Interesting Physics in 1+1 Dimensions: Electrodynamics, Entanglement and Propagation

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collaborators
(over the last few years)

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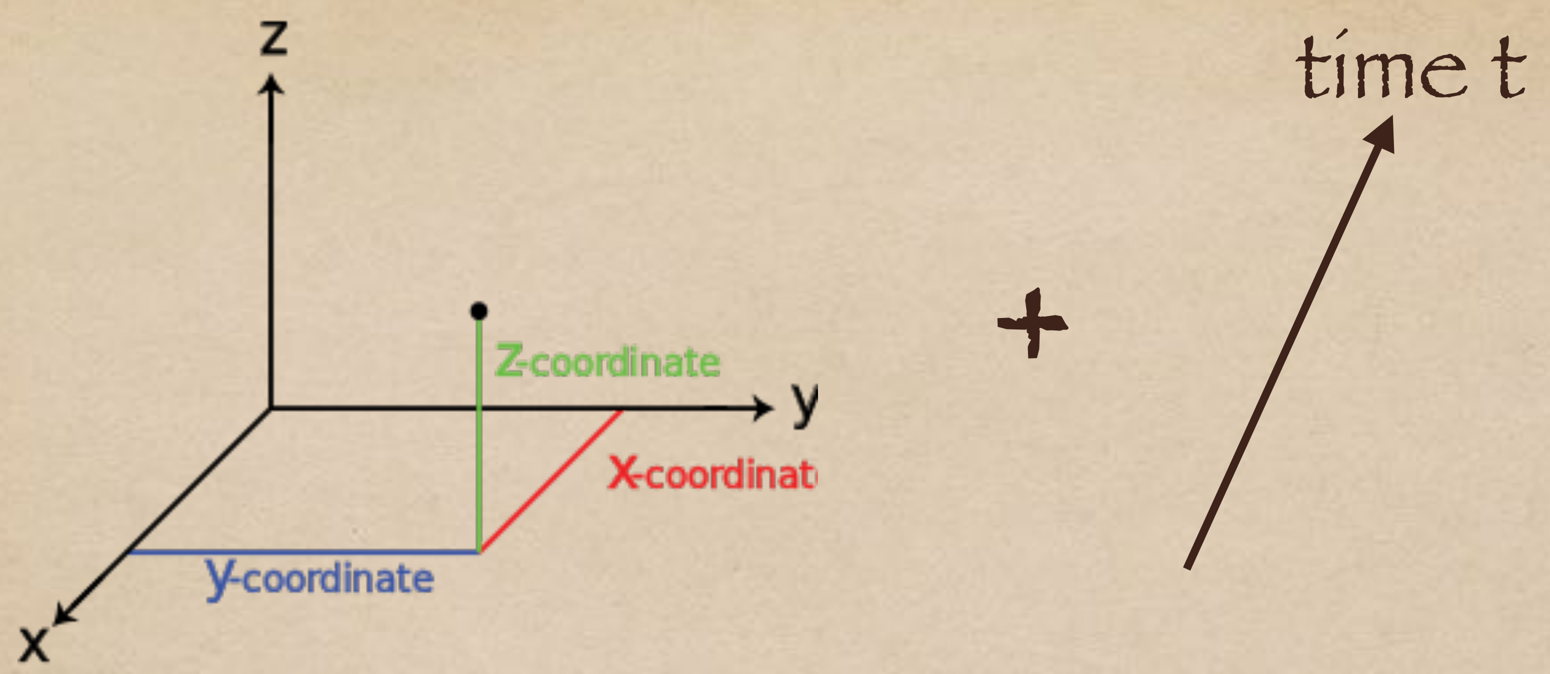


we live in 3D (+ time)

why do we live in 3+1 dimensions?

what would the world look like in 1+1 D?

in 2+1 D?

in 4+1 D?

...

difficult questions

strategy

in physics, when a question is too difficult, one simplifies it

usually one only gets partial answers

these answers are often satisfactory (even sufficient fapp)

and one is content with what one gets

new question:

why do we live in 3+1 dimensions?

what would the world look like in 1+1 D?

in 2+1 D?

in 4+1 D?

...

what would electrodynamics look like in 1 or 2 (+1) D?

even such a (much) simpler question requires attention

what would electrodynamics look like in 1 +1 D?

1D + time

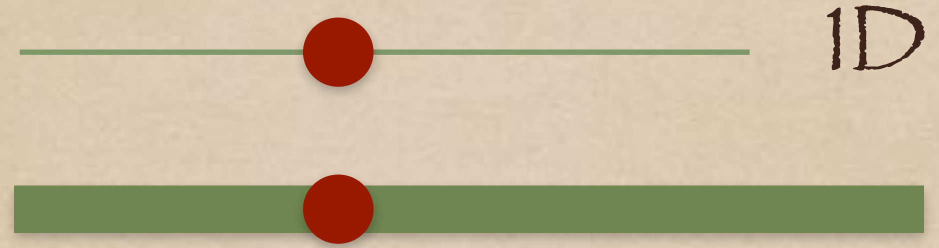
“1D” + time

what do we mean by 1D?

do we mean a “line” (with points)?

or do we mean a limiting procedure by which
all other dimensions can be neglected?

(in physics even the most naive questions require some kind of “definition”) in physics even the most naive questions require some kind of “definition”



1D + time

“1D” + time

we will place “atoms”and charges in our 1D “world”

quantum physics in 1D

- quantum emitters in 1D waveguides
- simulations of quantum field theories

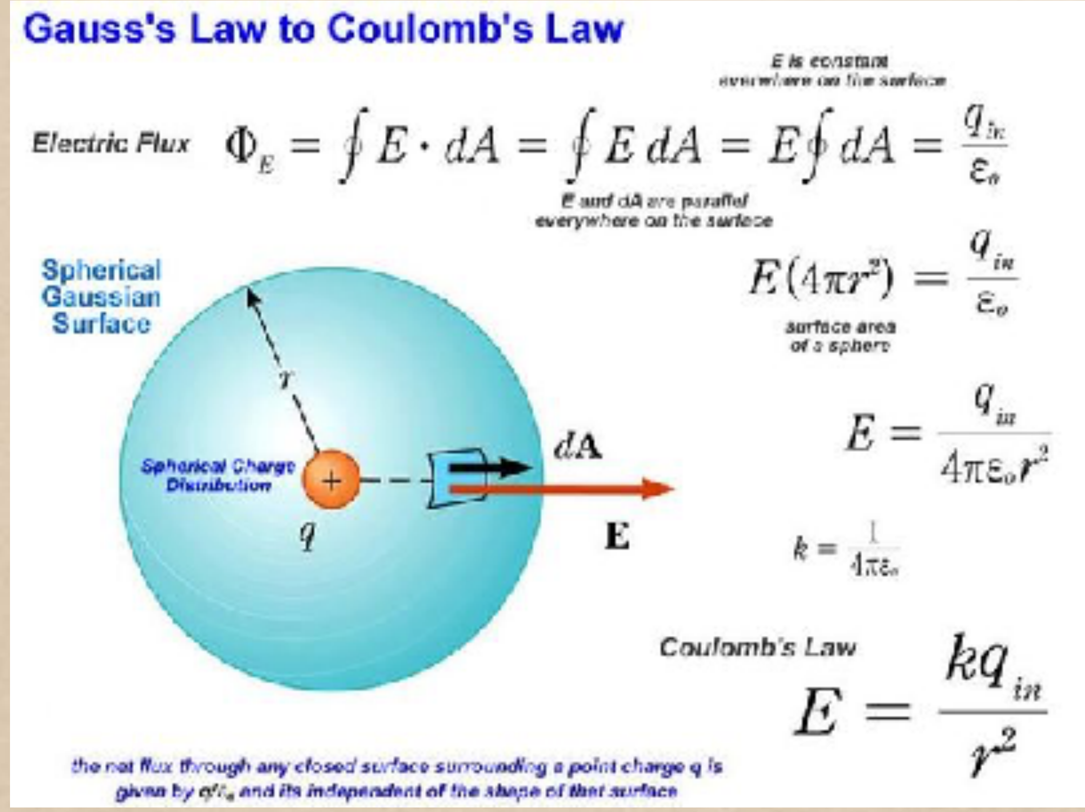
quantum field theory

T. Giamarchi, Quantum Physics in One Dimension, 2004

Y. Kuramoto and Y. Kato, Dynamics of One-Dimensional Quantum Systems: Inverse-Square Interaction Models, 2009

nice testbed: Gauss

3D

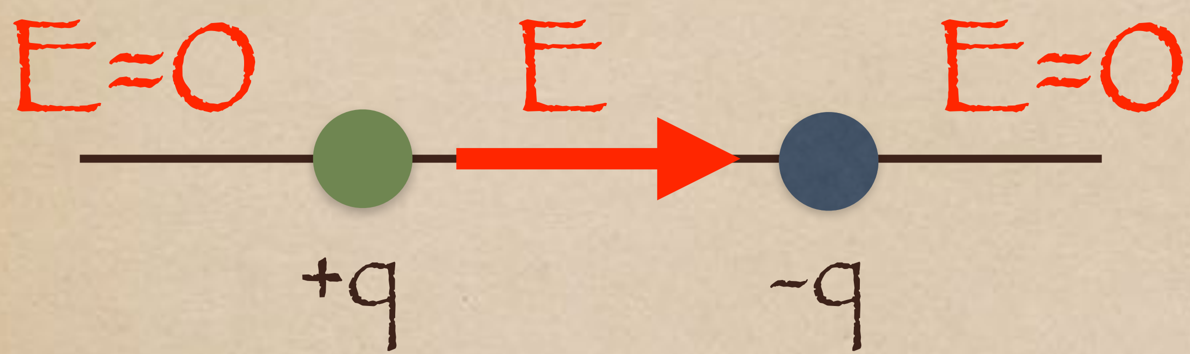


2D $$A = \text{circumference} = 2\pi r \to E \sim \frac{q}{r}$$

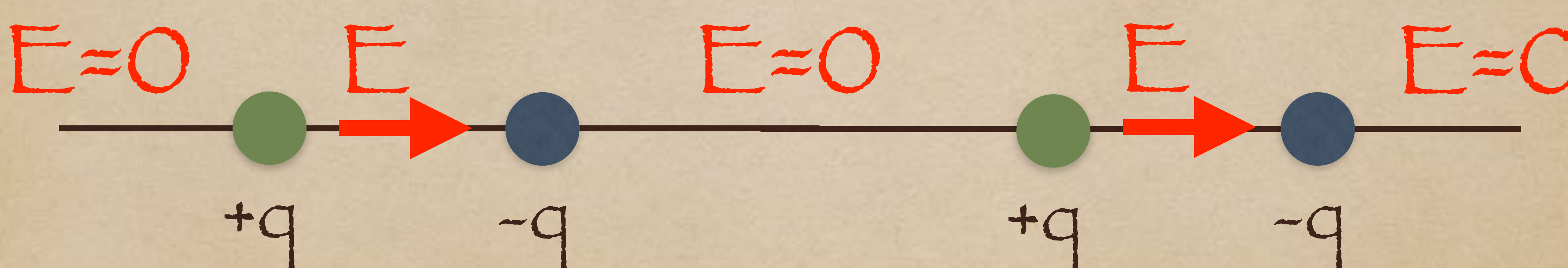
1D $$A = \{\text{two points}\} \to E \sim q$$

