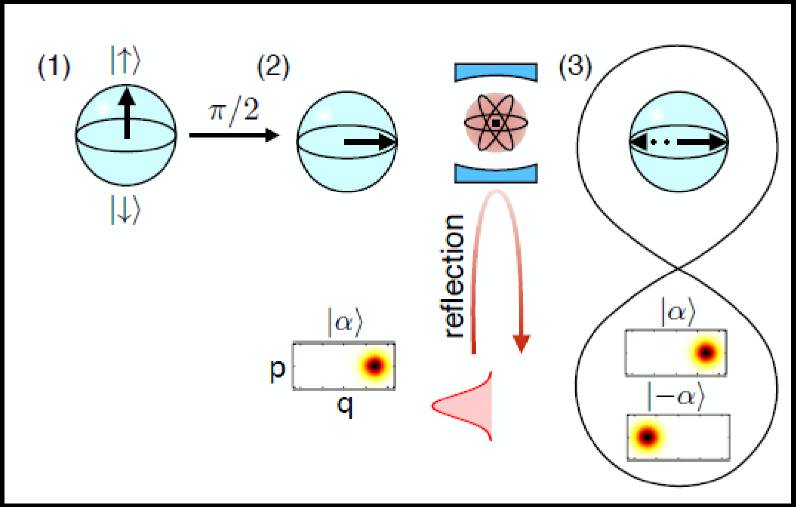

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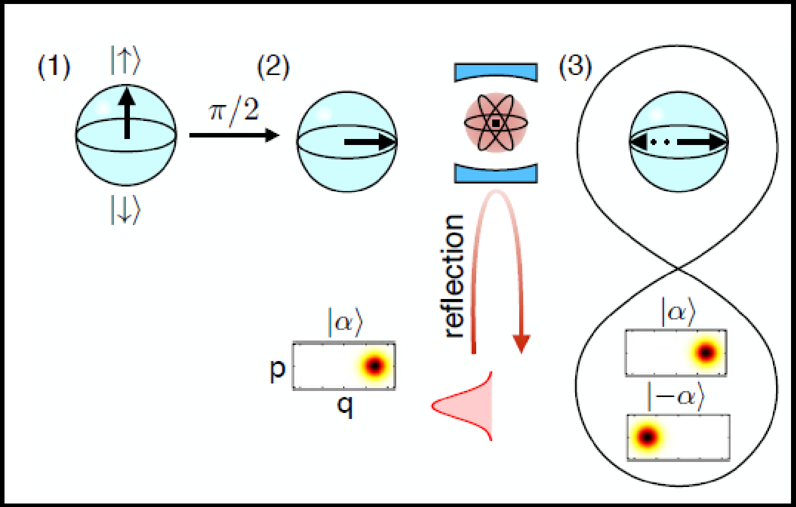
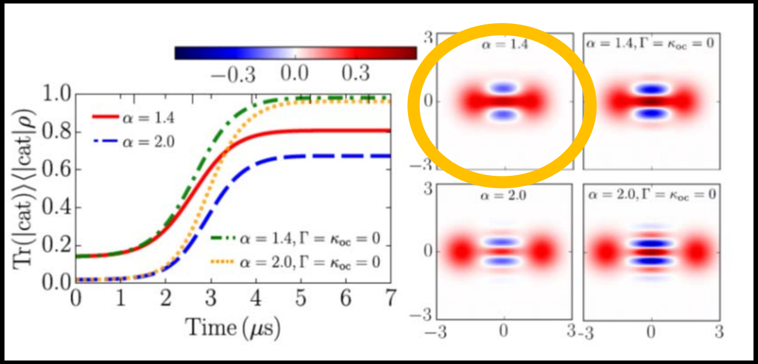
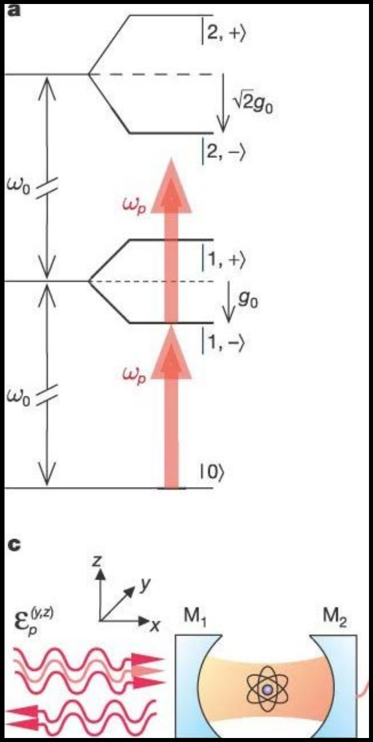
Schrödinger's cat





Hacker et al (Rempe group), Nature Photonics 2019.

