AQIS 2020 Sydney – online
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Qu*d*it

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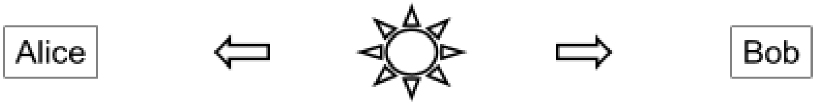
JK, J. Lee, S.-W. Ji, H. Nha, P. Anisimov, J. P. Dowling,
Opt Comm 337, 79 (2015)
arXiv:1012.5872

Interesting developments

- Entanglement
- Decoherence
- Quantum Thermodynamics

Entanglement
EPR & Nonlocality

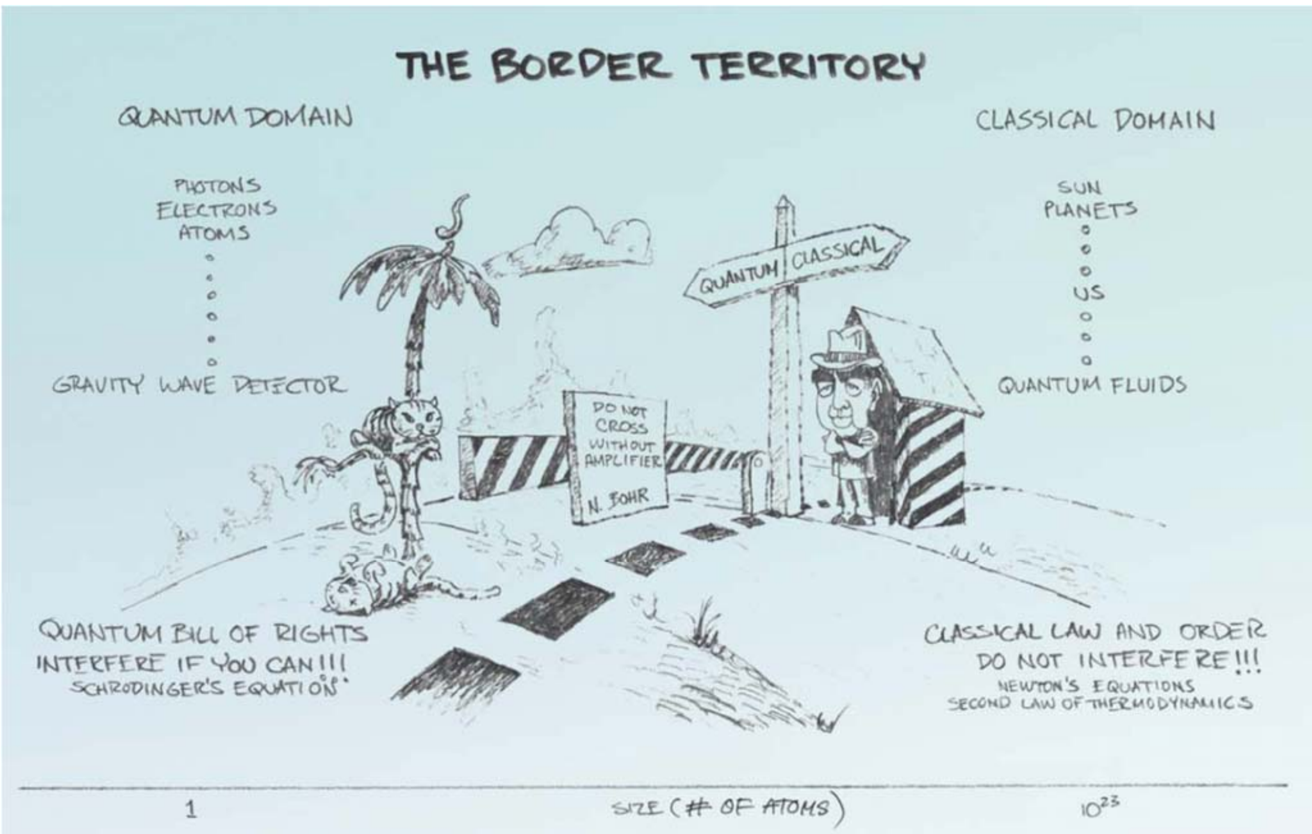


$$\frac{1}{\sqrt{2}}\left(|1\rangle_A|0\rangle_B - |0\rangle_A|1\rangle_B\right) \neq |\psi\rangle_A \otimes |\phi\rangle_B$$

Local Hidden Variable → Bell's Inequality

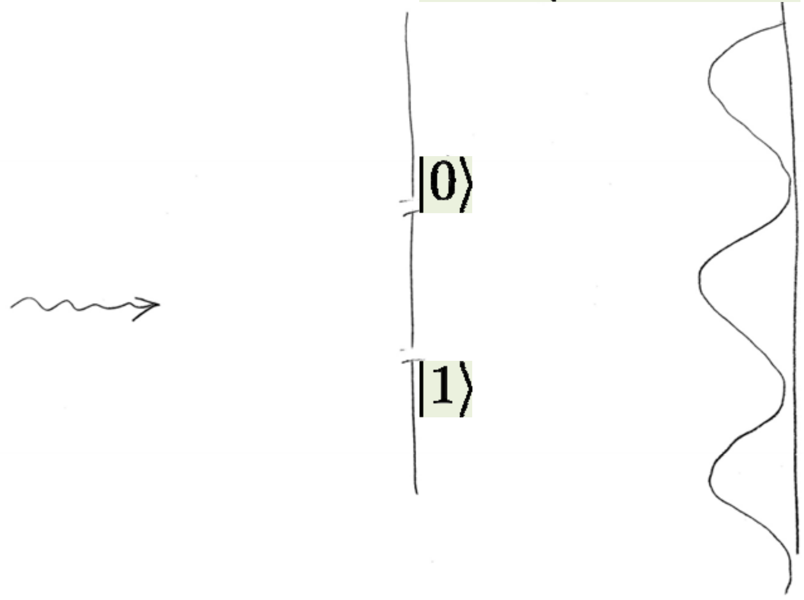
Aspect's Experiment → Quantum Mechanics is nonlocal!

GHZ State: $\frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C\right)$

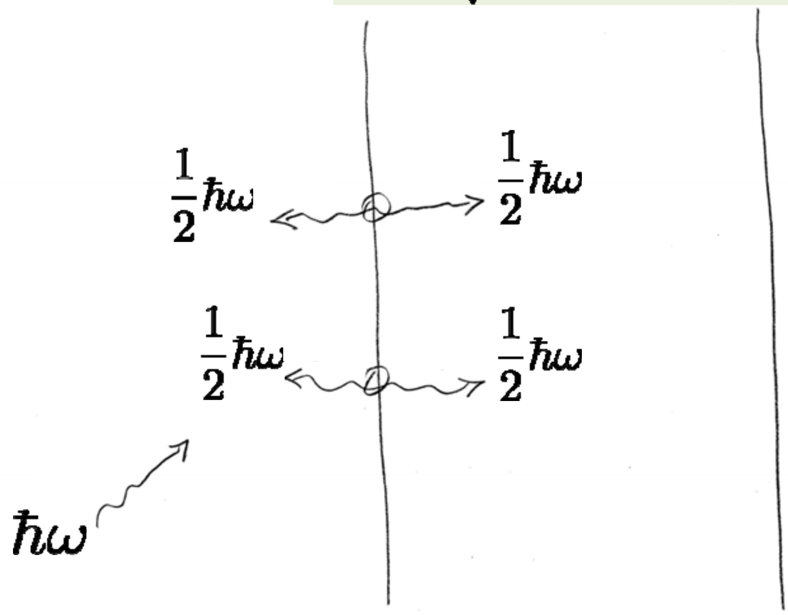


Zurek, QP/0306072

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

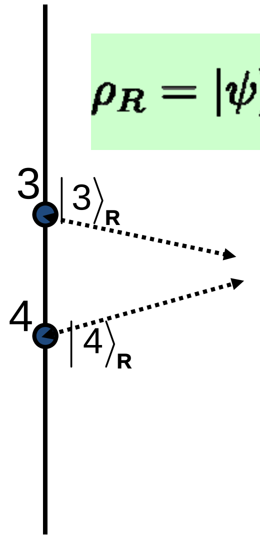


$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_L|0\rangle_R + |1\rangle_L|1\rangle_R\right)$$



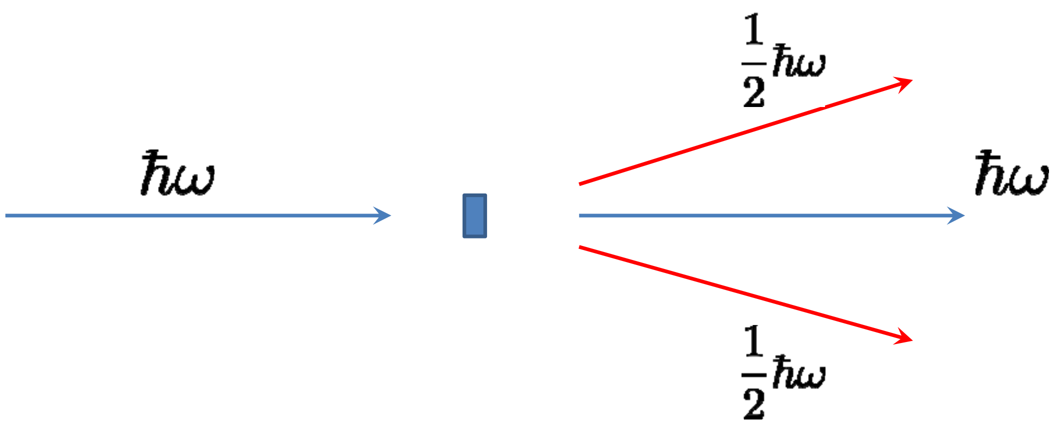
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_R + |4\rangle_R\right)$$

$$\rho_R = |\psi\rangle_R\langle\psi|_R = \begin{array}{c} \\ |3\rangle \\ |4\rangle \end{array}\!\!\begin{array}{c} \begin{array}{cc} \langle 3| & \langle 4| \end{array} \\ \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \end{array}$$