1D + time

$$A = \{\text{two points}\} \rightarrow E \sim q$$



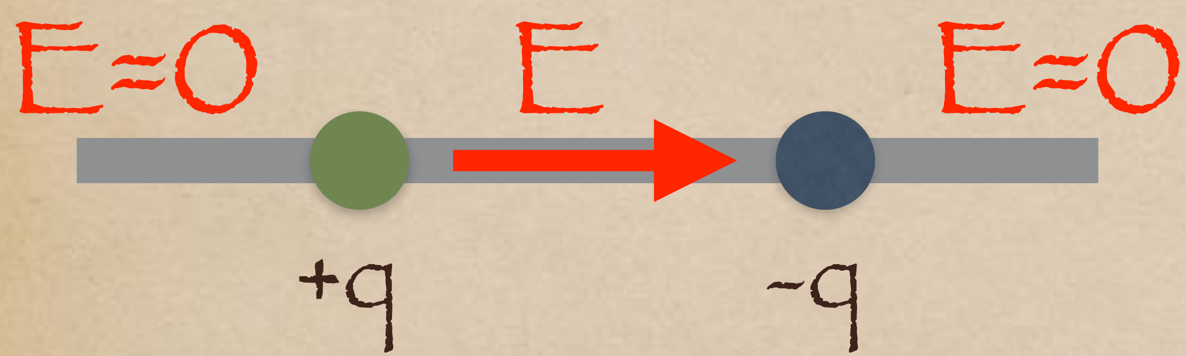
dipoles?



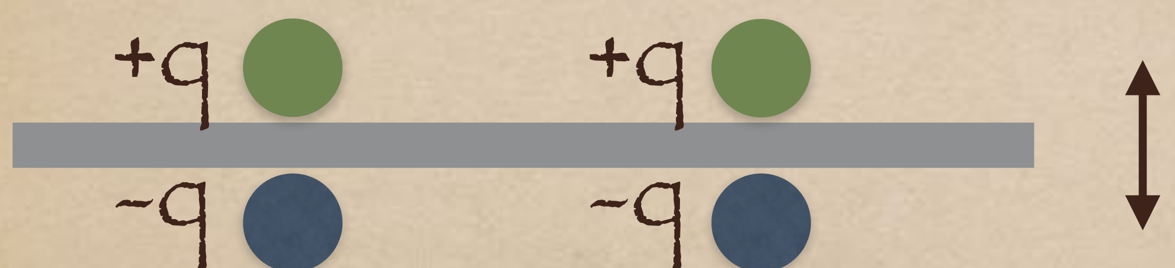
dipoles do not interact

intermezzo

"1D" + time ← is VERY different



remains valid



this limit to zero can be conceived,
so dipoles can be
"perpendicular" to the
"world" (1D line)

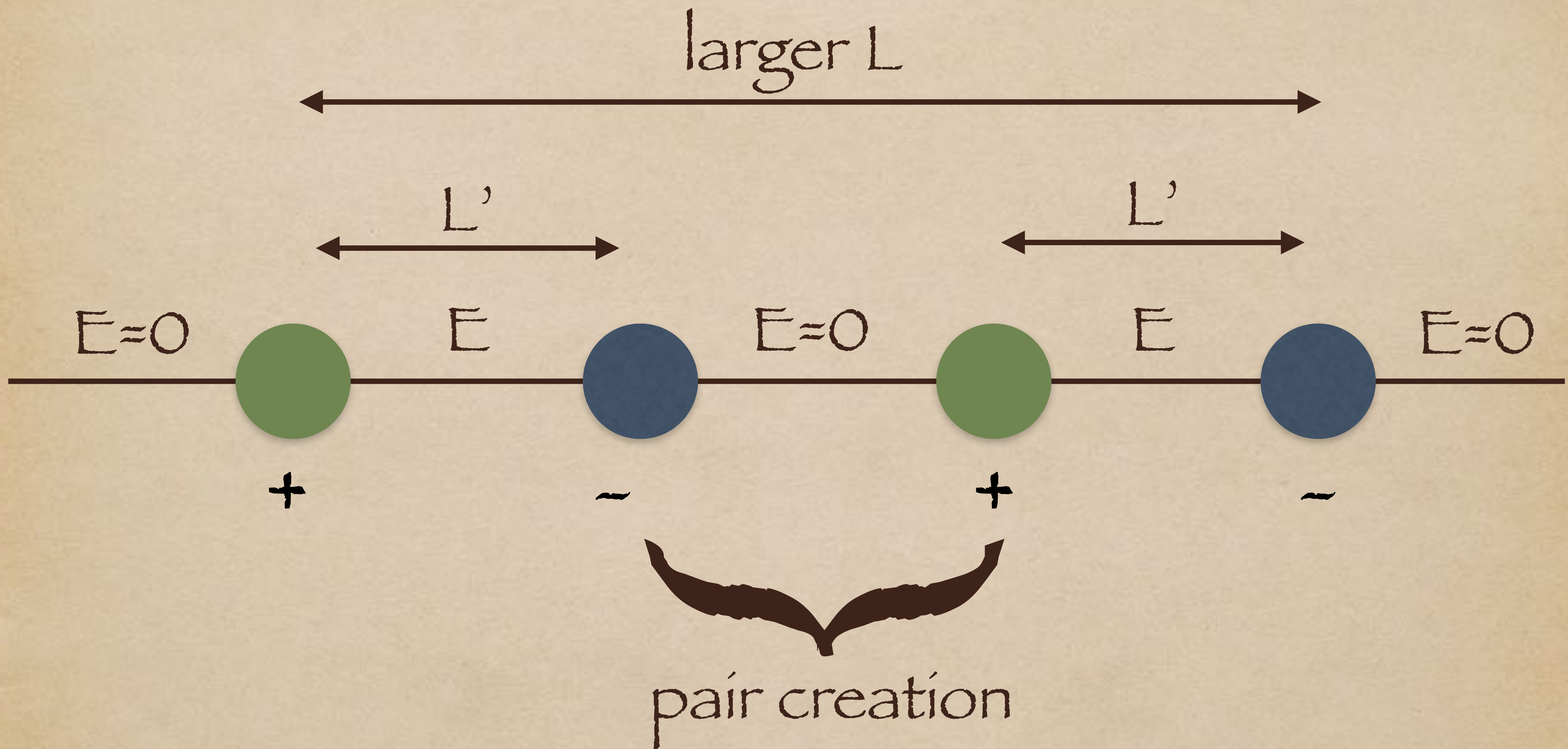
(Q)ED in 1D

(electric field E confined here)

$$\text{energy} = \frac{1}{2} L \times E^2 \propto \text{volume}$$

Schwinger, Coleman, Kogut, Susskind, Casher, 't Hooft, Parisi thinking about confinement mechanisms