The ”photon bandwidth dilemma”



Photon blockade in cavity QED

Wave packet incident on cavity with a single atom may be fully transmitted for 1 photon (resonant with eigenstate) and reflected for 2 or more photons (non resonant).

arXiv:2003.04573
Quantum interactions with pulses of radiation
A. Kiilerich, KM

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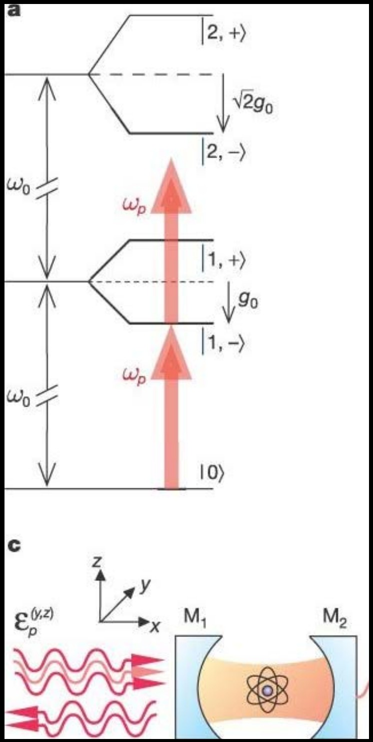

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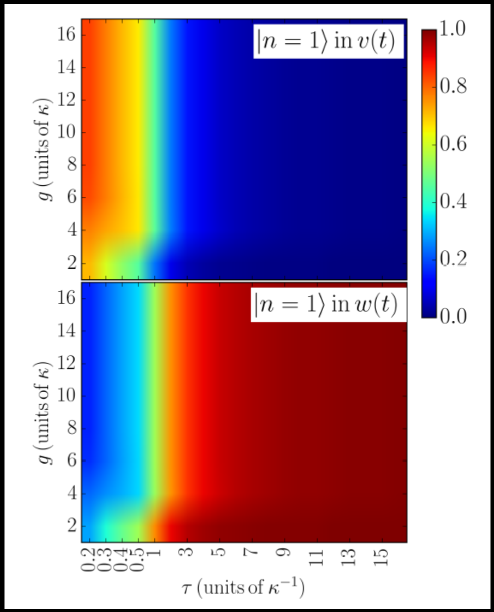
One photon, reflected

One photon, transmitted



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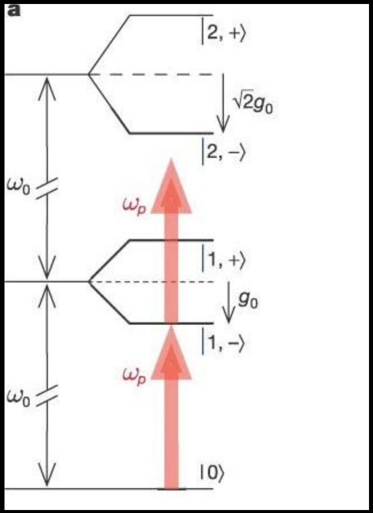

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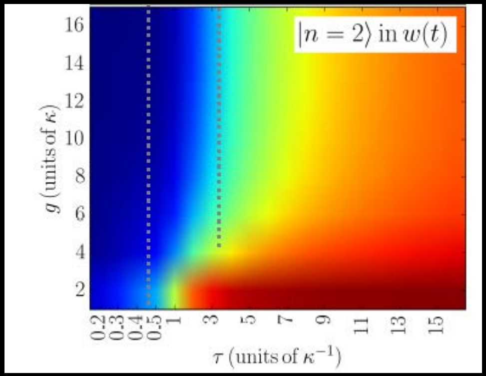