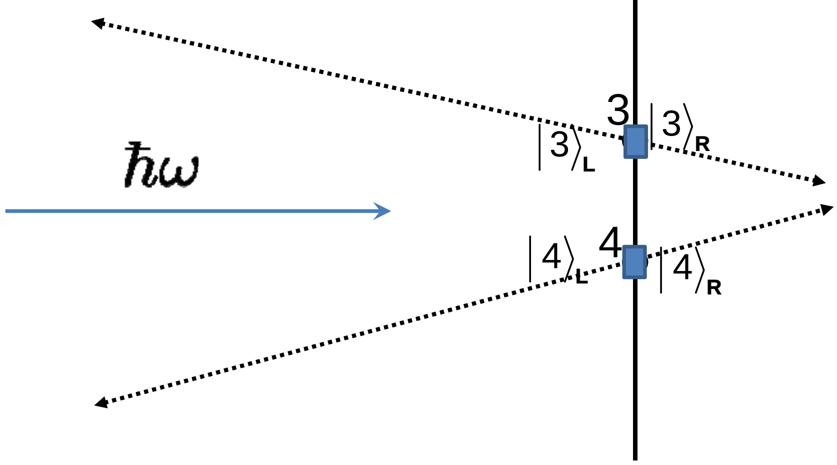
Parametric Down Conversion

: Splitting a photon into two



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_L|3\rangle_R + |4\rangle_L|4\rangle_R\right)$$

$$\rho_{LR} = |\Psi\rangle_{LR}\,\langle\Psi|_{LR}$$

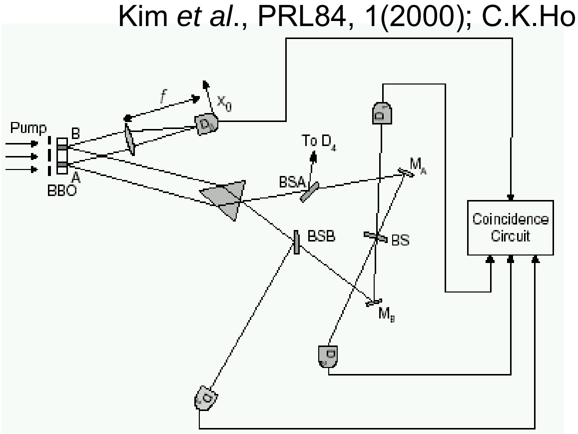


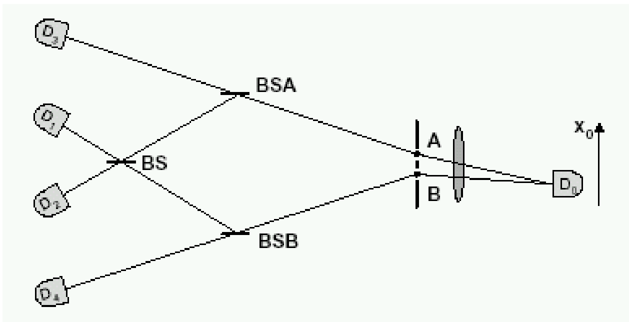
C K Hong and T Noh (1998)
Y-H Kim and Y Shi (2000)

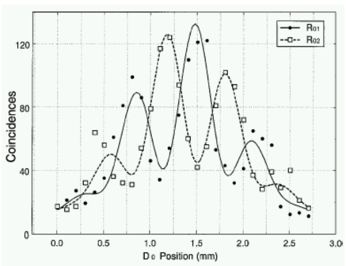
Delayed Choice Quantum Erasure

REVISITED

Kim *et al*., PRL84, 1(2000); C.K.Hong and T.G.Noh, JOSA B15, 1192(1998)







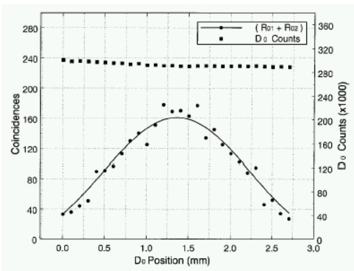


FIG. 4. $R_{01} + R_{02}$ is shown. The solid line is a fit to the sinc function given in Eq. (6). The single counting rate of D_0 is constant over the scanning range.

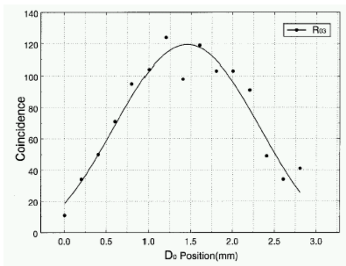
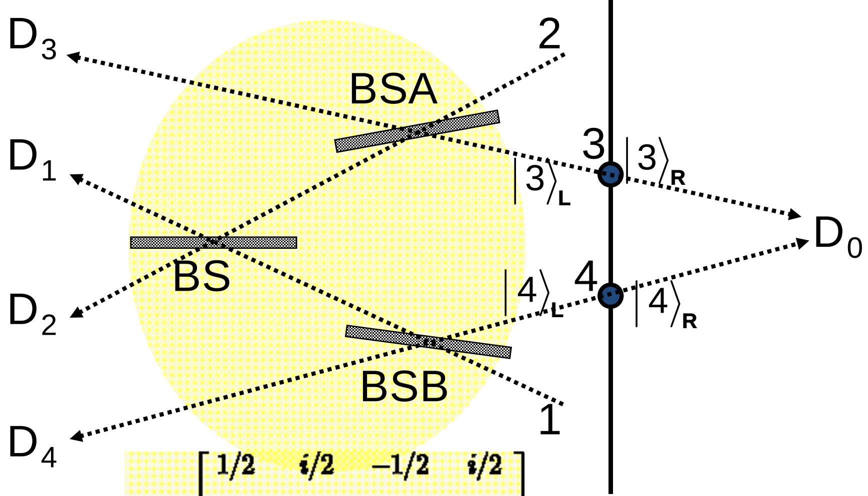


FIG. 5. R_{03} is shown. Absence of interference is clearly demonstrated. The solid line is a fit to the sinc function given in Eq. (6).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_L|3\rangle_R + |4\rangle_L|4\rangle_R\right)$$

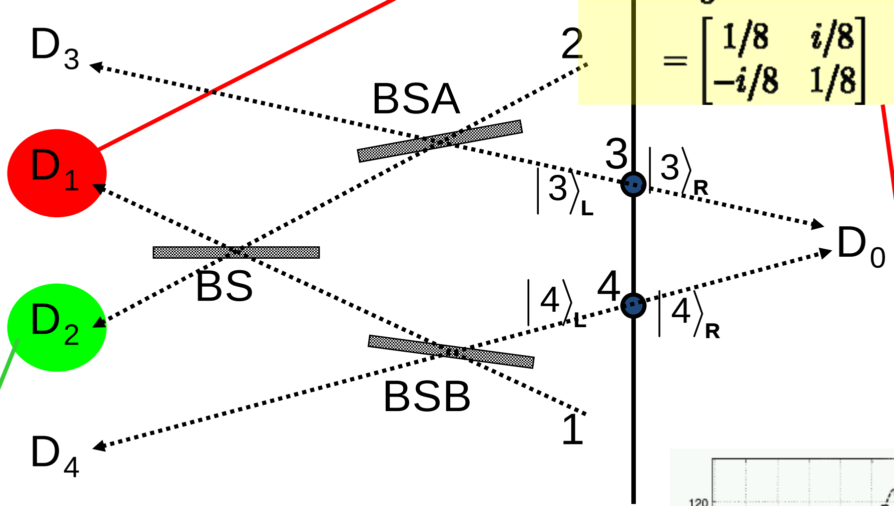
$$\rho_{LR} = |\Psi\rangle_{LR}\,\langle\Psi|_{LR}$$



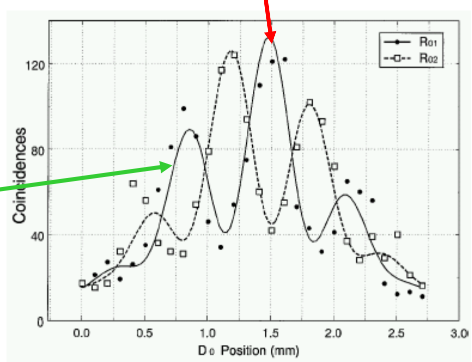
$$U_L = \begin{bmatrix} 1/2 & i/2 & -1/2 & i/2 \\ i/2 & 1/2 & i/2 & -1/2 \\ 0 & i/\sqrt{2} & 1/\sqrt{2} & 0 \\ i/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{bmatrix}$$

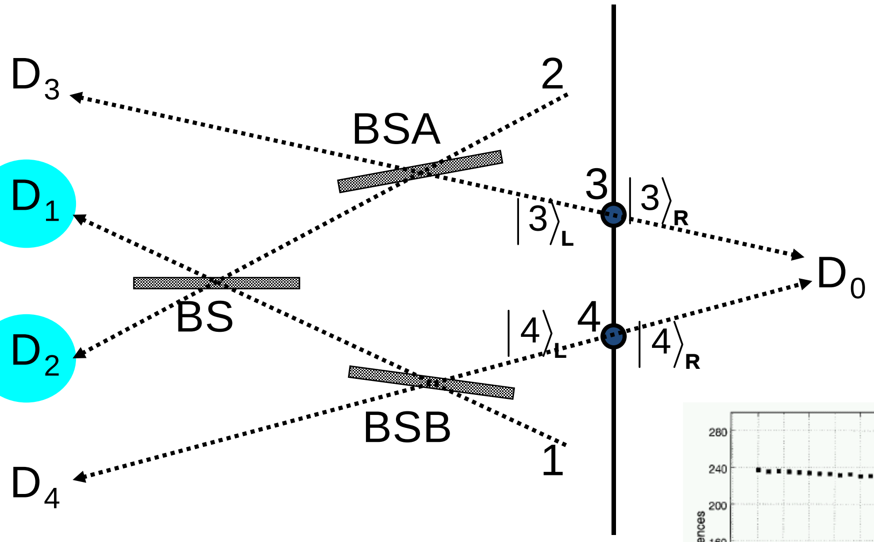
$$\rho'_{LR} = U_L|\Psi\rangle_{LR}\langle\Psi|_{LR}U_L^{\dagger} = \frac{1}{2}\left[\left(-\frac{|1\rangle_L}{2} + i\frac{|2\rangle_L}{2} + \frac{|3\rangle_L}{\sqrt{2}}\right)|3\rangle_R + \left(i\frac{|1\rangle_L}{2} - \frac{|2\rangle_L}{2} + \frac{|4\rangle_L}{\sqrt{2}}\right)|4\rangle_R\right][h.c.]$$

$$\begin{aligned}\rho'_{R|D_1} &= {}_L\langle 1|U_L|\Psi\rangle_{LR}\,\langle\Psi|_{LR}U_L^{\dagger}|1\rangle_L \\ &= \frac{1}{8}\left(-|3\rangle_R + i|4\rangle_R\right)\left(-{}_R\langle 3| - i\,{}_R\langle 4|\right) \\ &= \begin{bmatrix} 1/8 & i/8 \\ -i/8 & 1/8 \end{bmatrix}\end{aligned}$$



$$\begin{aligned}\rho'_{R|D_2} &= {}_L\langle 2|U_L|\Psi\rangle_{LR}\,\langle\Psi|_{LR}U_L^{\dagger}|2\rangle_L \\ &= \frac{1}{8}\left(i|3\rangle_R - |4\rangle_R\right)\left(-i_R\langle 3| - {}_R\langle 4|\right) \\ &= \begin{bmatrix} 1/8 & -i/8 \\ i/8 & 1/8 \end{bmatrix}\end{aligned}$$





$$\rho'_{R|(D_1+D_2)} = \begin{bmatrix}1/8 & i/8\\ -i/8 & 1/8\end{bmatrix} + \begin{bmatrix}1/8 & -i/8\\ i/8 & 1/8\end{bmatrix} = \begin{bmatrix}1/4 & 0\\ 0 & 1/4\end{bmatrix}$$

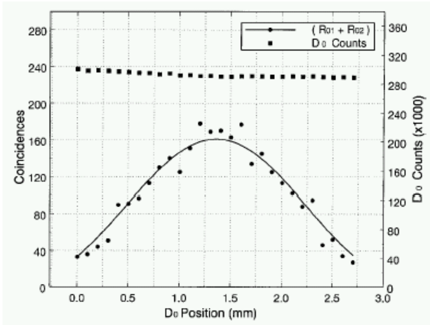


FIG. 4. $R_{01} + R_{02}$ is shown. The solid line is a fit to the sinc function given in Eq. (6). The single counting rate of D_0 is constant over the scanning range.