QED in 1D: string breaking

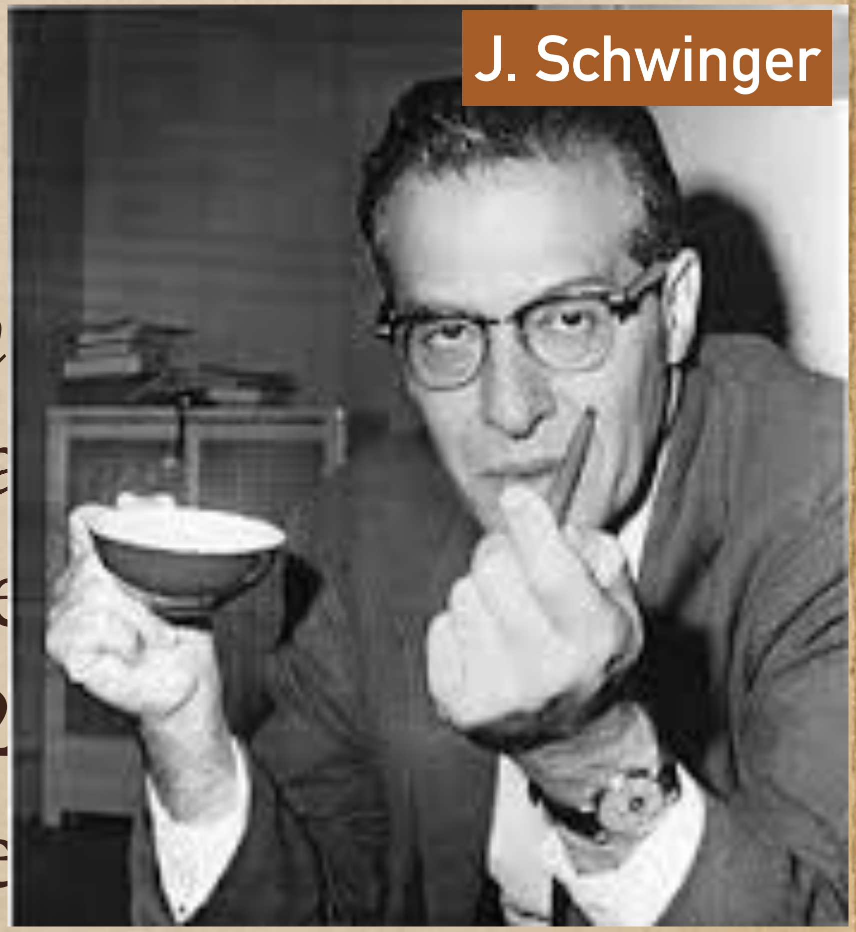


$$\text{energy} = \frac{1}{2} 2L' \times E^2 \propto \text{volume}$$

QED HAMILTONIAN

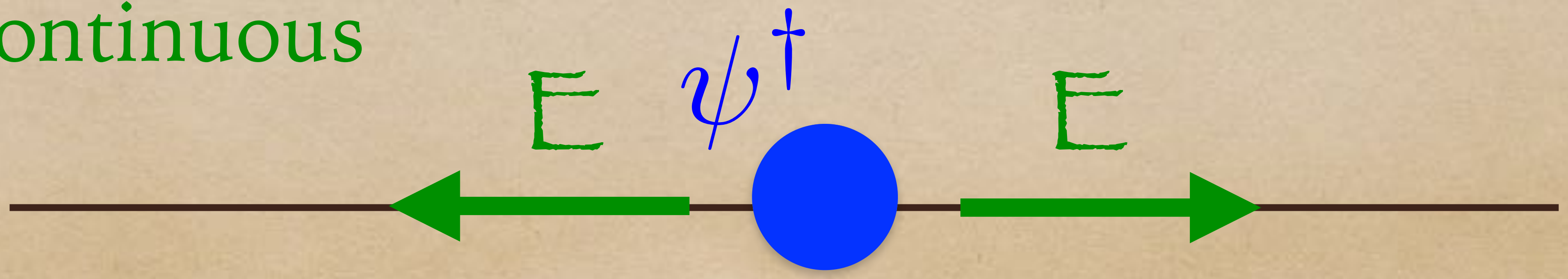
$$H = \int dx \left\{ \psi^\dagger \gamma^0 \left[-\gamma^1 (i\partial_1 + A) + m \right] \psi(x) + \frac{g^2}{2} E^2 \right\}$$

$$G(x) = \partial_1 E(x) - \psi^\dagger(x)\psi(x) = 0$$

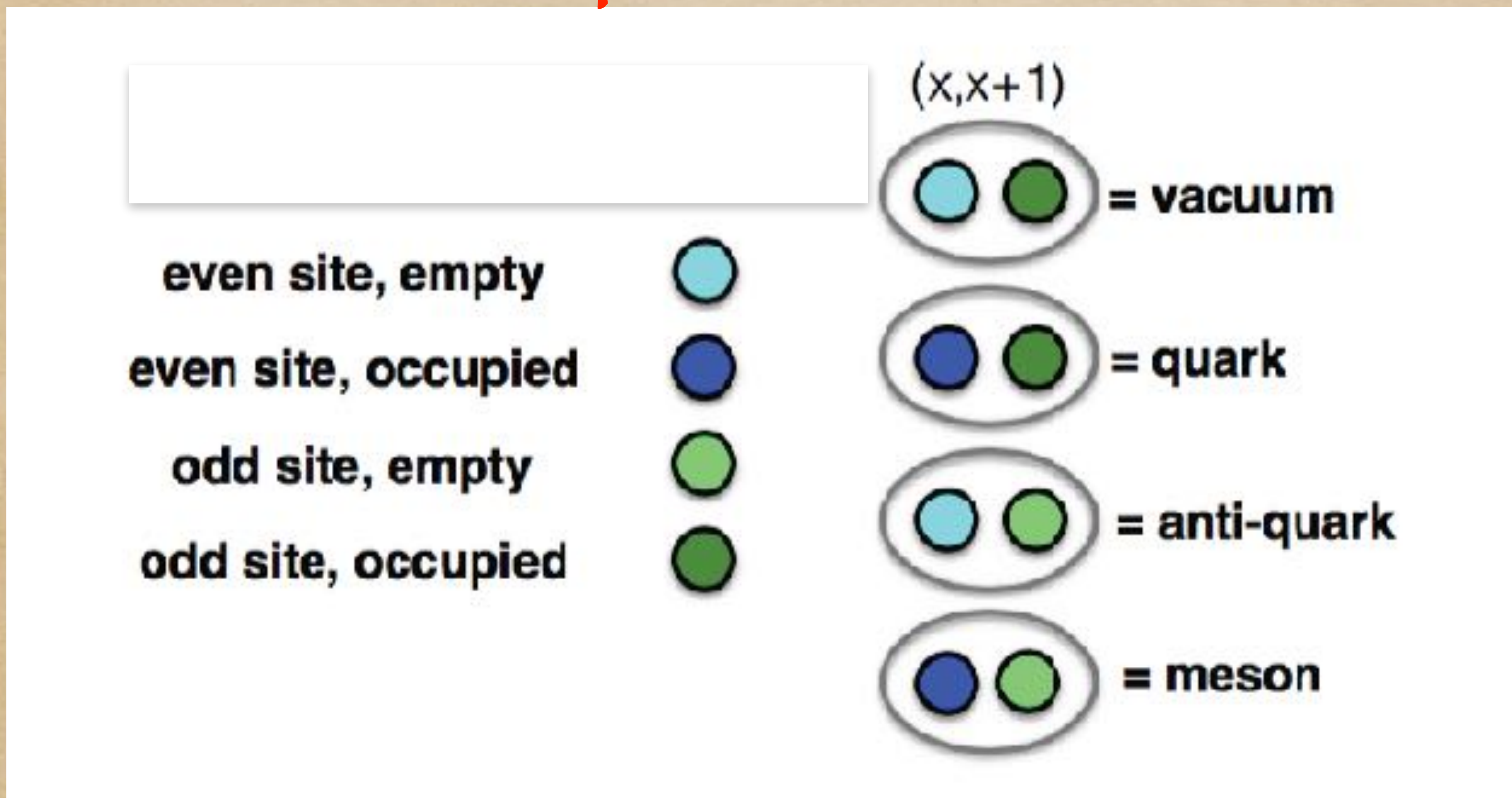


"Classical" discretization

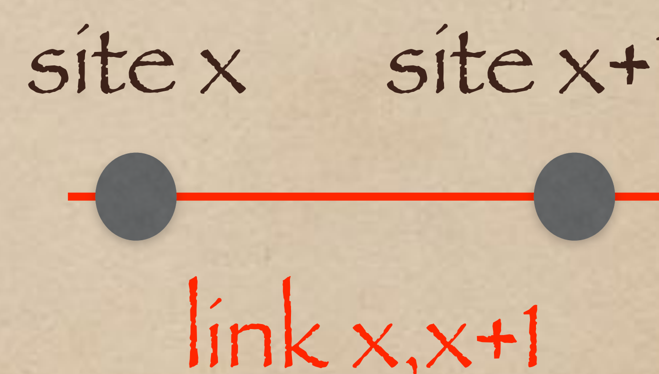
- continuum space replaced by regula
- fermionic degrees of freedom are de sites: dim of local Hilbert space is 2
- e.m. field degrees of freedom are de are continuous



discretize space: lattice



staggered fermions



$$[E_{x,x+1}, A_{y,y+1}] = i\delta_{x,y}$$

or

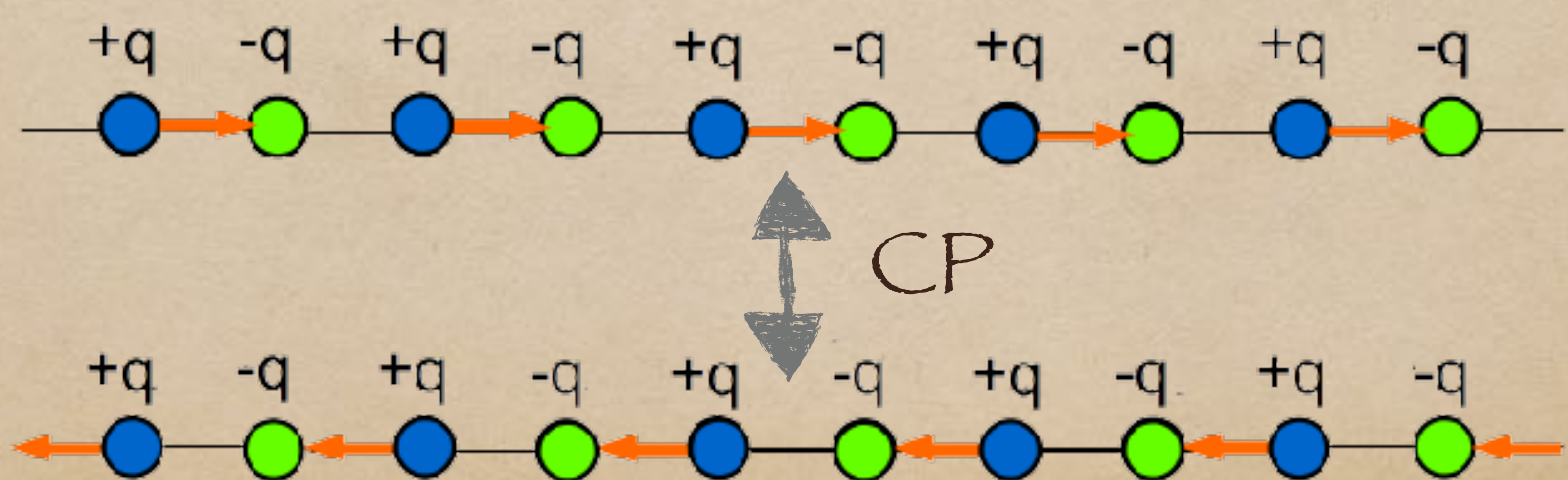
$$U_{x,x+1} = e^{-iA_{x,x+1}} \text{ with } [E_{x,x+1}, U_{y,y+1}] = \delta_{x,y} U_{x,x+1}$$