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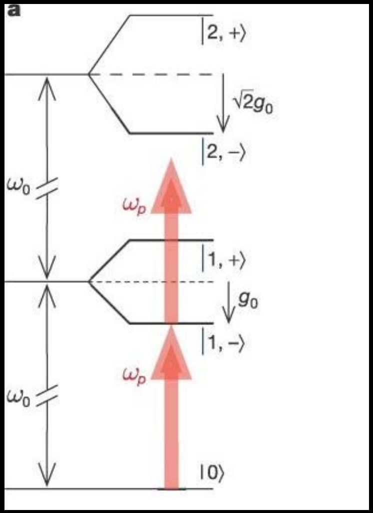

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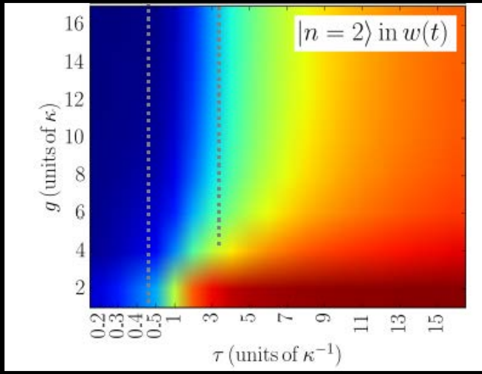

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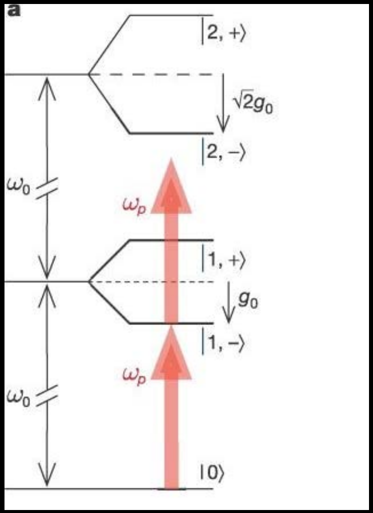

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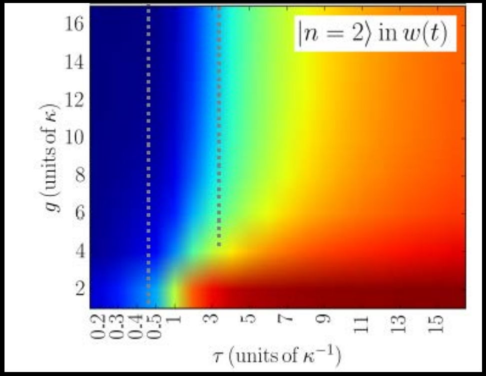

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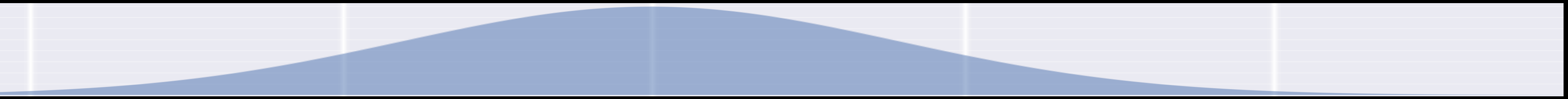

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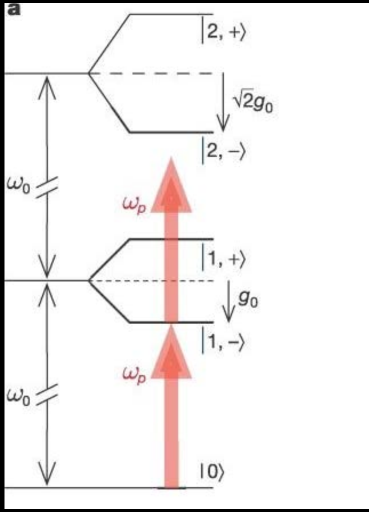

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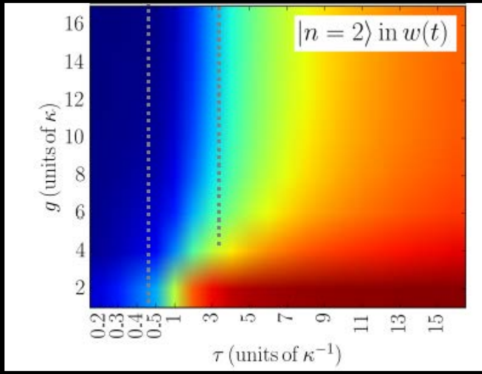