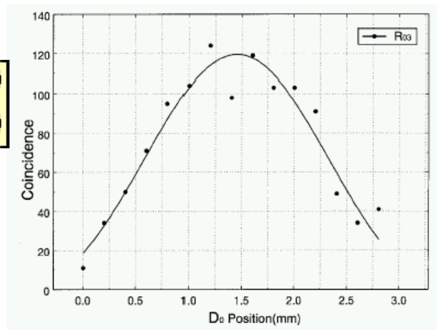
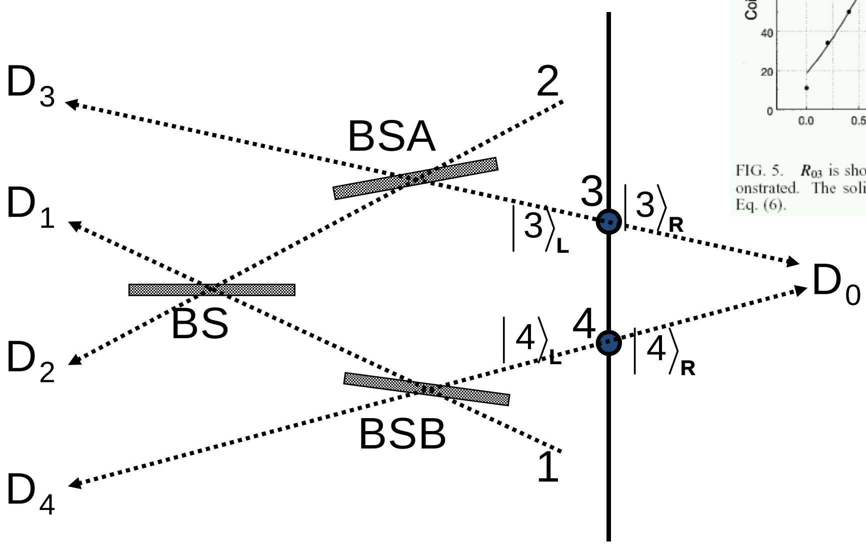


FIG. 5. R_{03} is shown. Absence of interference is clearly demonstrated. The solid line is a fit to the sinc function given in Eq. (6).

$$\rho'_{R|D_3} = {}_L\langle 3|U_L|\Psi\rangle_{LR}\ {}_{LR}\langle\Psi|U_L^\dagger|3\rangle_L = \frac{1}{4}|3\rangle_R\ {}_R\langle 3| = \begin{bmatrix}1/4 & 0\\ 0 & 0\end{bmatrix}$$

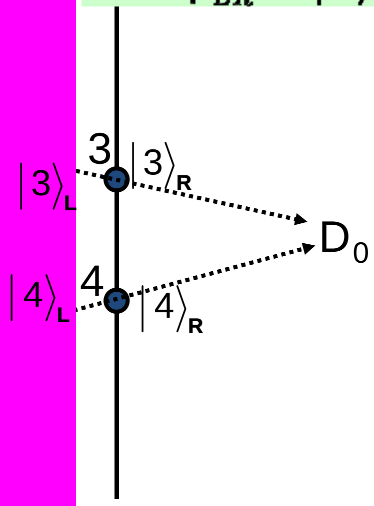


$$\rho'_{R|D_4} = {}_L\langle 4|U_L|\Psi\rangle_{LR}\ {}_{LR}\langle\Psi|U_L^\dagger|4\rangle_L = \frac{1}{4}|4\rangle_R\ {}_R\langle 4| = \begin{bmatrix}0 & 0\\ 0 & 1/4\end{bmatrix}$$

Environment Decoherence

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_L|3\rangle_R + |4\rangle_L|4\rangle_R\right)$$

$$\rho_{LR} = |\Psi\rangle_{LR}\,\langle\Psi|_{LR}$$



$$\rho_R = \mathrm{tr}_L\,\rho_{LR} = \frac{1}{2}\left\{|3\rangle_R\,\langle 3|_R + |4\rangle_R\,\langle 4|_R\right\} = \frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\rho'_{R|(D_1+D_2+D_3+D_4)} = \begin{bmatrix}1/4 & 0\\ 0 & 1/4\end{bmatrix} + \begin{bmatrix}1/4 & 0\\ 0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 1/4\end{bmatrix} = \begin{bmatrix}1/2 & 0\\ 0 & 1/2\end{bmatrix}$$

Objectivity, retrocausation, and the experiment of Englert, Scully, and Walther

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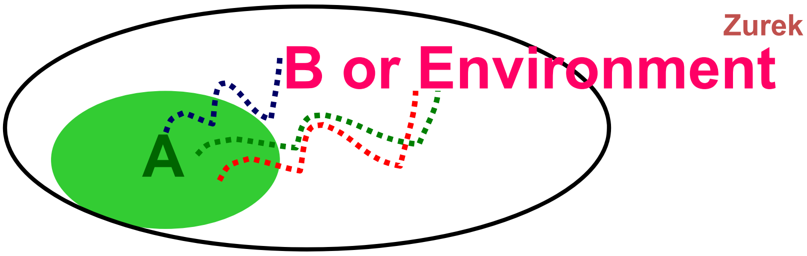
In a recent contribution to this journal [Am. J. Phys. **64**, 1468–1475 (1996)] I wrongly asserted that retrocausation in the Englert, Scully, and Walther (ESW) experiment (a double-slit interference experiment with atoms) can occur only until the atom arrives at the screen. In their response, Englert, Scully, and Walther [preceding paper] point out my fallacy but give an incomplete analysis of its origin. In this paper I trace this fallacy to a deep-seated preconception about time and reality. I show that among the two possible realistic interpretations of standard quantum mechanics, the reality-of-states view and the reality-of-phenomena view, only the latter is viable. It follows that retrocausation is a necessary feature of any realistic account of the ESW experiment based on standard quantum mechanics. Finally I elucidate the sense in which the spatial properties of quantum systems are objective, and show that they are extrinsic rather than intrinsic.

I. INTRODUCTION

In a recent article[1] I analyzed the thought experiment of Englert, Scully, and Walther[2,3] (ESW) from two ''meta-physical'' perspectives, the reality-of-states view and the reality-of-phenomena view. In that article I arrived at a wrong conclusion, for which I wish to express my sincere apologies to the readers of this journal. I compounded my mistake by attributing my views to Englert, Scully, and Walther. My apologies also to these authors! It ought to be mentioned, however, that I was argued into misrepresenting their views by the anonymous referee of my article. He/she not only agreed with my erroneous conclusion but also thought that ESW would likewise agree with it. For this the referee cannot be blamed, however, for it is only in their

slits. The cavities are designed to force each atom to emit a microwave photon. They are separated from each other by a pair of shutters between which a photosensor is placed (see Fig. 1). Each atom leaves a mark where it hits the screen. If one simply looks at the distribution of marks created by a large number of atoms, no interference pattern is seen. But quantum mechanics predicts that if the experimenters open the shutters (this can be done well after the corresponding atom has reached the screen) and consider separately the cases in which the sensor responds and the cases in which it does not respond, they will be able to discern two complementary interference patterns. Alternatively, they can leave the shutters closed and ascertain the cavity that contains the photon. They thus appear to have a choice between either

Entanglement and Decoherence



When system A is entangled with environment, state of A cannot be described by a state vector, but by a density matrix.

$$|\Psi\rangle_{AB} = a\,|0\rangle_A\,|0\rangle_B + b\,|1\rangle_A\,|1\rangle_B \neq |\psi\rangle_A\,|\phi\rangle_B$$

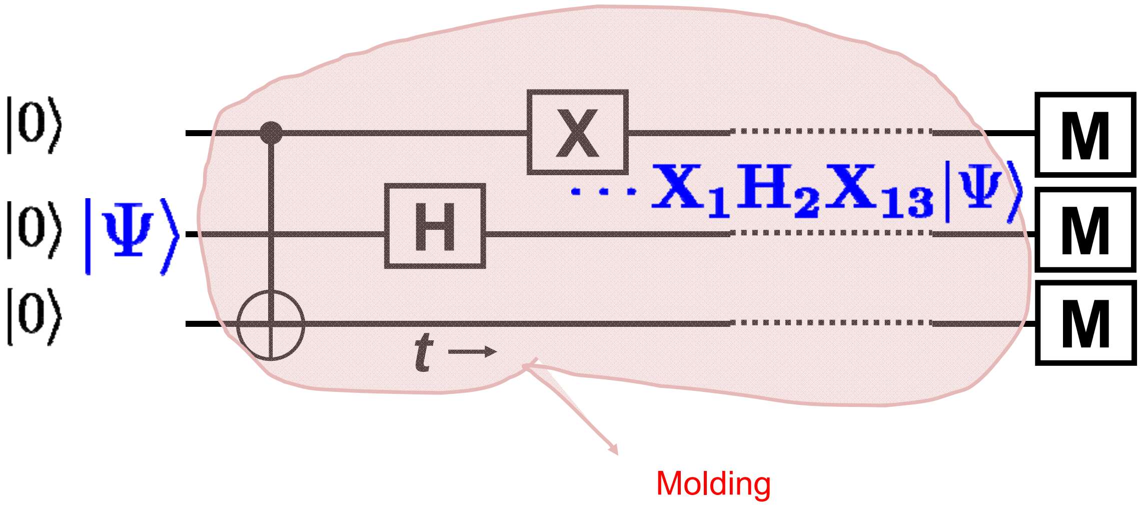
ab*

$$\rho_A = Tr_B\,\rho_{AB} = Tr_B\,|\Psi\rangle_{AB\;AB}\langle\Psi| = \boxed{\begin{matrix} |a|^2 & 0 \\ 0 & |b|^2 \end{matrix}}$$

ba*

$$\neq |\psi\rangle_{A\;A}\langle\psi|$$

MOLDING a Quantum State



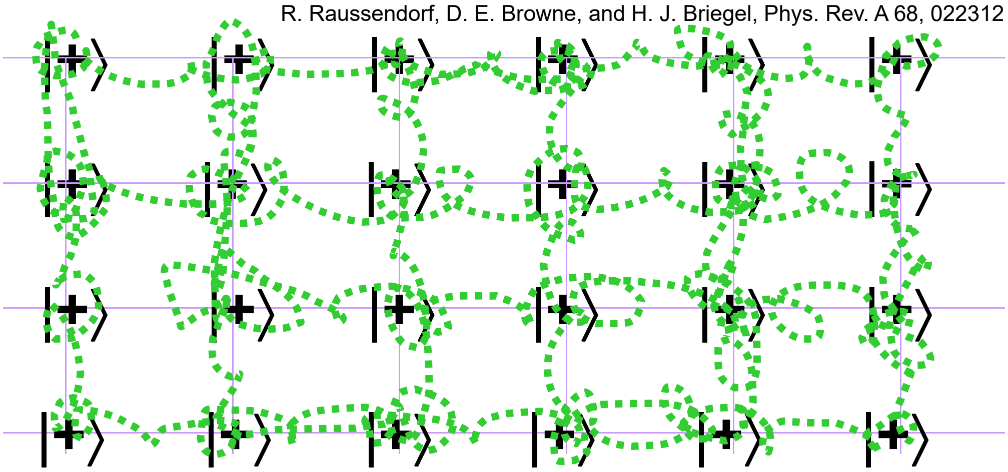
E. Knill, R. Laflamme, and G. J. Milburn, Nature 409, 46 (2001).
M. A. Nielsen, Phys. Rev. Lett. 93, 040503 (2004).
M. A. Nielsen and C. M. Dawson, Phys. Rev. A 71, 042323 (2005).

SCULPTURING a Quantum State

- Cluster State [One-way] Quantum Computing -

R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. 86, 5188 (2001).