order parameter $$\Sigma = \frac{1}{N}\sum_x \langle E_{x,x+1}\rangle$$

- Large positive m: GS = filled Dirac sea invariant under C and P

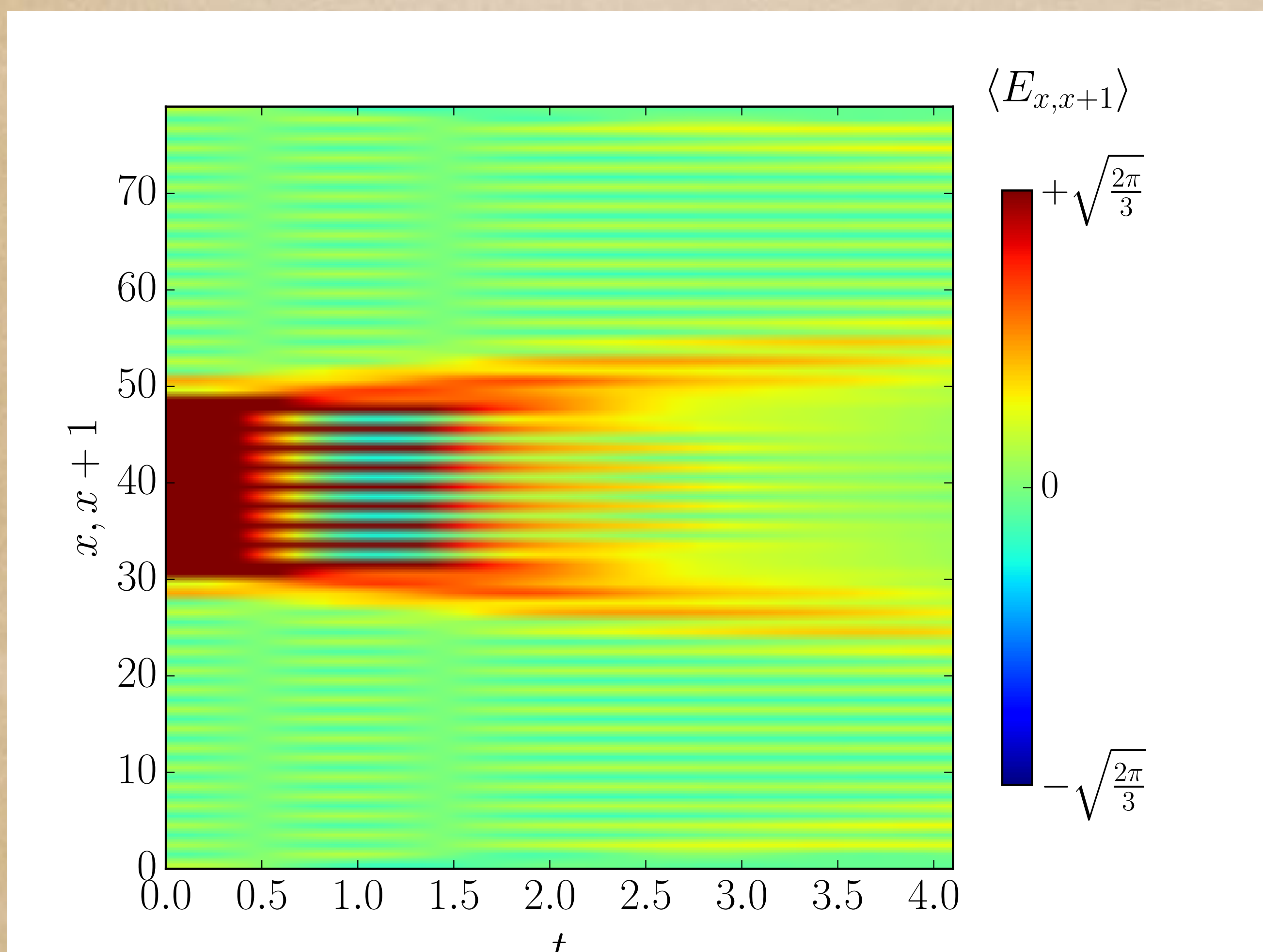
$$\Sigma = 0$$

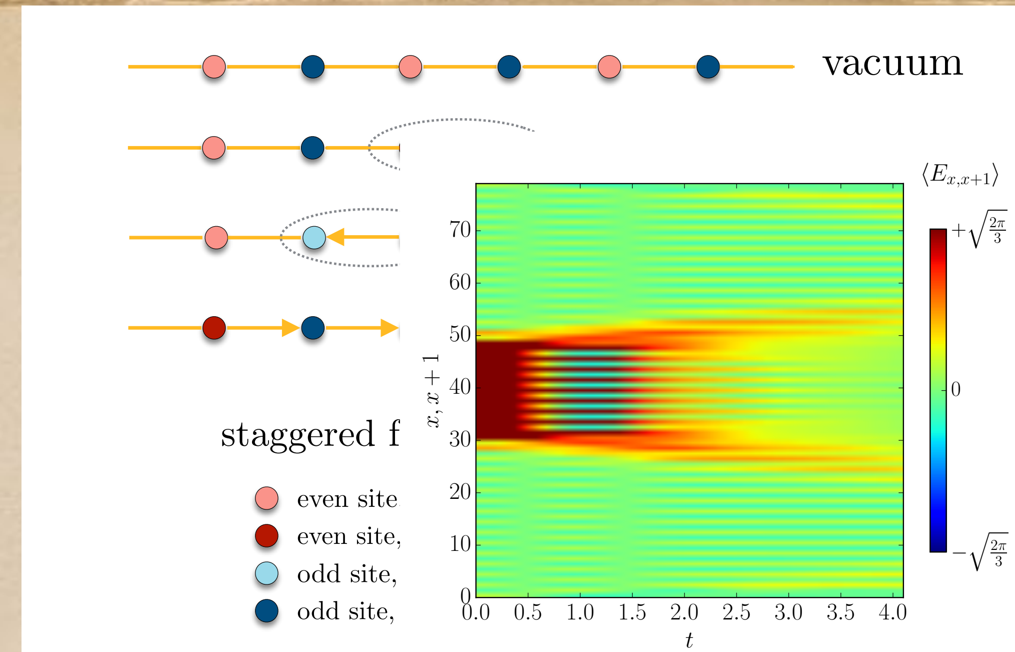
- Large negative m: GS = meson/antimeson state



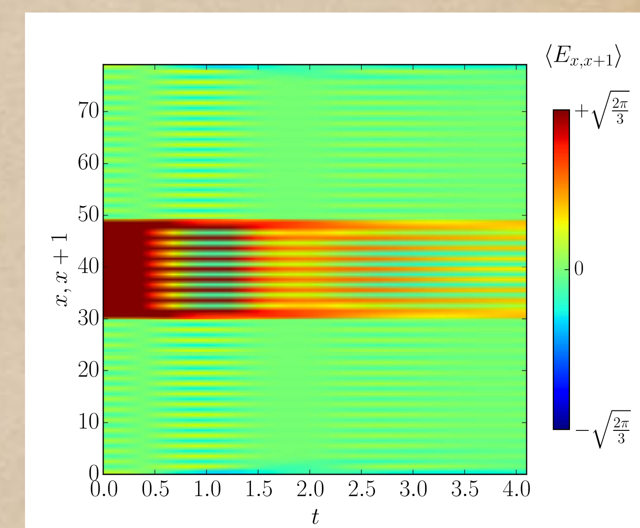
$$\Sigma \neq 0$$

$$H = -\sum_x (\psi^\dagger_{x+1} U_{x,x+1} \psi_x + h.c.) + m \sum_x (-1)^x \psi^\dagger_x \psi_x + \frac{g^2}{2} \sum_x E^2_{x,x+1}$$

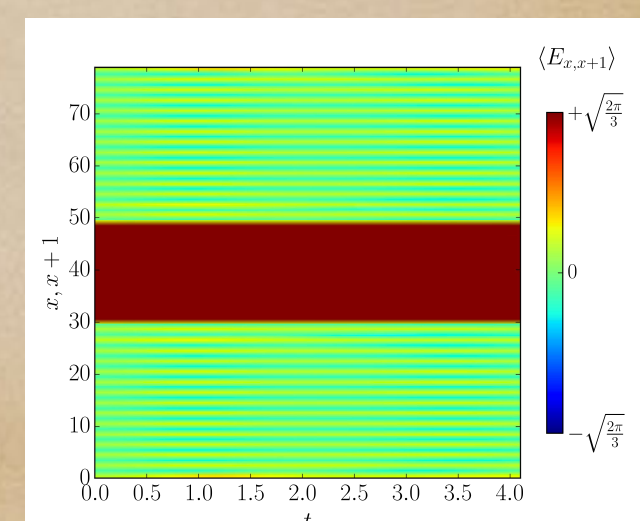




(a)



(b)



(c)

see: Pichler, Dalmonte, Rico, Zoller, Montangero

real-time dynamics (with Tensor Networks) from 1D + time

to "1D" + time

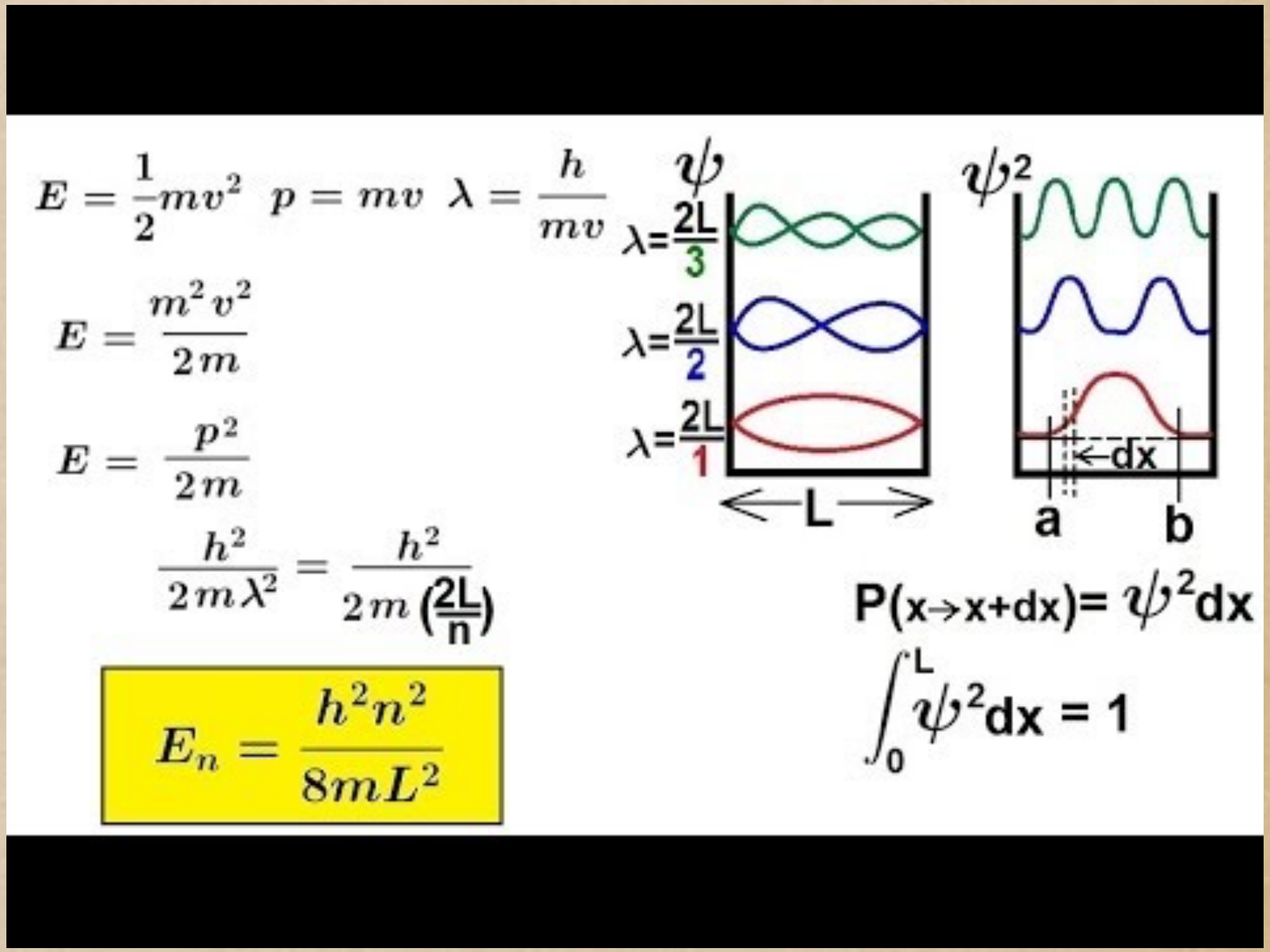
Theoretical physics at its zenith!