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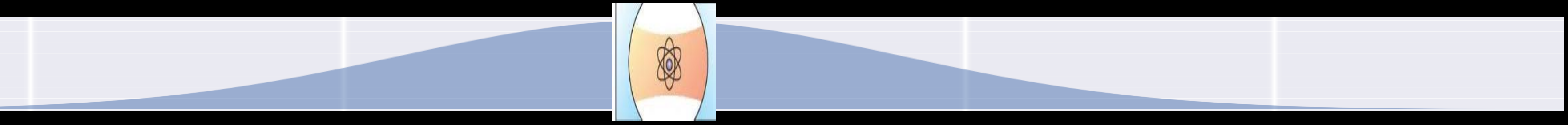

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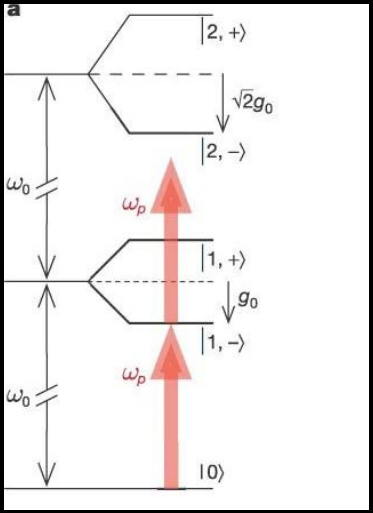

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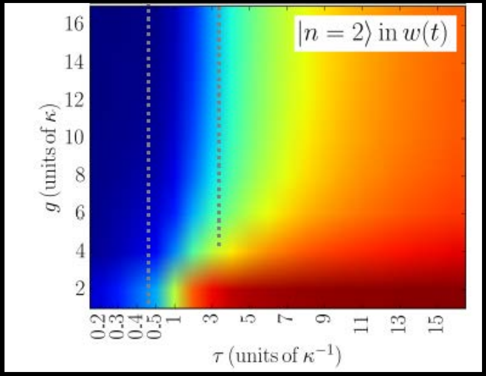

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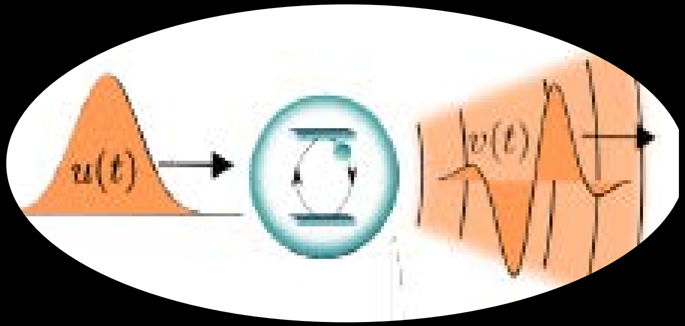
What is the output mode $v(t)$?



Output field:

$L = g_u^*(t) a_u + \sqrt{\gamma}\, \sigma$

$$g^1(t,t') = < L^+(t) L(t') >$$
$$= \Sigma_i\, n_i\, v_i^*(t) v_i(t')$$

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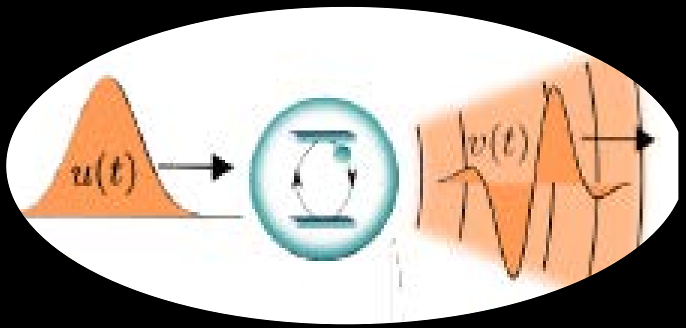