R. Raussendorf, D. E. Browne, and H. J. Briegel, Phys. Rev. A 68, 022312 (2003).



1. Initialize each qubit in the state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$
2. Contolled-Phase(CZ) between the neighboring qubits.
3. Single qubit manipulations and single qubit measurements only [Sculpturing].
   No two qubit operations!

Cont-Z and $|+\rangle$

- Commuting with each other
- Symmetric w.r.t. control and target

Even superposition
of computational basis states

$$
\begin{aligned}
Z_{12}|+\rangle_1|+\rangle_2 &= Z_{12}\left(\frac{|0\rangle_1+|1\rangle_1}{\sqrt{2}}\right)\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right) \\
&= Z_{12}\frac{|0\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right) + Z_{12}\frac{|1\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right) \\
&= \frac{|0\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right) + \frac{|1\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2-|1\rangle_2}{\sqrt{2}}\right) \\
&= \frac{1}{\sqrt{2}}|0\rangle_1|+\rangle_2 + \frac{1}{\sqrt{2}}|1\rangle_1|-\rangle_2
\end{aligned}
$$

$$
\begin{aligned}
Z_{12}Z_{23}|+\rangle_1|+\rangle_2|+\rangle_3 &= Z_{12}Z_{23}|+\rangle_1\frac{|0\rangle_2}{\sqrt{2}}|+\rangle_3 + Z_{12}Z_{23}|+\rangle_1\frac{|1\rangle_2}{\sqrt{2}}|+\rangle_3 \\
&= \frac{1}{\sqrt{2}}|+\rangle_1|0\rangle_2|+\rangle_3 + \frac{1}{\sqrt{2}}|-\rangle_1|1\rangle_2|-\rangle_3
\end{aligned}
$$

B. C. Sanders and G. J. Milburn, Phys. Rev. A 45, 1919 (1992).
M. Paternostra et al., Phys. Rev. A 67, 023811 (2003).
Wang W.-F. et al., Chin. Phys. Lett. 25, 839 (2008)
Nguyen B. A. and J. Kim, Phys. Rev. A 80, 042316 (2009).

Exponential Function

$$
\begin{aligned}
e^x &= 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^{d-1}}{(d-1)!} \\
&\quad + \frac{x^d}{d!} + \frac{x^{d+1}}{(d+1)!} + \frac{x^{d+2}}{(d+2)!} + \cdots + \frac{x^{2d-1}}{(2d-1)!} \\
&\quad + \frac{x^{2d}}{(2d)!} + \frac{x^{2d+1}}{(2d+1)!} + \frac{x^{2d+2}}{(2d+2)!} + \cdots + \frac{x^{3d-1}}{(3d-1)!} \\
&\quad + \cdots \\
&= f_0(x) + f_1(x) + f_2(x) + \cdots + f_{d-1}(x) \\
&= \sum_{k=0}^{d-1} f_k(x)
\end{aligned}
$$

$$f_k(x) = \sum_{m=0}^{\infty} \frac{x^{k+md}}{(k+md)!} \qquad \text{for } k = 0, 1, 2, \cdots, d-1$$

$$
\begin{aligned}
f_0(x) &= 1 + \frac{x^d}{d!} + \frac{x^{2d}}{(2d)!} + \cdots \\
f_1(x) &= \frac{x}{1!} + \frac{x^{d+1}}{(d+1)!} + \frac{x^{2d+1}}{(2d+1)!} + \cdots \\
f_2(x) &= \frac{x^2}{2!} + \frac{x^{d+2}}{(d+2)!} + \frac{x^{2d+2}}{(2d+2)!} + \cdots \\
&\vdots \\
f_{d-1}(x) &= \frac{x^{d-1}}{(d-1)!} + \frac{x^{2d-1}}{(2d-1)!} + \frac{x^{3d-1}}{(3d-1)!} + \cdots
\end{aligned}
$$

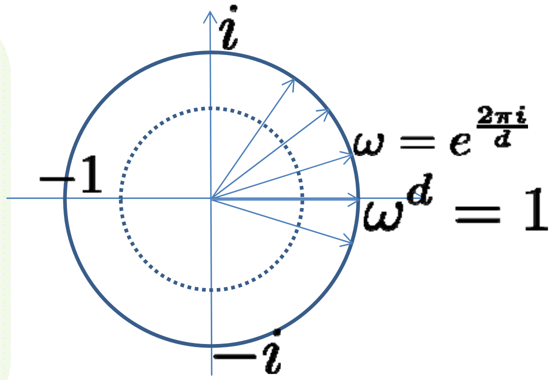
$$f'_{d-1}(x) = f_{d-2}(x),\ f'_{d-2}(x) = f_{d-3}(x),\ \cdots,\ f'_0(x) = f_{d-1}(x)$$

$$\lim_{x\to\infty} \frac{f_k(x)}{e^x} = \frac{1}{d} \qquad O\left(e^{-\frac{2\pi^2}{d^2}x}\right)$$

Conjugate Relations

$$e_s(x) \equiv e^{\omega^s x} \quad \text{with } \omega = e^{\frac{2\pi i}{d}}.$$

$$\begin{cases} e_s(x) = \displaystyle\sum_{k=0}^{d-1} \omega^{sk} f_k(x) \\ f_k(x) = \displaystyle\frac{1}{d}\sum_{s=0}^{d-1} \omega^{-ks} e_s(x) \end{cases}$$



Coherent State

$$\begin{aligned}
|\alpha\rangle &= e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \\
&= e^{-\frac{|\alpha|^2}{2}} \Big(\frac{\alpha^0}{\sqrt{0!}}|0\rangle + \frac{\alpha^1}{\sqrt{1!}}|1\rangle + \frac{\alpha^2}{\sqrt{2!}}|2\rangle + \cdots + \frac{\alpha^{d-1}}{\sqrt{(d-1)!}}|d-1\rangle \\
&\quad + \frac{\alpha^d}{\sqrt{d!}}|d\rangle + \frac{\alpha^{d+1}}{\sqrt{(d+1)!}}|d+1\rangle + \frac{\alpha^{d+2}}{\sqrt{(d+2)!}}|d+2\rangle + \cdots + \frac{\alpha^{2d-1}}{\sqrt{(2d-1)!}}|2d-1\rangle \\
&\quad + \frac{\alpha^{2d}}{\sqrt{(2d)!}}|2d\rangle + \frac{\alpha^{2d+1}}{\sqrt{(2d+1)!}}|2d+1\rangle + \frac{\alpha^{2d+2}}{\sqrt{(2d+2)!}}|2d+2\rangle + \cdots + \frac{\alpha^{3d-1}}{\sqrt{(3d-1)!}}|3d-1\rangle \\
&\quad + \cdots \Big) \\
&= \frac{1}{\sqrt{d}} \Big(|0_d\rangle + |1_d\rangle + |2_d\rangle + \cdots + |(d-1)_d\rangle \Big) \\
&= \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k_d\rangle
\end{aligned}$$

Pseudo-Number State

$$|k_d\rangle = \sqrt{d}\cdot e^{-\frac{|\alpha|^2}{2}} \sum_{m=0}^{\infty} \frac{\alpha^{k+md}}{\sqrt{(k+md)!}}|k+md\rangle$$

$$|\alpha\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k_d\rangle$$

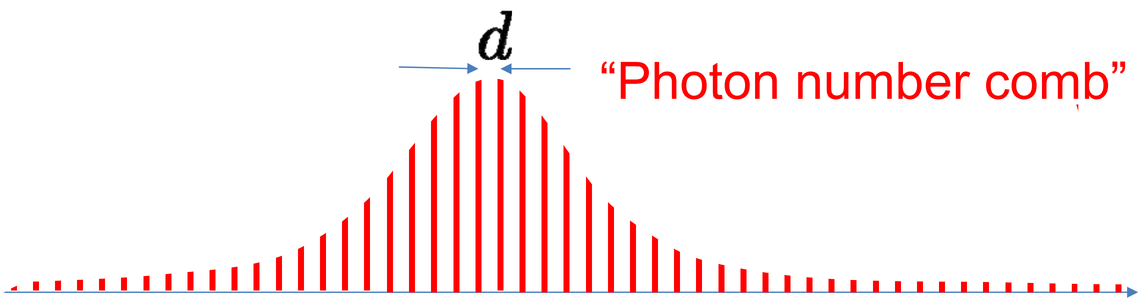
$$\begin{aligned} \langle k_d|k_d\rangle &= d\cdot e^{-|\alpha|^2}\cdot \sum_{m=0}^{\infty} \frac{|\alpha|^{2(k+md)}}{(k+md)!} \\ &= d\cdot \frac{f_k(|\alpha|^2)}{e^{|\alpha|^2}} \\ &\xrightarrow{|\alpha|^2\to\infty} 1 \end{aligned}$$

$$O\left(e^{-\frac{2\pi^2}{d^2}|\alpha|^2}\right)$$

As $|\alpha|^2$ tends to ∞, $\langle k_d|l_d\rangle \longrightarrow \delta_{kl}$.

Pseudo-number State

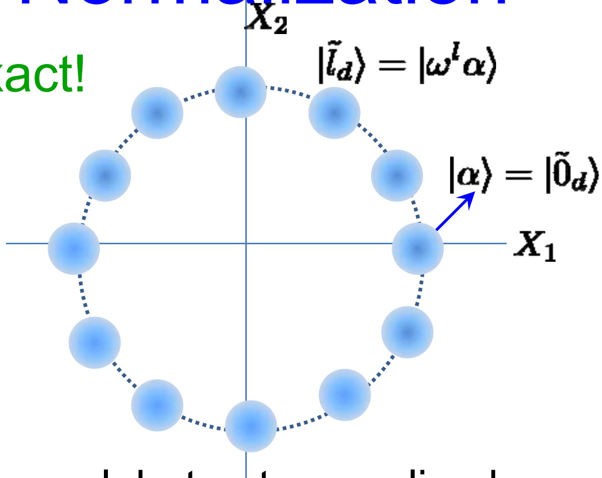
$$|k_d\rangle = \sqrt{d}\cdot e^{-\frac{|\alpha|^2}{2}} \sum_{m=0}^{\infty} \frac{\alpha^{k+md}}{\sqrt{(k+md)!}}|k+md\rangle$$



Orthogonality & Normalization

Conjugate Relations are exact!

$$\begin{cases} |\tilde{l}_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{l=0}^{d-1} \omega^{lk} |k_d\rangle \\ |k_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{l=0}^{d-1} \omega^{-kl} |\tilde{l}_d\rangle \end{cases}$$



Pseudo-number states: Orthogonal, but not normalized.
Pseudo-phase states: Normalized, but not orthogonal.
As |α| gets bigger, they become ortho-normalized.

$$O\left(e^{-\frac{2\pi^2}{d^2}|\alpha|^2}\right)$$

Practically $|\alpha| \geq d$.