Make use of intuitions acquired in the 1D world and apply them in "1D"

Quantum systems confined in effectively one-dimensional geometries

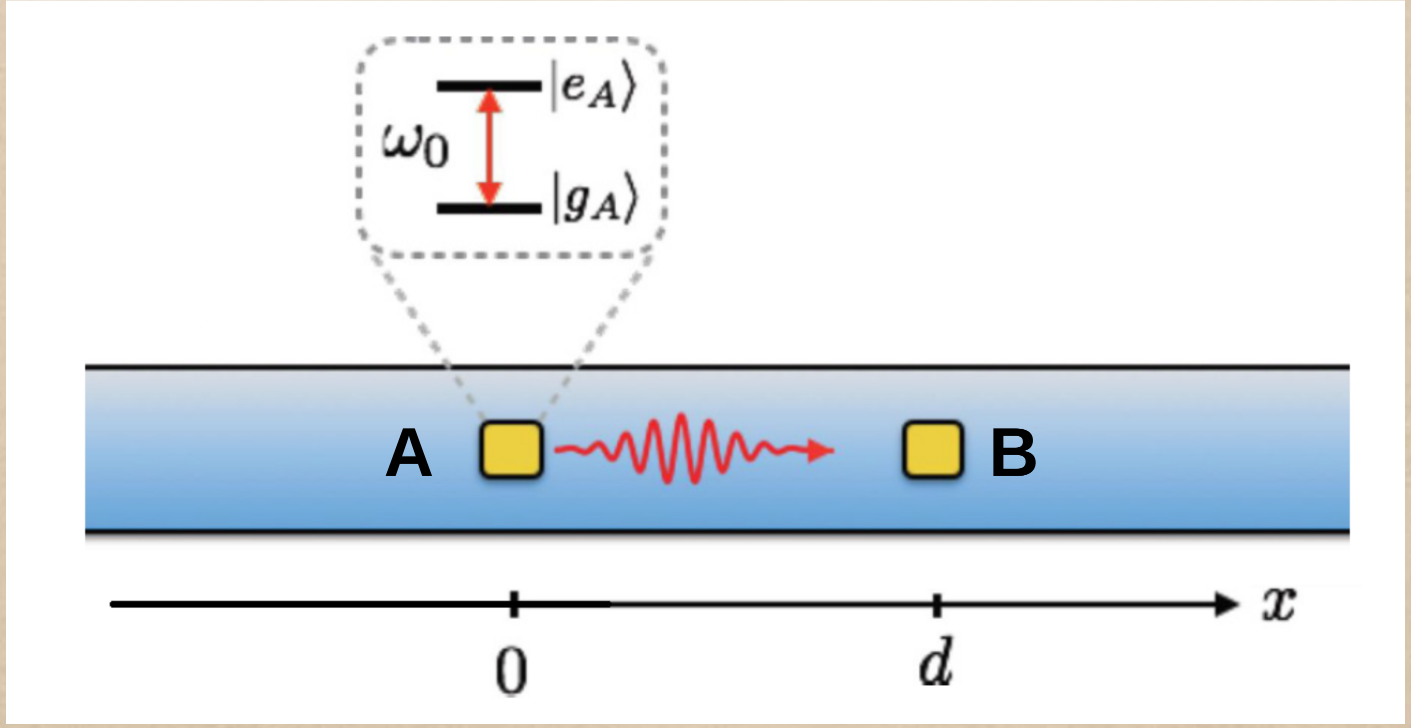
“1D” + time

exercise for students



compute energy levels in 3D box and send one (or two) $L \to 0$

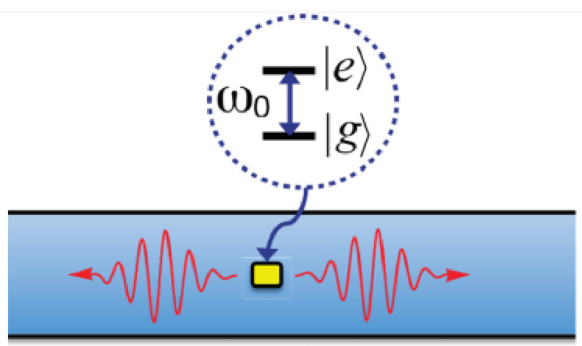
a pair of two-level (artificial) atoms in a waveguide



lowest energy mode, one-excitation sector

start from one atom

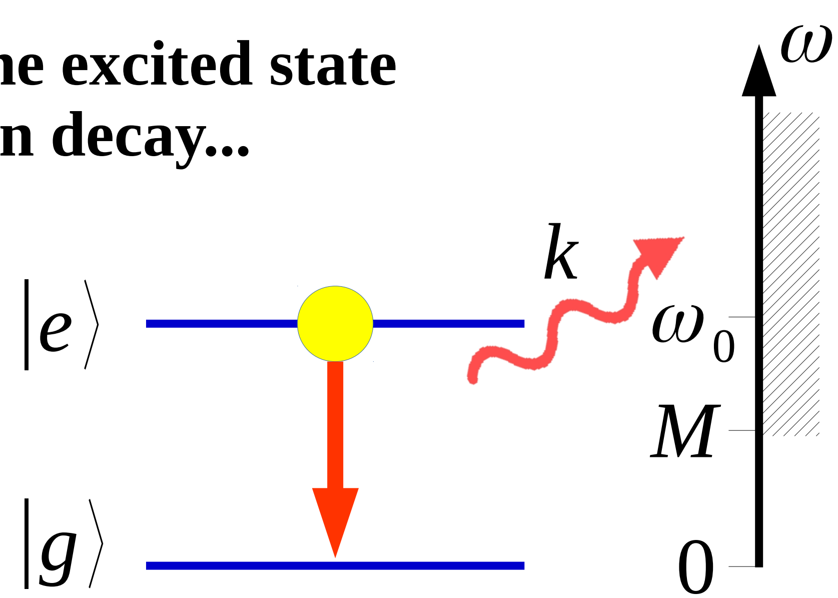
Coupling with the lowest-energy mode in a linear waveguide



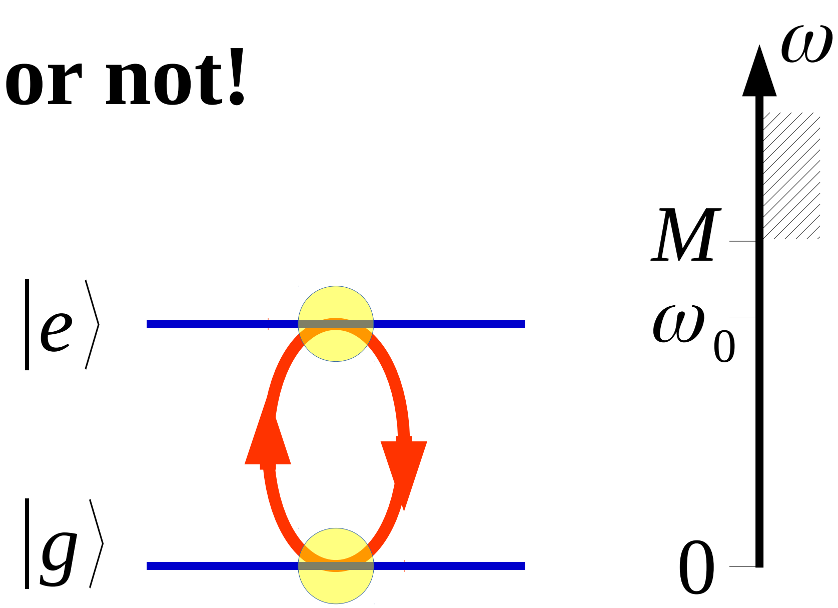
Dispersion relation (massive)