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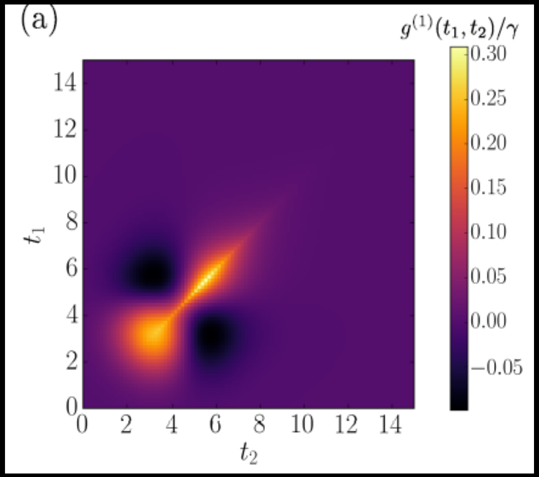
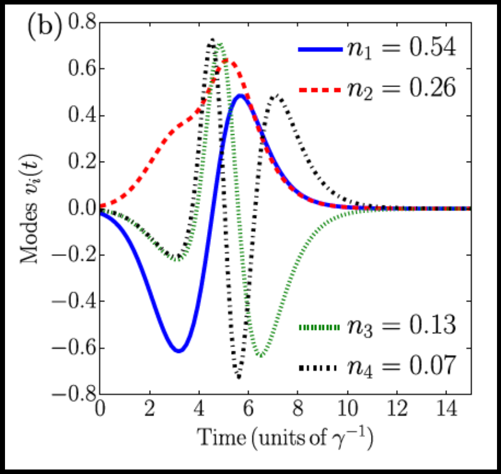
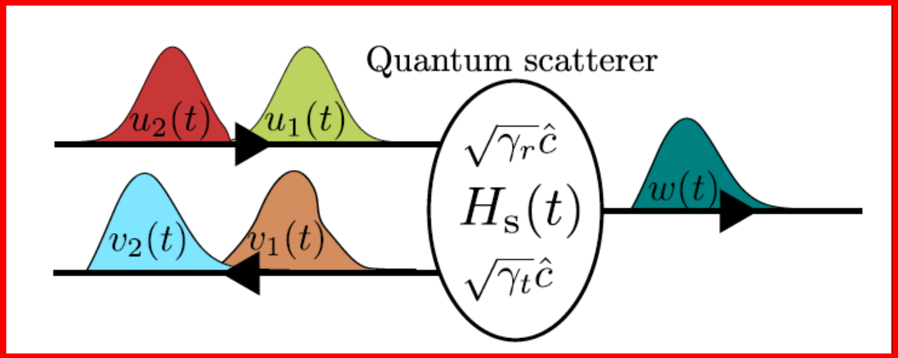
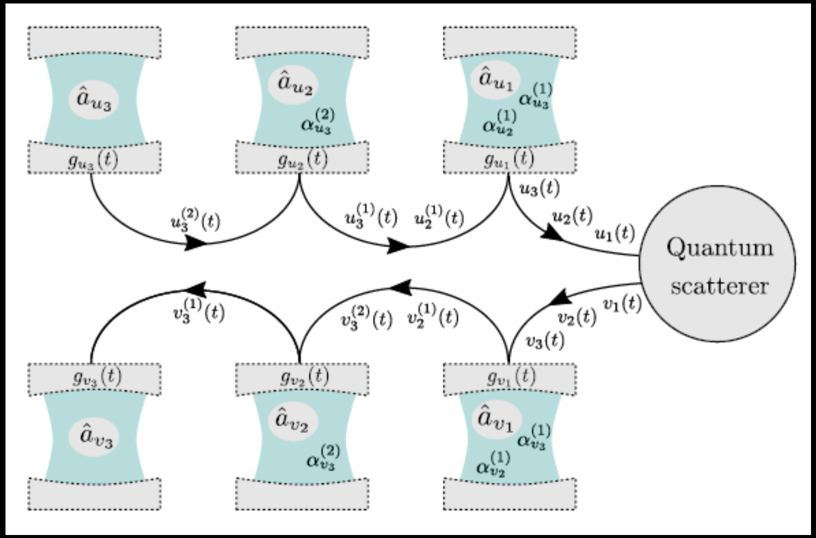
Coherent state on cavity with phase noise (shaking mirror)





Multiple input and output modes





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Conclusion

Sometimes photons move, and that is what we like about them ;=)

Moving photons occupy a continuum of modes, and their quantum states and dynamics are non-trivial.

Quantum information protocols rely on precise handling of the modes.

If we may restrict to few incident and outgoing travelling modes (solutions of classical wave equation), we can apply usual master equation theory.

Theory applies to any wave (is exact - but assumes Markov approximation and linear dispersion)

Thank you for your Attention !