Qubits and Qu*d*its

Computational Basis

$\{|0\rangle, |1\rangle\}$

Conjugate Basis

$\{|+\rangle, |-\rangle\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Computational Basis

: pseudo-number basis

$\{|0_d\rangle, |1_d\rangle, \cdots, |(d-1)_d\rangle\}$

Conjugate Basis

: pseudo-phase basis

$\{|\tilde{0}_d\rangle = |\alpha\rangle, |\tilde{1}_d\rangle = |\omega\alpha\rangle, \cdots, |\widetilde{(d-1)}_d\rangle\}$

$\omega = e^{\frac{2\pi i}{d}}$

$$\begin{cases} |k_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{s=0}^{d-1} \omega^{-kl} |\tilde{l}_d\rangle \\ |\tilde{l}_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{k=0}^{d-1} \omega^{lk} |k_d\rangle \end{cases}$$

Qubit Operators and Qu*d*it Operators

$$X=\begin{bmatrix}0&1\\1&0\end{bmatrix}$$

$$Z=\begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

$$\mathrm{CNOT}=X_{12}=|0\rangle_{1\,1}\langle 0|\otimes I_2+|1\rangle_{1\,1}\langle 1|\otimes X_2$$

$$=\left[\begin{array}{cc|cc}1&0&&\\0&1&\multicolumn{2}{c}{0}\\\hline &&0&1\\\multicolumn{2}{c|}{0}&1&0\end{array}\right]$$

$$\text{C-Z}=Z_{12}=|0\rangle_{1\,1}\langle 0|\otimes I_2+|1\rangle_{1\,1}\langle 1|\otimes Z_2$$

$$=\left[\begin{array}{cc|cc}1&0&&\\0&1&\multicolumn{2}{c}{0}\\\hline &&1&0\\\multicolumn{2}{c|}{0}&0&-1\end{array}\right]$$

$$H=\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}=|+\rangle\langle 0|+|-\rangle\langle 1|$$

$$X=\begin{bmatrix}0&1&0&&&\\&0&1&\ddots&&\\&&0&\ddots&\ddots&\\&&&\ddots&1&0\\&&&&0&1\\1&&&&&0\end{bmatrix}$$

$$\omega=e^{\frac{2\pi i}{d}}$$

$$Z=\begin{bmatrix}1&0&0&\cdots&&0\\&\omega&0&\ddots&&\\&&\omega^2&\ddots&\ddots&\vdots\\&&&\ddots&0&0\\&&&&\omega^{d-2}&0\\0&&&&&\omega^{d-1}\end{bmatrix}=\sum_{k=0}^{d-1}\omega^k|k_d\rangle\langle k_d|=\omega^{\hat{n}}$$

$$\text{C-Z}=Z_{12}=|0_d\rangle_{1\,1}\langle 0_d|\otimes I_2+|1_d\rangle_{1\,1}\langle 1_d|\otimes Z_2+|2_d\rangle_{1\,1}\langle 2_d|\otimes Z^2{}_2+\cdots$$

$$=\sum_{k=0}^{d-1}\sum_{l=0}^{d-1}|k_d\rangle_{1\,1}\langle k_d|\,\omega^{kl}\,|l_d\rangle_{2\,2}\langle l_d|=\omega^{\hat{n}_1\hat{n}_2}$$

$$H=\frac{1}{\sqrt{d}}\begin{bmatrix}1&1&1&\cdots&\cdots&1\\1&\omega&\omega^2&\omega^3&\cdots&\\1&\omega^2&\omega^4&\omega^6&&\\\vdots&\omega^3&\omega^6&\ddots&&\\\vdots&\vdots&&&\ddots&\\1&&&&&\omega^{(d-1)^2}\end{bmatrix}=\sum_{k=0}^{d-1}|\tilde{k}_d\rangle\langle k_d|$$

Cf. D. L. Zhou et al., Phys. Rev. A 68, 062303 (2003).

D. T. Pegg and S. M. Barnett,
Phys. Rev. A 39, 1665 (1989).

Generalized X Operator
Pseudo-Phase Operator
~ Pegg-Barnett Phase Operator

$$X=\sum_{l=0}^{d-1}\left|(l-1)_d\right\rangle\langle l_d|\quad\text{with }\left|(-1)_d\right\rangle\equiv\left|(d-1)_d\right\rangle$$

$$Z=\sum_{l=0}^{d-1}\omega^l|l_d\rangle\langle l_d|=\omega^{\hat{n}}$$

Generalized Z Operator
Pseudo-Number Operator
~ Pegg-Barnett Number Operator
→ Phase shifter

$$H=\sum_{l=0}^{d-1}|\tilde{l}_d\rangle\langle l_d|$$

Generalized Hadamard Operator
→ One-step teleportation

$$CZ=Z_{\mathrm{ct}}=\sum_{l=0}^{d-1}\sum_{m=0}^{d-1}\omega^{lm}|l_d\rangle_{\mathrm{c\,c}}\langle l_d|\otimes|m_d\rangle_{\mathrm{c\,c}}\langle m_d|=\omega^{\hat{n}_1\hat{n}_2}$$

Generalized Cont-Z Operator
→ Cross Kerr Interaction

Generalized Controlled-Z Operator

$$H = -\chi \hat{n}_1 \hat{n}_2$$ Cross Kerr Interaction

$$|\chi L| = \frac{2\pi}{d},\ d \lesssim |\alpha|$$

($d \cong$ 10~1000 ?!)

$$U = e^{i\chi L \hat{n}_1 \hat{n}_2} = e^{\frac{2\pi i}{d}\hat{n}_1 \hat{n}_2} = \omega^{\hat{n}_1 \hat{n}_2} = Z_{12}$$

$$Z_{12}|\alpha\rangle_1|\alpha\rangle_2 = \omega^{\hat{n}_1\hat{n}_2}\frac{1}{d}\sum_{k=0}^{d-1}|k_d\rangle\sum_{l=0}^{d-1}|l_d\rangle$$
$$= \frac{1}{d}\sum_{k=0}^{d-1}\sum_{l=0}^{d-1}\omega^{kl}|k_d\rangle|l_d\rangle$$
$$= \frac{1}{\sqrt{d}}\sum_{l=0}^{d-1}|\tilde{l}_d\rangle|l_d\rangle = \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|k_d\rangle|\tilde{k}_d\rangle$$

Maximal Entanglement of
Pseudo-Number State and Pseudo-Phase State

Jeffrey H. Shapiro, Phys. Rev. A 73, 062305 (2006)
"Single-photon Kerr nonlinearities do not help quantum computation"

Nature Light Science and Applications **1** e40 (2012)

Memory-enhanced noiseless cross-phase modulation

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Abstract

Large nonlinearity at the single-photon level can pave the way for the implementation of universal quantum gates. However, realizing large and noiseless nonlinearity at such low light levels has been a great challenge for scientists in the past decade. Here, we propose a scheme that enables substantial nonlinear interaction between two light fields that are both stored in an atomic memory. Semiclassical and quantum simulations demonstrate the feasibility of achieving large cross-phase modulation (XPM) down to the single-photon level. The proposed scheme can be used to implement parity gates from which CNOT gates can be constructed. Furthermore, we present a proof of principle experimental demonstration of XPM between two optical pulses: one stored and one freely propagating through the memory medium.

Keywords:
CNOT gate; cross-phase modulation; electromagnetically induced transparency; parity gate; quantum memory

Cross-Kerr vs. Self-Kerr

$$Z_{12}|\alpha\rangle_1|\alpha\rangle_2 = \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|\tilde{k}_d\rangle_1|k_d\rangle_2 = \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|k_d\rangle_1|\tilde{k}_d\rangle_2$$

$$Z_{12}Z_{23}|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3 = \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|\tilde{k}_d\rangle_1|k_d\rangle_2|\tilde{k}_d\rangle_3$$

$$\text{measurement on 2, } |\tilde{m}_d\rangle_2 \rightarrow \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|\tilde{k}_d\rangle_1\,\omega^{-km}|\tilde{k}_d\rangle_3$$

$$= \frac{1}{\sqrt{d}}\sum_{l=0}^{d-1}|l_d\rangle_1|(m-l)_d\rangle_3$$

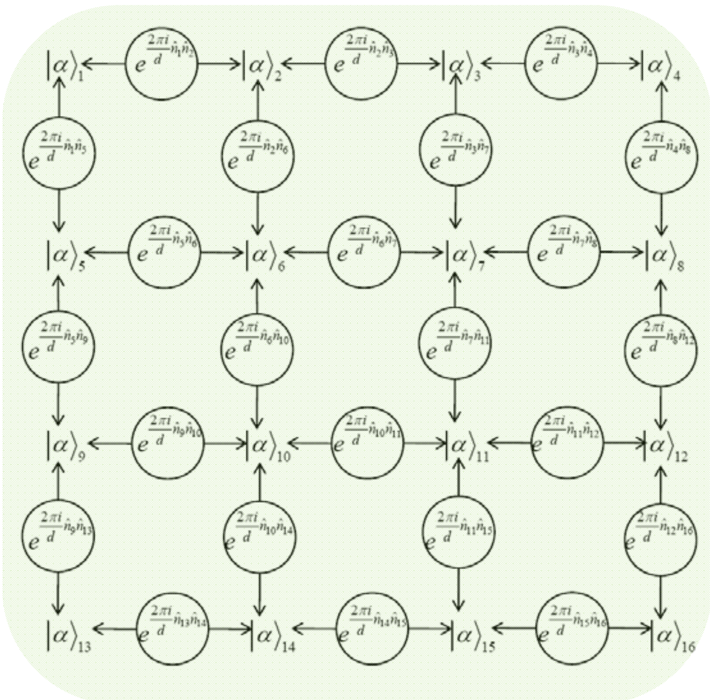
Van Enk

$$|\alpha\rangle \xrightarrow[\text{Self-Kerr}]{} \xrightarrow[\text{Beam-splitting}]{} \frac{1}{\sqrt{d}}\sum_{l=0}^{d-1}|\tilde{l}_d\rangle|\tilde{l}_d\rangle = \frac{1}{\sqrt{d}}\sum_{l=0}^{d-1}|\frac{\omega^l\alpha}{\sqrt{2}}\rangle|\frac{\omega^l\alpha}{\sqrt{2}}\rangle$$

S. J. van Enk, PRL 91, 017902 (2003)

"Deterministic" Generation of a Qu*d*it Cluster State

$$\prod_{\langle p,q\rangle}\omega^{\hat{n}_p\hat{n}_q}\prod_{r\in\text{lattice}}|\alpha\rangle_r$$

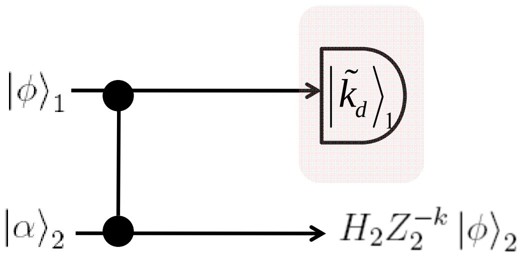


Tayloring

Scissors:
measurements in pseudo-number basis (Z)

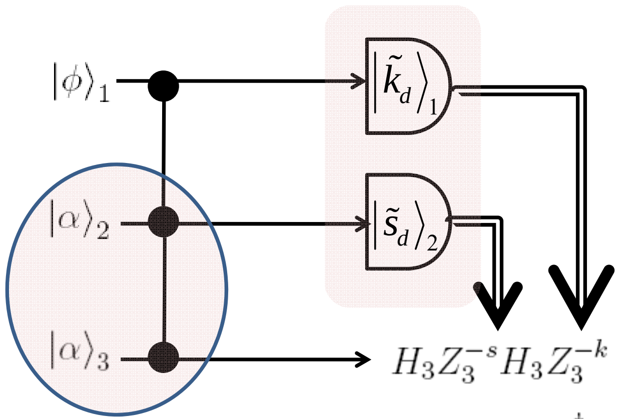
Stitches:
measurements in pseudo-phase basis (X)