$$\omega(k) = \sqrt{k^2 + M^2}$$

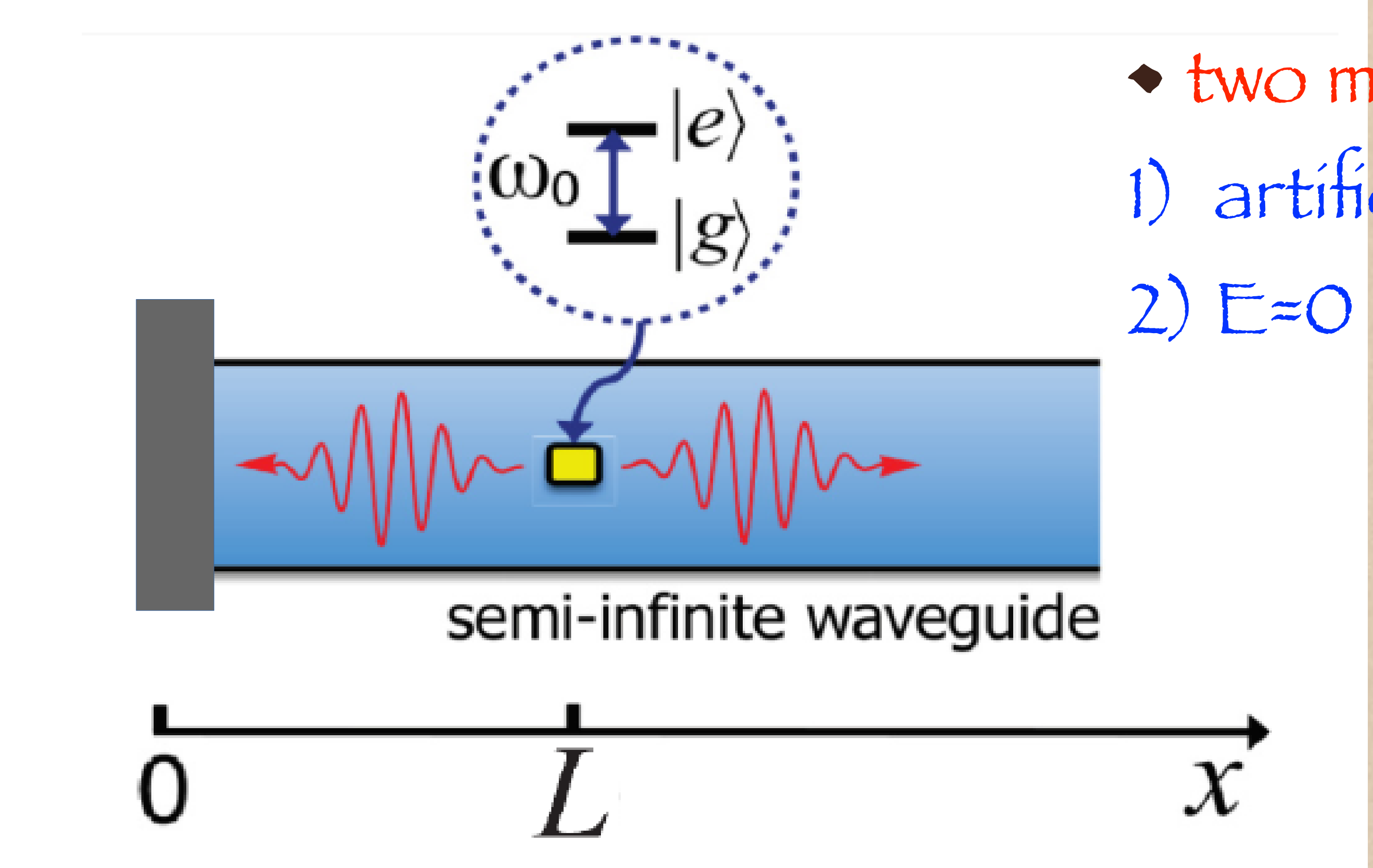
The excited state can decay...



...or not!



+ mirror



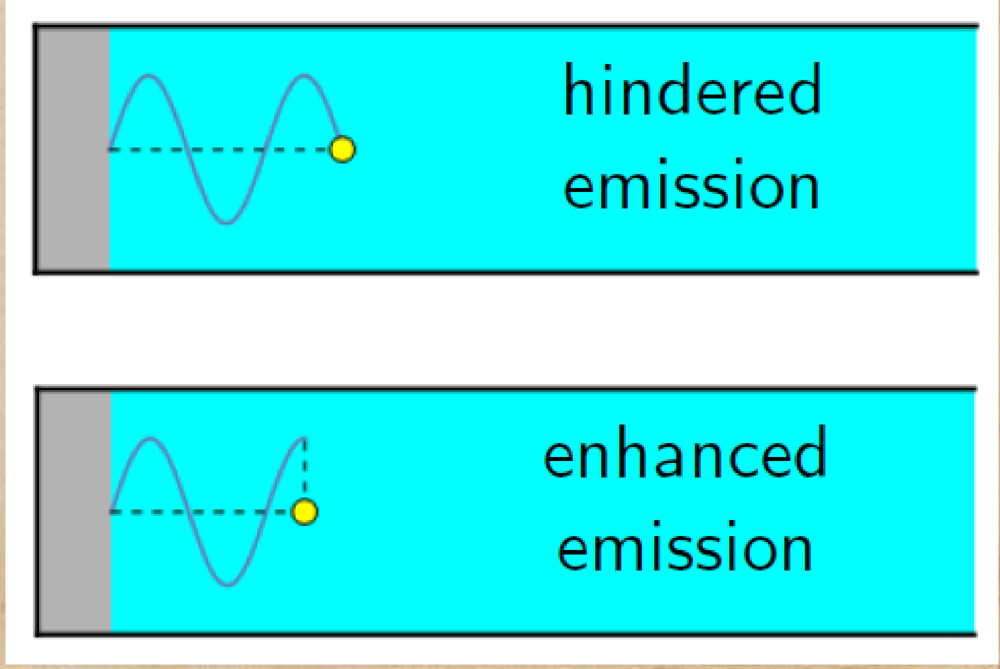
- two main ingredients:

1) artificial dimensional reduction

2) E=0 at mirror

Fermi golden rule:

$$\gamma \propto \left|\langle g;\bar{k}|H_{\text{int}}|e\rangle\right|^2 \propto \sin^2 \bar{k}L$$

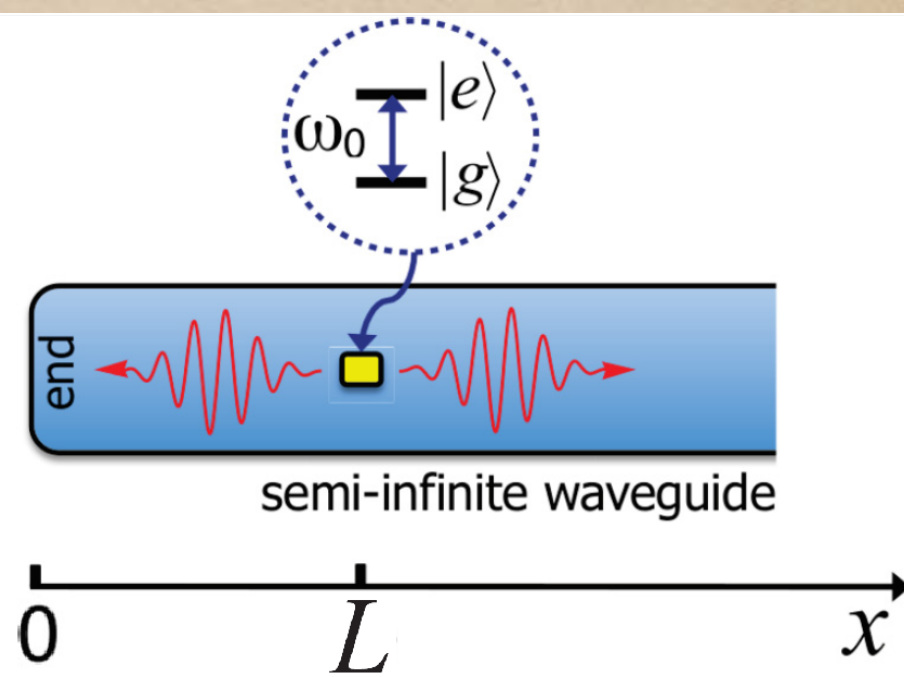


- Dorner & Zoller 2002
- Shen & Fan 2005
- Gonzales-Tudela, , Martin-Cano, Moreno, Martin-Moreno, Tejedor & Garcia-Ripoll, Vidal 2011
- Tufarello, Ciccarello & Kim 2013

$$|g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad |e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

by a Rotating Wave Hamiltonian:

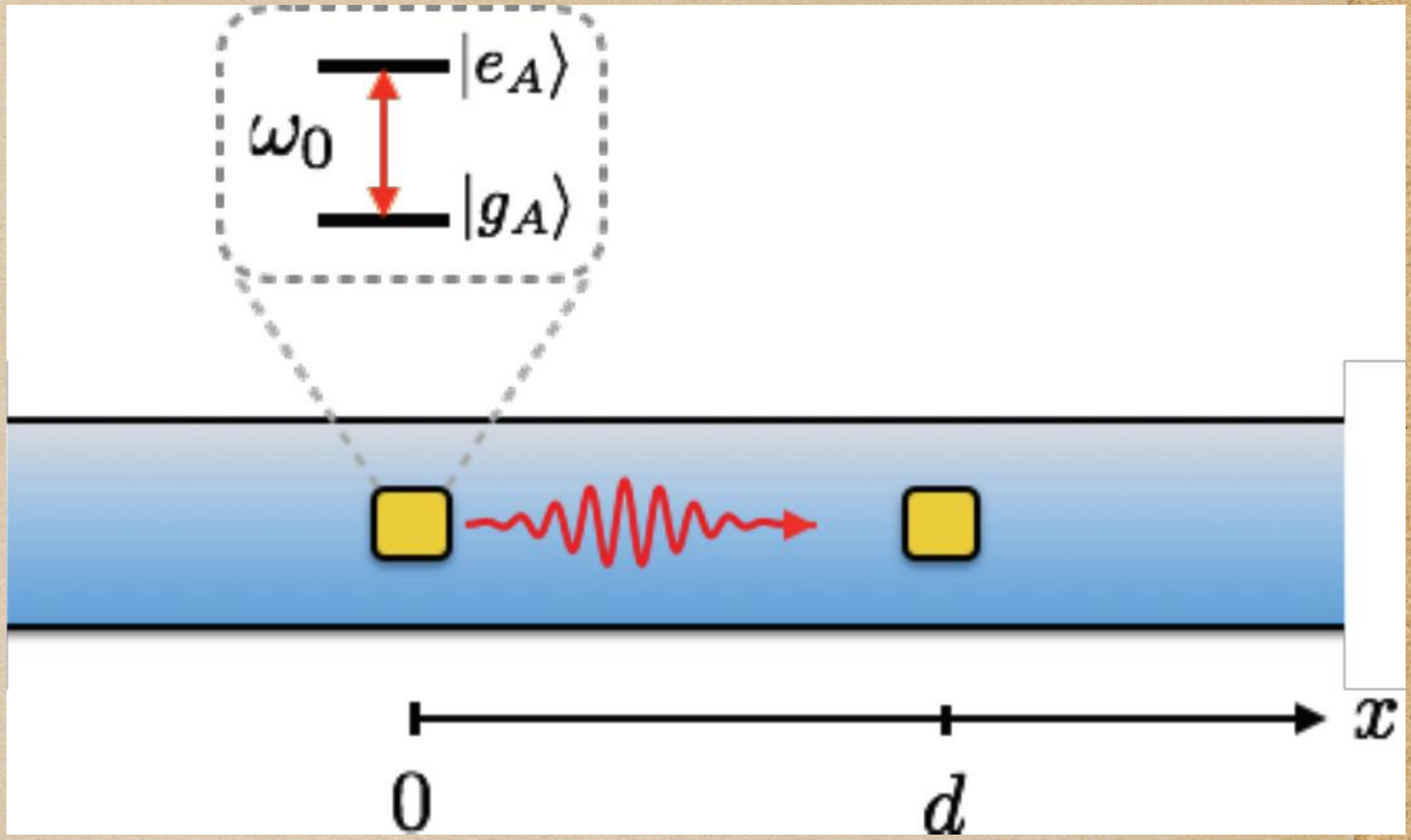
$$\begin{aligned}\hat{\mathcal{H}} &= \hat{\mathcal{H}}_0 + d\left(\hat{E}^\dagger(L)\hat{\sigma}_- + \hat{\sigma}_+\hat{E}(L)\right) \\ &= \hat{\mathcal{H}}_0 + \int dk\ g(k)(\hat{a}_k^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a}_k)\end{aligned}$$



System and Hamiltonian

No need to have mirror!