One-step
d-dim Teleportation



$$Z_{12}|\phi\rangle_1|\alpha\rangle_2 = \omega^{\hat{n}_1\hat{n}_2}\sum_{l=0}^{d-1}|l_d\rangle_1\sum_{m=0}^{d-1}\frac{|m_d\rangle_2}{\sqrt{d}}$$
$$= \sum_l\sum_m \omega^{lm}a_l|l_d\rangle_1\frac{|m_d\rangle_2}{\sqrt{d}}$$
$$= \sum_l a_l|l_d\rangle_1|\tilde{l}_d\rangle_2$$
$$= \sum_l a_l\left\{\sum_k \omega^{-lk}\frac{|\tilde{k}_d\rangle_1}{\sqrt{d}}\right\}|\tilde{l}_d\rangle_2$$
$$\xrightarrow[\text{into } |\tilde{k}_d\rangle_1]{\text{Projective Measurement}} \sum_l a_l\omega^{-lk}|\tilde{l}_d\rangle_2$$
$$= \sum_l a_l\omega^{-lk}H_2|l_d\rangle_2$$
$$= H_2\sum_l a_l\omega^{-lk}|l_d\rangle_2$$
$$= H_2Z_2^{-k}\sum_l a_l|l_d\rangle_2$$
$$= H_2Z_2^{-k}|\phi\rangle_2$$

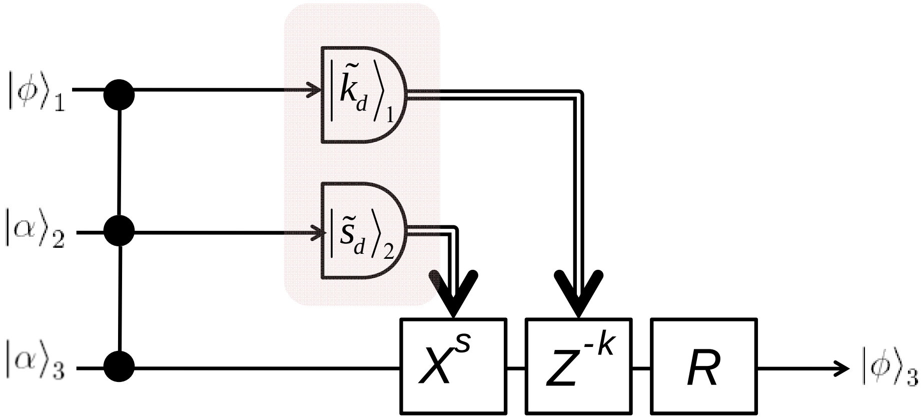
d-dim Teleportation



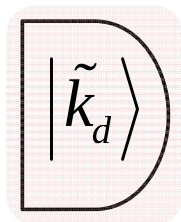
d-dim Bell state

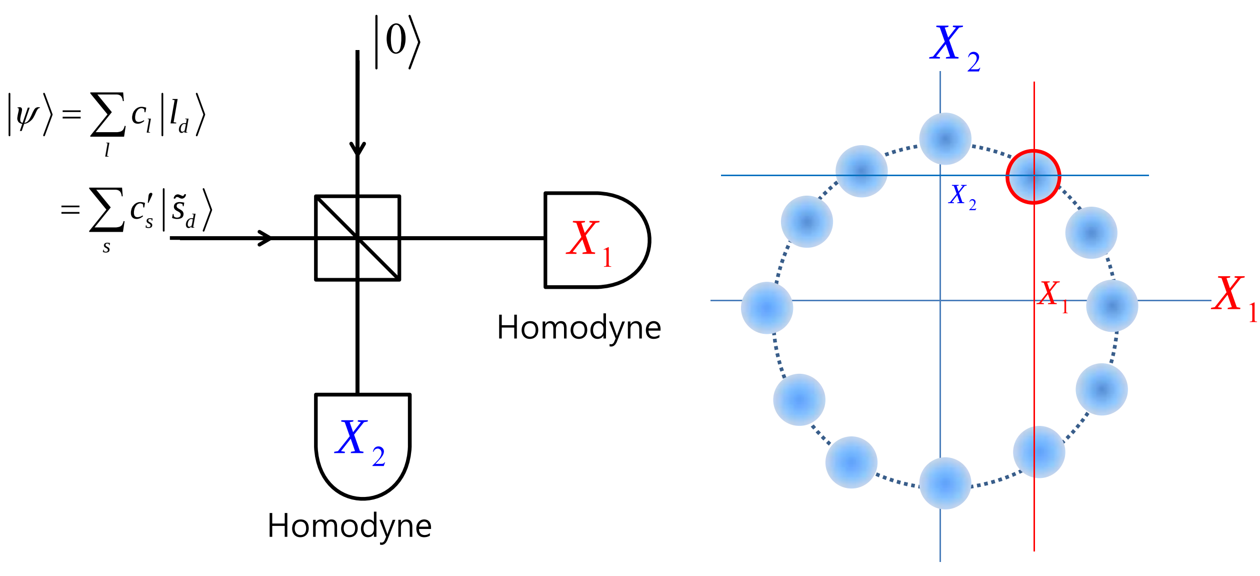
$$H_3Z_3^{-s}H_3Z_3^{-k}|\phi\rangle_3$$
$$= H_3Z_3^{-s}H_3^{\dagger}H_3H_3Z_3^{-k}|\phi\rangle_3$$
$$= X_3^{-s}R_3Z_3^{-k}|\phi\rangle_3$$
$$= X_3^{-s}Z_3^{k}R_3|\phi\rangle_3$$

d-dim Teleportation

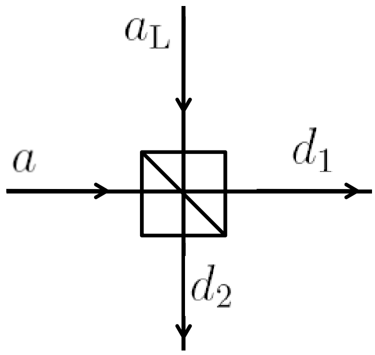


Pseudo-Phase Measurement by Homodyne Detection





Homodyne Detection



$$d_1 = \frac{1}{\sqrt{2}}\left(ia_{\mathrm{L}} + a\right)$$

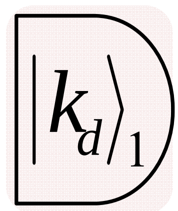
$$d_2 = \frac{1}{\sqrt{2}}\left(a_{\mathrm{L}} + ia\right)$$

$$a = \frac{X_1 + iX_2}{\sqrt{2}}$$

$$a_{\mathrm{L}} = \frac{Y_1 + iY_2}{\sqrt{2}}$$

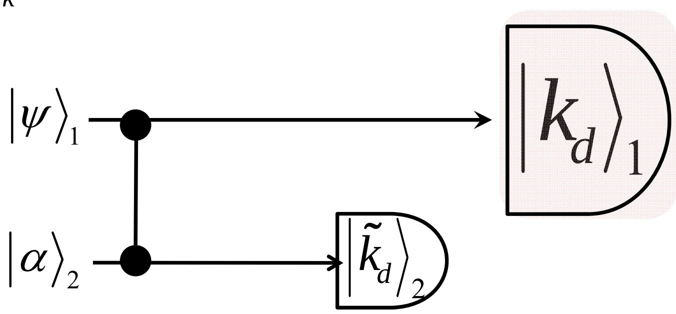
$$d_1^\dagger d_1 - d_2^\dagger d_2 = i(a^\dagger a_{\mathrm{L}} - a_{\mathrm{L}} a) = Y_1 X_2 - Y_2 X_1$$

Pseudo-Number *Measurement*



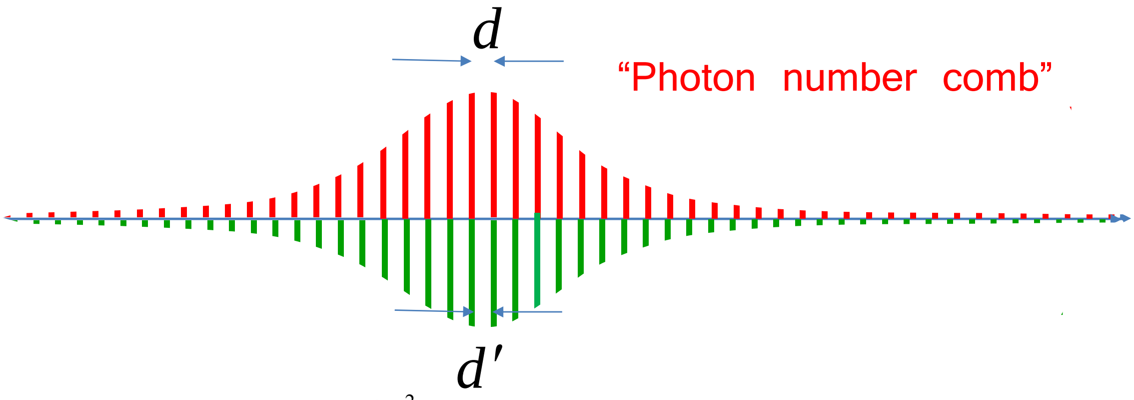
$$|\psi\rangle = \sum_k c_k |k_d\rangle = \sum_s c'_s |\tilde{s}_d\rangle$$

$$Z_{12}|\psi\rangle_1 |\alpha\rangle_2 = \sum_k c_k |k_d\rangle_1 |\tilde{k}_d\rangle_2$$



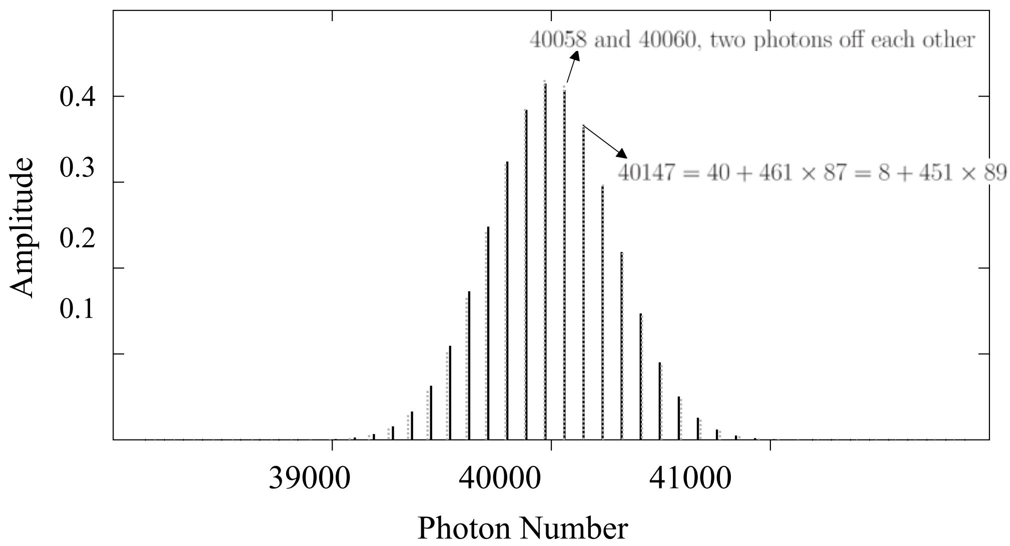
Postselection of high Number state

$$\left|k_d\right\rangle = \sqrt{d} \cdot e^{-\frac{|\alpha|^2}{2}} \sum_{m=0}^{\infty} \frac{\alpha^{k+md}}{\sqrt{(k+md)!}} \left|k+md\right\rangle$$



$$\left|l_{d'}\right\rangle = \sqrt{d'} \cdot e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^{l+nd'}}{\sqrt{(l+nd')!}} \left|l+nd'\right\rangle$$

$$k + md = l + nd' = N$$



One-step teleportation

$$\begin{aligned}
\omega^{\hat{n}_1\hat{n}_2}|\phi\rangle_1|\alpha\rangle_2 &= \omega^{\hat{n}_1\hat{n}_2}\sum_l a_l|l_d\rangle_1\sum_k\frac{|k_d\rangle_2}{\sqrt{d}} \\
&= \sum_l a_l|l_d\rangle_1\sum_k \omega^{lk}\frac{|k_d\rangle_2}{\sqrt{d}} \\
&= \sum_l a_l|l_d\rangle_1|\tilde{l}_d\rangle_2 \quad |\tilde{p}_d\rangle_1 \\
\xrightarrow{\text{measure qudit 1 into } |\tilde{p}_d\rangle_1} & \sum_l a_l\omega^{-pl}|\tilde{l}_d\rangle_2 \\
&= \sum_l a_l\omega^{-pl}H_2|l_d\rangle_2 \\
&= H_2\sum_l a_l\omega^{-pl}|l_d\rangle_2 \\
&= H_2\sum_l a_l Z_2^{-p}|l_d\rangle_2 \\
&= H_2 Z_2^{-p}\sum_l a_l|l_d\rangle_2 \\
&= H_2 Z_2^{-p}|\phi\rangle_2
\end{aligned}$$

Quantum Repeater

$$|\tilde{p}_d\rangle_1$$

$$\omega^{\hat{n}_0\hat{n}_1}\omega^{\hat{n}_1\hat{n}_2}|\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2 = \sum_l |\tilde{l}_d\rangle_0|l_d\rangle_1|\tilde{l}_d\rangle_2$$

$$\xrightarrow{\text{Project qudit 1 into } |\tilde{p}_d\rangle_1} \sum_l |\tilde{l}_d\rangle_0\, \omega^{-pl}\, |\tilde{l}_d\rangle_2$$

$$= \sum_l |\tilde{l}_d\rangle_0\, HZ^{-p}|l_d\rangle_2$$

Quantum Repeater

4 qudits in series

$|\tilde{p}_d\rangle_1 \; |\tilde{q}_d\rangle_2$

$$\omega^{\hat{n}_0\hat{n}_1}\omega^{\hat{n}_1\hat{n}_2}\omega^{\hat{n}_2\hat{n}_3}|\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3$$

Project qudits 1&2 into $|\tilde{p}_d\rangle_1|\tilde{q}_d\rangle_2$

$$\longrightarrow \sum_l |\tilde{l}_d\rangle_0 \, \omega^{-pl} \, |(q-l)_d\rangle_3$$

Quantum Repeater

5 qudits in series

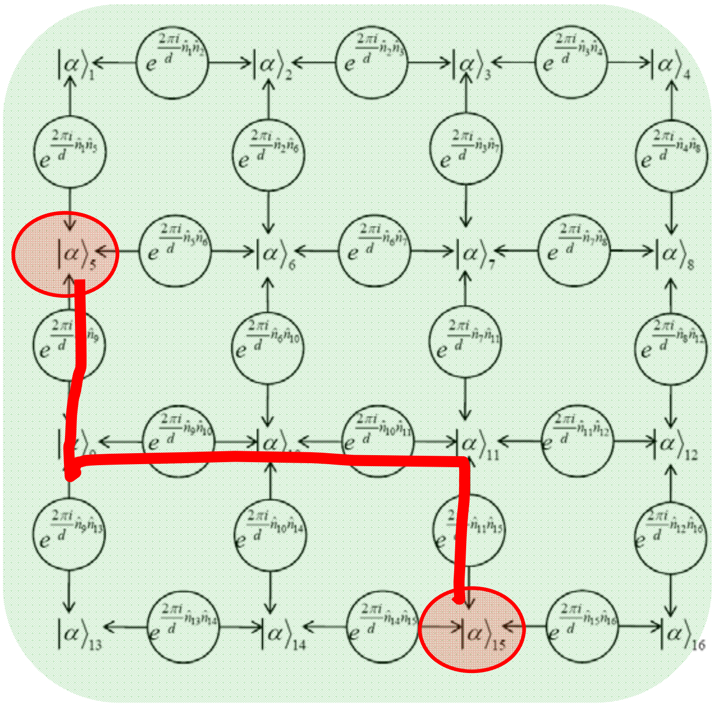
$|\tilde{p}_d\rangle_1 \; |\tilde{q}_d\rangle_2 \; |\tilde{r}_d\rangle_3$

$$\omega^{\hat{n}_0\hat{n}_1}\omega^{\hat{n}_1\hat{n}_2}\omega^{\hat{n}_2\hat{n}_3}\omega^{\hat{n}_3\hat{n}_4}|\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4$$

Project qudits 1,2&3 into $|\tilde{p}_d\rangle_1|\tilde{q}_d\rangle_2|\tilde{r}_d\rangle_3$

$$\longrightarrow \sum_l |\tilde{l}_d\rangle_0 \, \omega^{rl-pl-qr} \, |(\widetilde{q-l})_d\rangle_4$$

Bell State

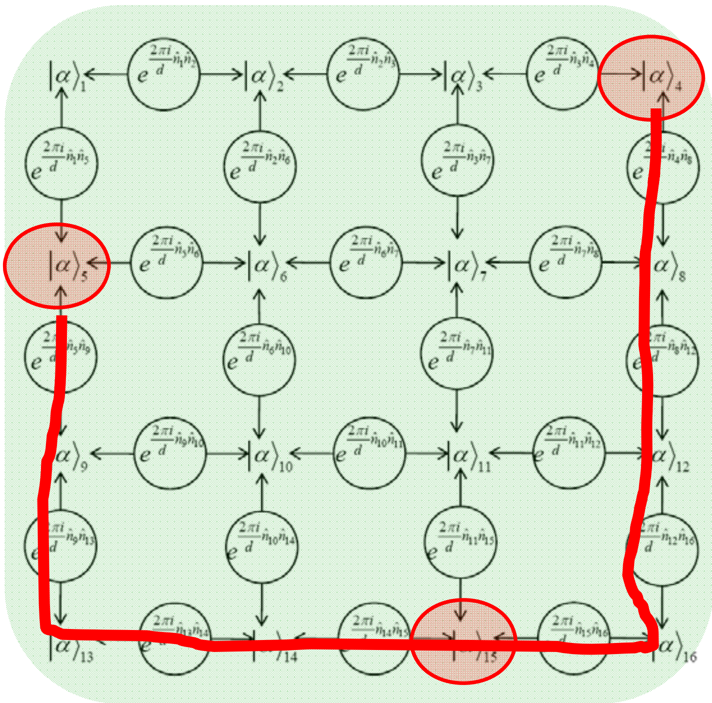


Tayloring

Scissors:
measurements in pseudo-number basis (Z)

Stitches:
measurements in pseudo-phase basis (X)

GHZ State



Tayloring

Scissors:
measurements in pseudo-number basis (Z)

Stitches:
measurements in pseudo-phase basis (X)

arXiv:1410.3217 [pdf, ps, other]

Title: Discrete-phase-randomized coherent state source and its application in quantum key distribution

Authors: Zhu Cao, Zhen Zhang, Hoi-Kwong Lo, Xiongfeng Ma

Comments: 4 figures, comments welcome

Journal-ref: New J. Phys. 17 053014 (2015)

In the case of general $N \geq 1$, the decomposition is similar but a bit more

$$
\begin{aligned}
|\Psi_N\rangle &= \sum_{k=0}^{N-1} |k\rangle_A \big|\sqrt{2}\alpha e^{2k\pi i/N}\big\rangle_B \\
&= \sum_{j=0}^{N-1} |j\rangle_A |\lambda_j\rangle_B
\end{aligned}
$$

where $|j\rangle_A$ can be understood as a quantum coin with N random outputs ar pure states are given by

$$|\lambda_j\rangle = \sum_{k=0}^{N-1} e^{-2kj\pi i/N} \big|e^{2k\pi i/N}\sqrt{2}\alpha\big\rangle.$$

By substituting Eq. (1.1), we have the following observations for $|\lambda_j\rangle$. superposition of Fock states whose photon numbers modulo N are the same j

$$|\lambda_j\rangle = \sum_{l=0}^{\infty} \frac{(\sqrt{2}\alpha)^{lN+j}}{\sqrt{(lN+j)!}} |lN+j\rangle.$$

Then, it is not hard to see that $|\lambda_j\rangle$ becomes close to a Fock state when N is la

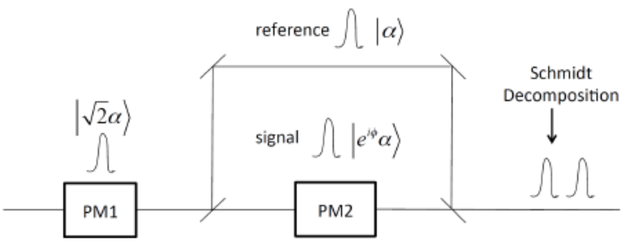
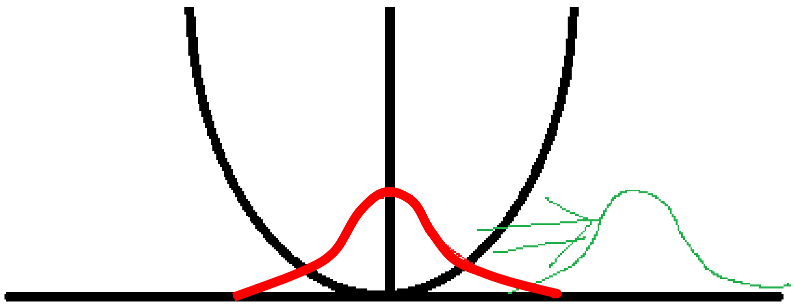


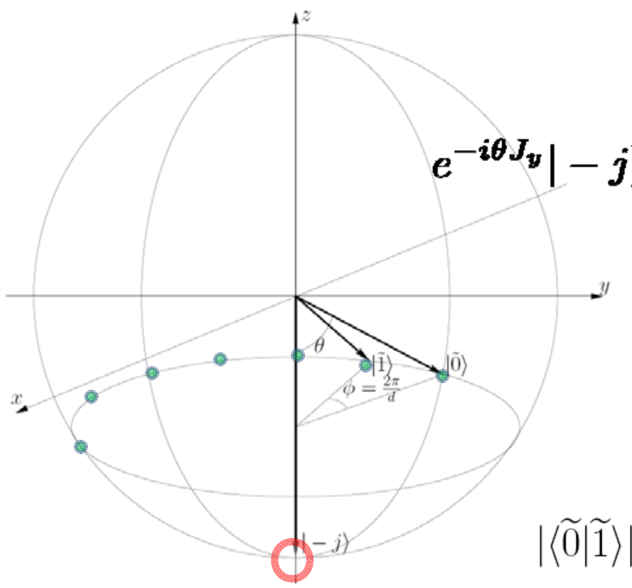
Figure 1: Schematic diagram for the phase-encoding QKD scheme with coherent states. The first phase modulator, $PM1$, is used for phase randomization according to Eq. (2.1), and the second one, $PM2$, is used for QKD encoding $\phi \in \{0, \pi/2, \pi, 3\pi/2\}$.