Atoms behave like "mirrors"



$$
\begin{aligned}
H &= H_0 + \lambda V \\
&= \omega_0(|e_A\rangle\langle e_A| + |e_B\rangle\langle e_B|) + \int \mathrm{d}k\, \omega(k) b^\dagger(k) b(k) \\
&\quad + \lambda \int \frac{\mathrm{d}k}{\omega(k)^{1/2}} \Big[|e_A\rangle\langle g_A| b(k) + |g_A\rangle\langle e_A| b^\dagger(k) \\
&\qquad + |e_B\rangle\langle g_B| b(k) \mathrm{e}^{\mathrm{i}kd} + |g_B\rangle\langle e_B| b^\dagger(k) \mathrm{e}^{-\mathrm{i}kd} \Big],
\end{aligned}
$$

$$\omega(k) = \sqrt{k^2 + M^2}$$

$$M \propto L_y^{-1}$$

Rotating Wave Approximation

The total number of excitations is a constant of motion

$$N = N_{\text{at}} + N_{\text{field}} = |e_A\rangle\langle e_A| + |e_B\rangle\langle e_B| + \int \mathrm{d}k\, b^\dagger(k) b(k)$$

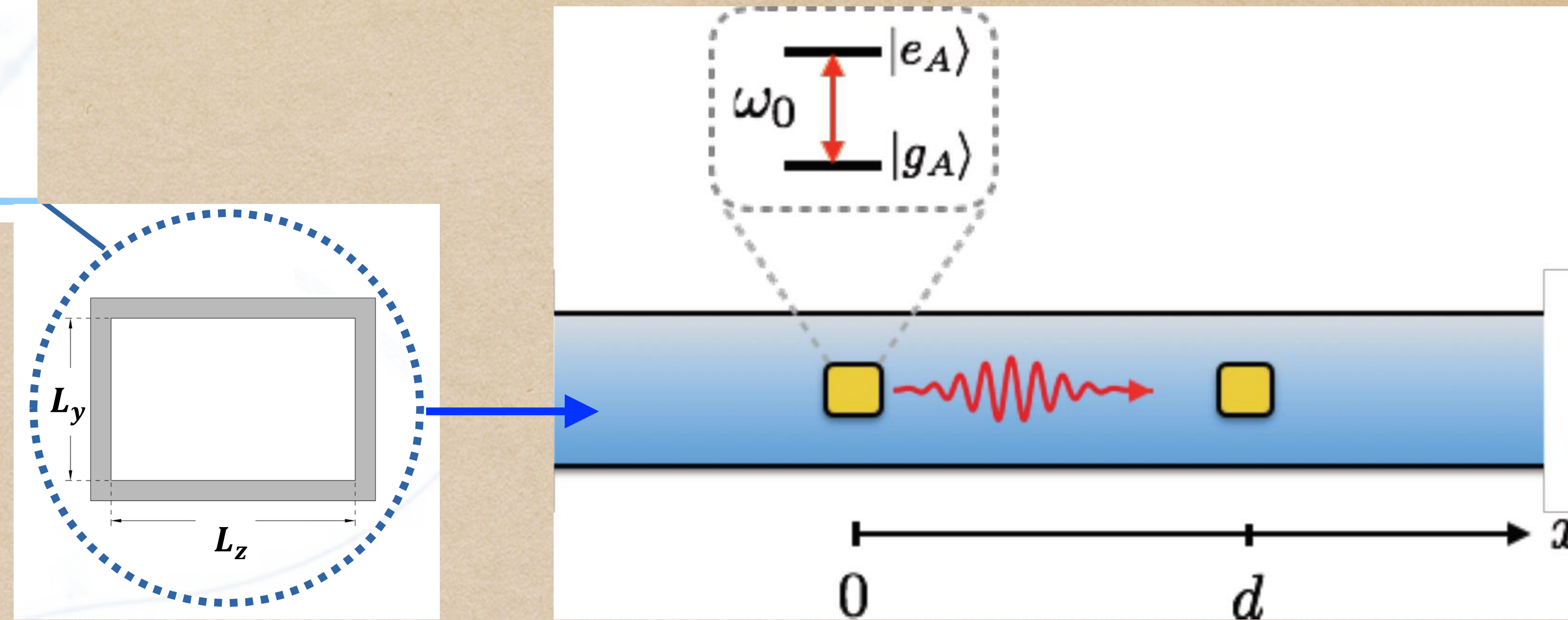
let N=1 (one-excitation sector)

General wavefunction in the sector

$$|\psi\rangle = \left(c_A |e_A, g_B\rangle + c_B |g_A, e_B\rangle\right)|\text{vac}\rangle + |g_A, g_b\rangle |1 \text{ photon}\rangle$$

Bound states $\quad H|\psi\rangle = E|\psi\rangle \quad$ with $\langle\psi|\psi\rangle = 1$

$$\omega_{nm}(k) = \sqrt{\frac{k^2}{\mu\epsilon} + M_{nm}^2}$$

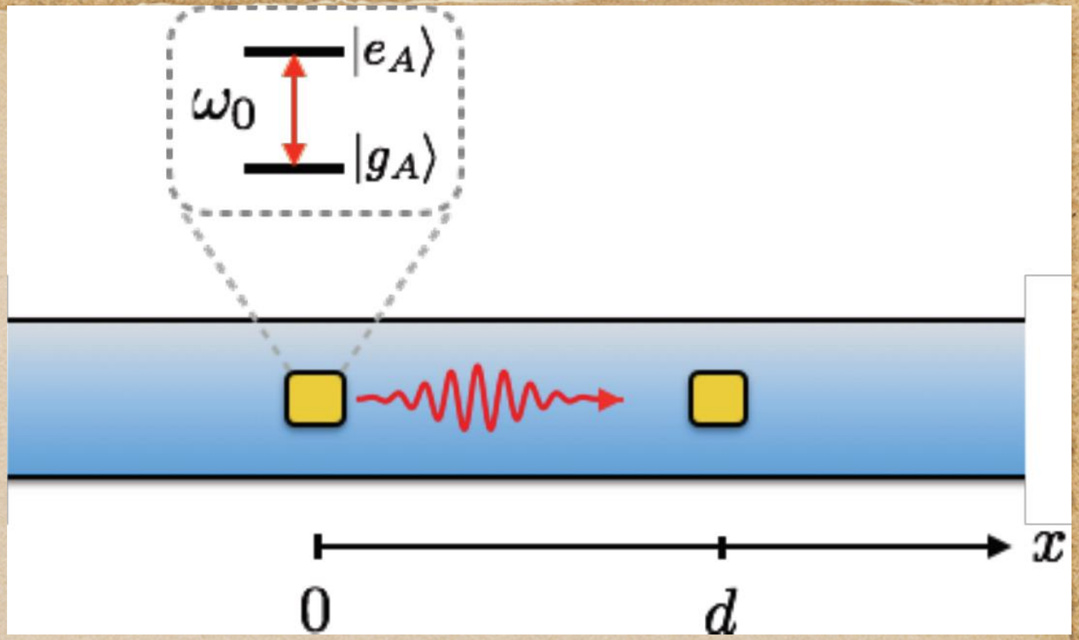


$\mathrm{TE}_{1,0}$ mode; role of boundary conditions

$$|\psi\rangle = \left(c_A|e_A, g_B\rangle + c_B|g_A, e_B\rangle\right) \otimes |\mathrm{vac}\rangle + |g_A, g_B\rangle \otimes |\varphi\rangle$$

$$d_n = \frac{n\pi}{\bar{k}}, \quad \text{with} \quad \bar{k} := \sqrt{\left(\omega_0 + \frac{2\lambda^2}{M}\right)^2 - M^2}.$$

Facchi, Kim, P, Pepe, Pomarico, Tufarelli
PRA 94, 043839 (2016)



structure of bound state

$$|\psi_n\rangle = \sqrt{p_n}|\Psi^s\rangle \otimes |\text{vac}\rangle + |g_A, g_B\rangle \otimes |\varphi_n\rangle$$