$$
\begin{aligned}
|0_x^L\rangle &= \sum_{k=0}^{N-1} e^{-2kj\pi i/N} \big|e^{2k\pi i/N}\alpha\big\rangle \big|e^{2k\pi i/N}\alpha\big\rangle \\
|1_x^L\rangle &= \sum_{k=0}^{N-1} e^{-2kj\pi i/N} \big|e^{2k\pi i/N}\alpha\big\rangle \big|-e^{2k\pi i/N}\alpha\big\rangle \\
|0_y^L\rangle &= \sum_{k=0}^{N-1} e^{-2kj\pi i/N} \big|e^{2k\pi i/N}\alpha\big\rangle \big|ie^{2k\pi i/N}\alpha\big\rangle \\
|1_y^L\rangle &= \sum_{k=0}^{N-1} e^{-2kj\pi i/N} \big|e^{2k\pi i/N}\alpha\big\rangle \big|-ie^{2k\pi i/N}\alpha\big\rangle.
\end{aligned}
$$

Coherent State



Spin Coherent State *Qudit*



$$e^{-i\theta J_y}|-j\rangle = |\tilde{0}_d\rangle = \sum_{\lambda=-j}^{j} |\lambda\rangle\langle\lambda|e^{-i\theta J_y}|-j\rangle$$

$$|\tilde{l}_d\rangle = e^{-i\phi J_z l} e^{-i\theta J_y}|-j\rangle$$

$$|\langle\tilde{0}|\tilde{1}\rangle| = \left|\frac{1+\hat{n}_{|\tilde{0}\rangle}\cdot\hat{n}_{|\tilde{1}\rangle}}{2}\right|^j \approx e^{-j\frac{\pi^2}{d^2}\sin^2\theta}$$

Modulo-*d* spin state & Spin coherent state

$$|k_d\rangle = \sqrt{d} \sum_{m}^{-j \le k+md \le j} |k+md\rangle\langle k+md| e^{-i\theta J_y} |-j\rangle,$$

$$\begin{cases} |\tilde{l}_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{l=0}^{d-1} \omega^{lk} |k_d\rangle \\ |k_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{l=0}^{d-1} \omega^{-kl} |\tilde{l}_d\rangle \end{cases}$$

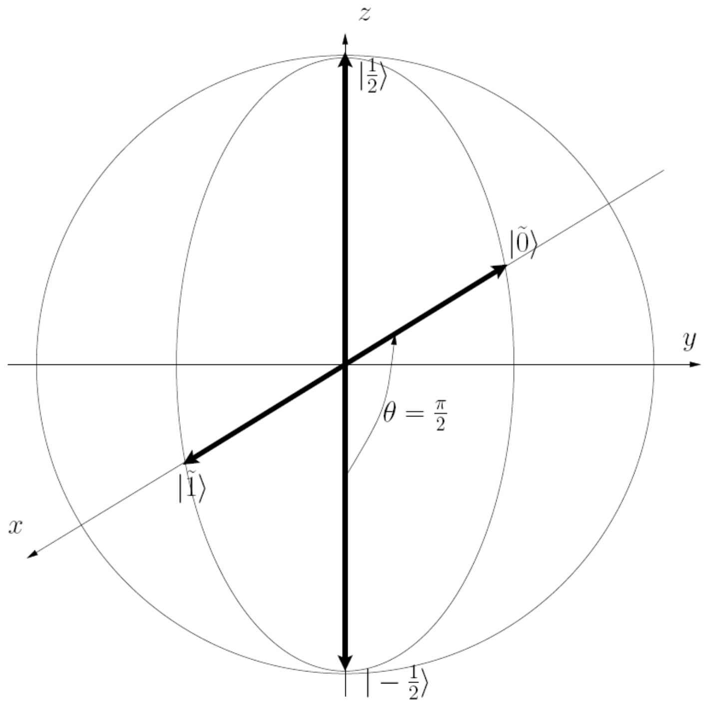
Ising Interaction→CZ

$$e^{-\frac{2\pi i}{d} J_{z_1} J_{z_2}} |\tilde{0}_d\rangle_1 |\tilde{0}_d\rangle_2$$

$$= \frac{1}{d} \sum_{k=0}^{d-1} \sum_{l=0}^{d-1} \omega^{-kl} |k_d\rangle_1 |l_d\rangle_2$$

$$= \frac{1}{\sqrt{d}} \sum_{k} |\tilde{k}_d\rangle_1 |k_d\rangle_2 \ \text{ or } \ \frac{1}{\sqrt{d}} \sum_{k} |k_d\rangle_1 |\tilde{k}_d\rangle_2$$

Spin-half Coherent State *Qubit*



Summary

JK, J. Lee, S.-W. Ji, H. Nha, P. Anisimov, J. P. Dowling, Opt Comm 337, 79 (2015)

- Optical Coherent State: Even superposition of *d*-dim pseudo-number computational basis states
- Generalized Cont-Z can be implemented by Cross-Kerr interaction ($d \cong$ 10~1000 ?!)
  → Max Entanglement ➔ Qu*d*it Cluster State
- *d*-dim teleportation
- Pseudo-Phase Measurement by Homodyne detection
- Pseudo-Number Measurement
- Network for Quantum Communication
- Spin coherent state qudit
- Qu*d*it Cluster Quantum Computation …

\# Decoherence

\# Single qu*d*it operation with non-integer power

\# Quantum Optics … Circuit QED …

Proof of Principle Test

X_0 X_2

$$\omega^{\hat{n}_0\hat{n}_1}\omega^{\hat{n}_1\hat{n}_2}|\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2 = \sum_l |\tilde{l}_d\rangle_0|l_d\rangle_1|\tilde{l}_d\rangle_2$$

Proof of Principle Test

Easier One

3 coherent states

$\omega \equiv e^{i\phi}$ with $\phi = \chi L$

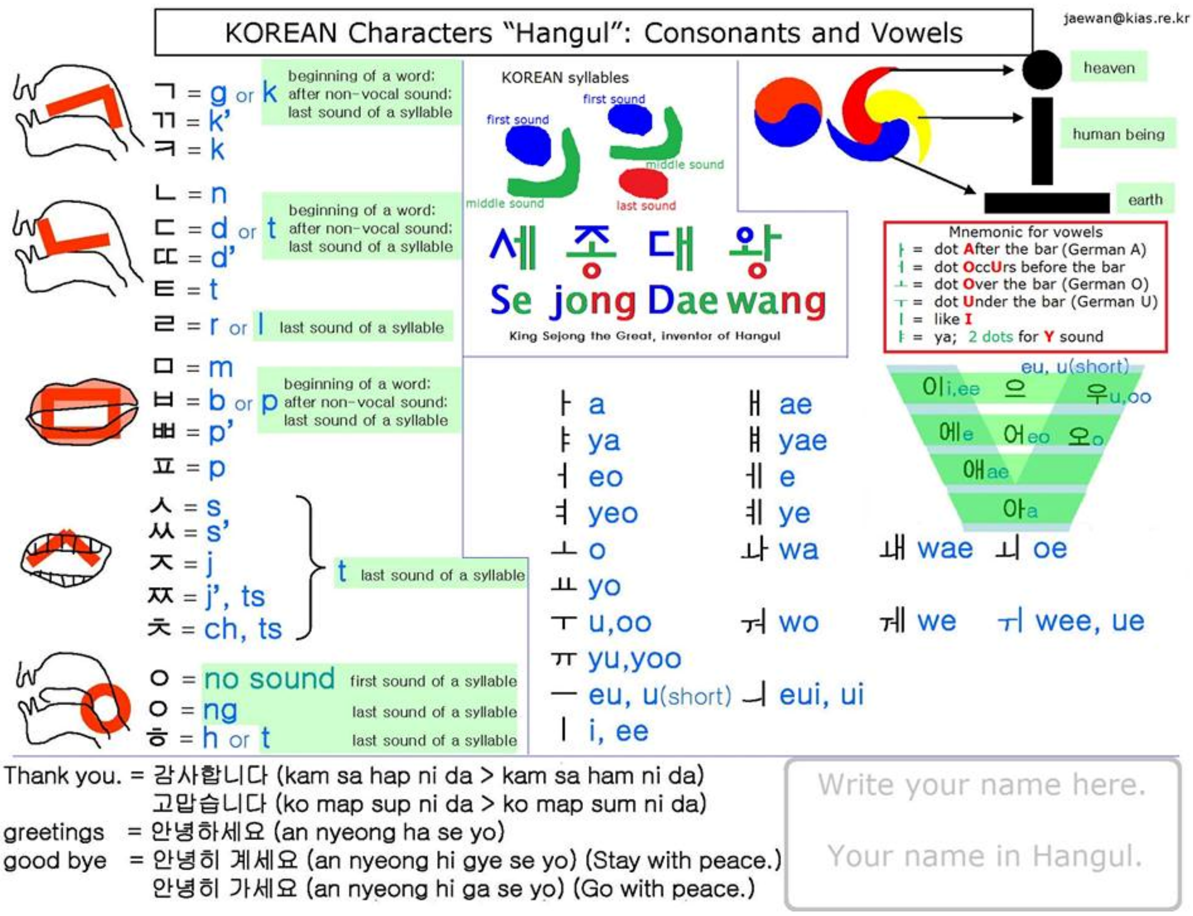
α, β; amplitudes of coherent states

$\beta\phi \gtrsim 10$

X_1 X_3

$$\omega^{\hat{n}_1\hat{n}_2}\omega^{\hat{n}_2\hat{n}_3}|\alpha\rangle_0|\beta\rangle_1|\alpha\rangle_2 = e^{-\frac{|\beta|^2}{2}}\sum_{l=0}^{\infty}|\omega^l\alpha\rangle_0|l\rangle_1|\omega^l\alpha\rangle_2$$

Thanks …


KOREAN Characters "Hangul": Consonants and Vowels
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ㄱ = g or k beginning of a word; after non-vocal sound; last sound of a syllable
ㄲ = k'
ㅋ = k
ㄴ = n
ㄷ = d or t beginning of a word; after non-vocal sound; last sound of a syllable
ㄸ = d'
ㅌ = t
ㄹ = r or l last sound of a syllable
ㅁ = m
ㅂ = b or p beginning of a word; after non-vocal sound; last sound of a syllable
ㅃ = p'
ㅍ = p
ㅅ = s
ㅆ = s'
ㅈ = j
ㅉ = j', ts
ㅊ = ch, ts
t last sound of a syllable
ㅇ = no sound first sound of a syllable
ㅇ = ng last sound of a syllable
ㅎ = h or t last sound of a syllable
KOREAN syllables
first sound
first sound
middle sound
middle sound
last sound
세 종 대 왕
Se jong Dae wang
King Sejong the Great, inventor of Hangul
heaven
human being
earth
Mnemonic for vowels
ㅏ = dot After the bar (German A)
ㅓ = dot OccUrs before the bar
ㅗ = dot Over the bar (German O)
ㅜ = dot Under the bar (German U)
ㅣ = like I
ㅑ = ya; 2 dots for Y sound
이 i,ee 으 eu, u(short) 우 u,oo
에 e 어 eo 오 o
애 ae
아 a
ㅏ a
ㅑ ya
ㅓ eo
ㅕ yeo
ㅗ o
ㅛ yo
ㅜ u,oo
ㅠ yu,yoo
ㅡ eu, u(short)
ㅣ i, ee
ㅐ ae
ㅒ yae
ㅔ e
ㅖ ye
ㅘ wa
ㅝ wo
ㅢ eui, ui
ㅙ wae
ㅚ oe
ㅞ we
ㅟ wee, ue
Thank you. = 감사합니다 (kam sa hap ni da > kam sa ham ni da)
고맙습니다 (ko map sup ni da > ko map sum ni da)
greetings = 안녕하세요 (an nyeong ha se yo)
good bye = 안녕히 계세요 (an nyeong hi gye se yo) (Stay with peace.)
안녕히 가세요 (an nyeong hi ga se yo) (Go with peace.)
Write your name here.
Your name in Hangul.