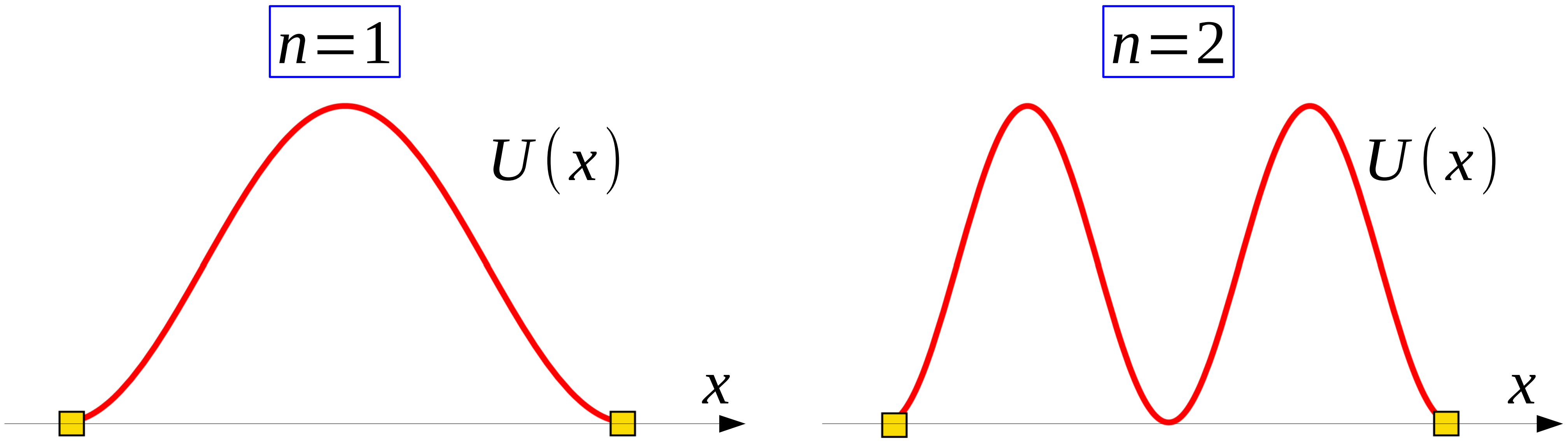
$$|\Psi^\pm\rangle = (|e_A, g_B\rangle \pm |g_A, e_B\rangle)/\sqrt{2}$$

$$p_n = \left(1 + n\frac{2\pi^2\lambda^2}{\bar{k}^2}\right)^{-1}$$

if state factorized at t=0
atomic density matrix

$$\rho_{\text{at}}(\infty) = \frac{p_n^2}{2}|\Psi^s\rangle\langle\Psi^s| + \left(1 - \frac{p_n^2}{2}\right)|g_A, g_B\rangle\langle g_A, g_B|$$

notice:



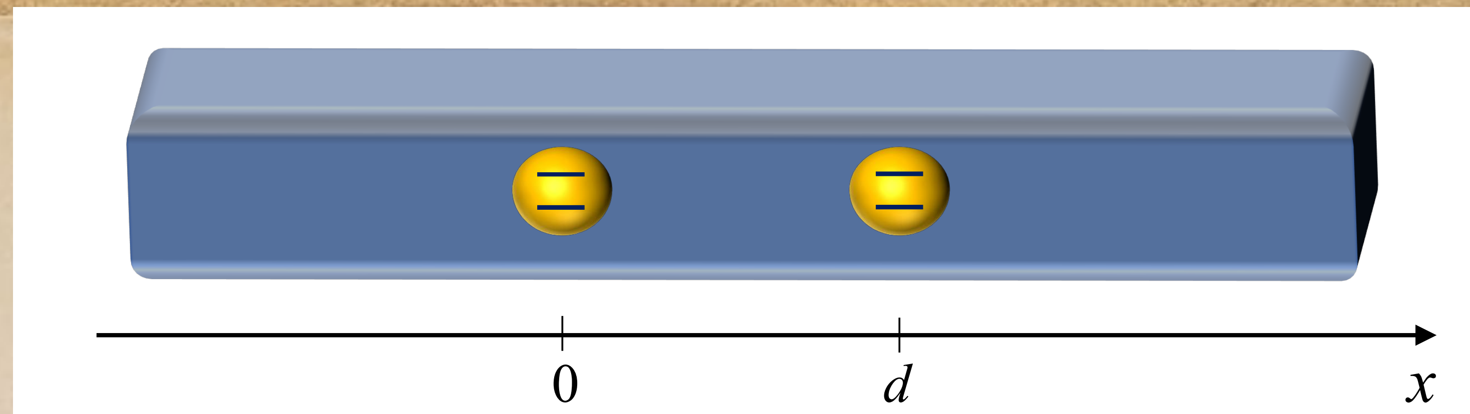
Field energy density

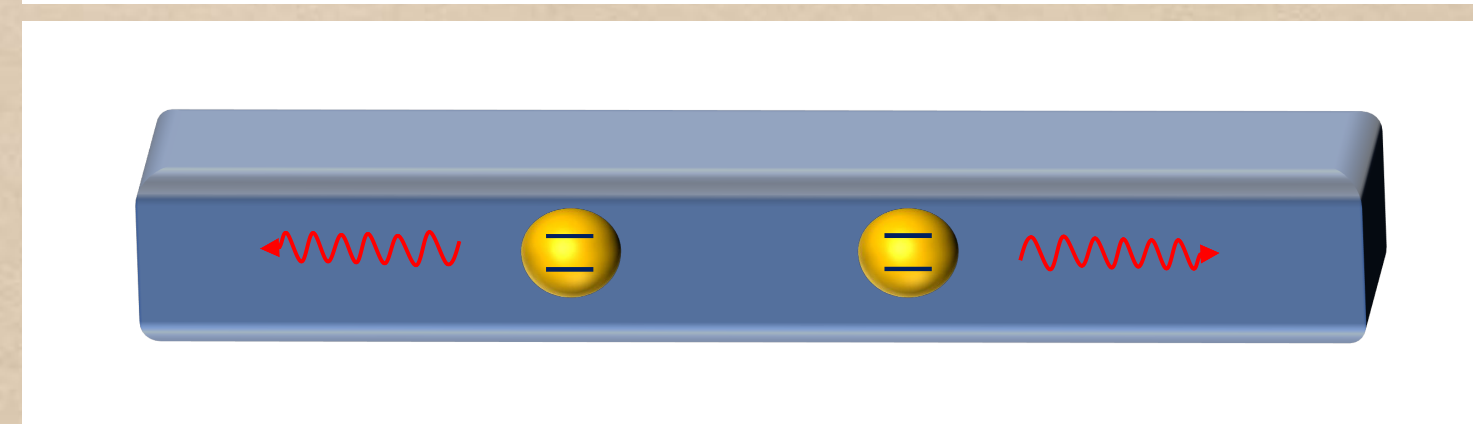
$$U(x) \propto \sin^2(\bar{k}\,x) \quad \text{if } x \in [0, d_n]$$

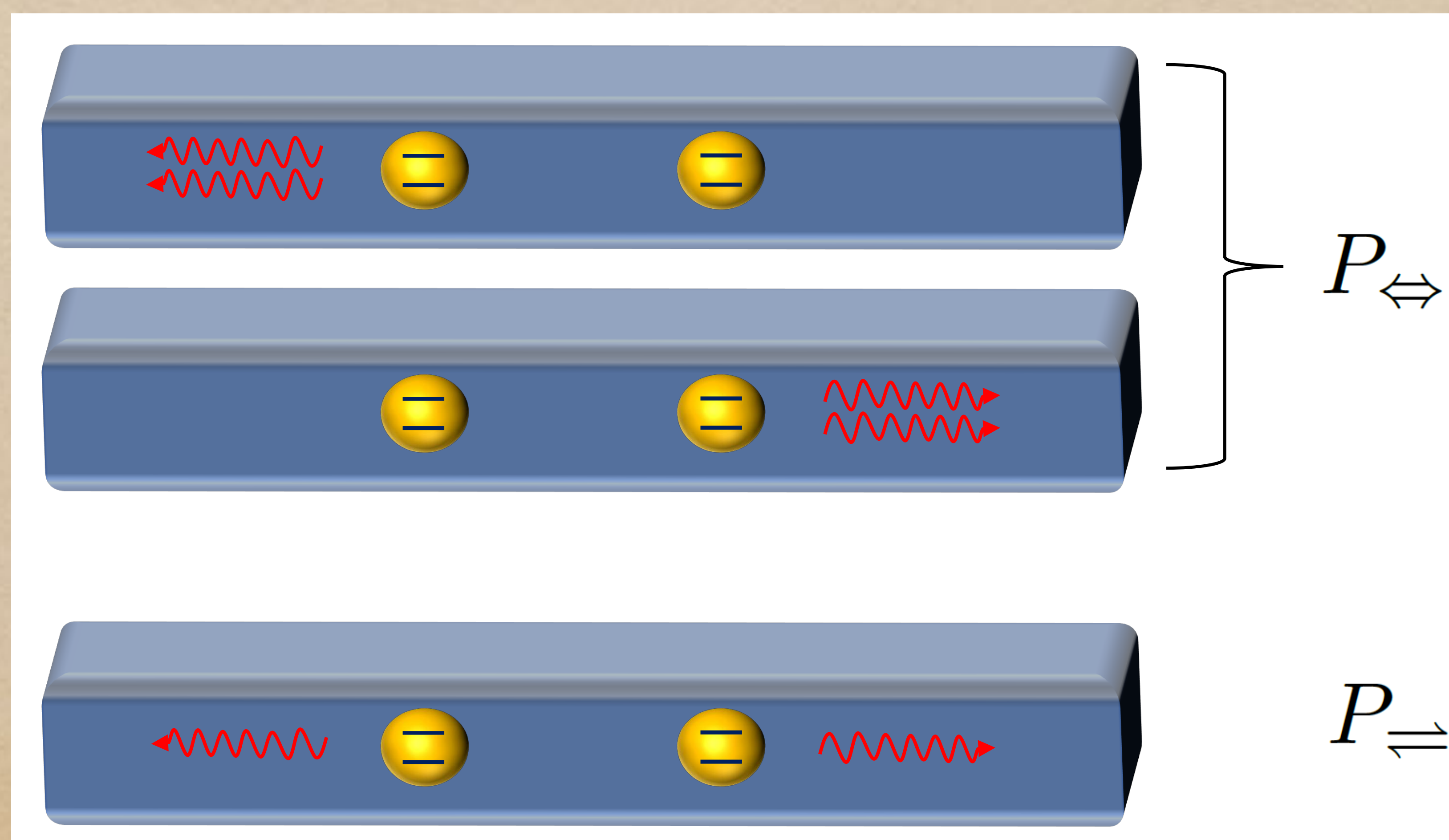
$$U(x) \simeq 0 \quad \text{elsewhere}$$

The atoms behave as dynamical mirrors

retardation effects automatically taken into account if we properly handle the underlying effective field theory


0
d
x




$P_{\Leftrightarrow}$
$P_{\rightleftharpoons}$

- Shen, Fan (2005)
  Gonzales-Tudela et al (2011)

- set of N two-level atoms in optical waveguide: presence of bound states affects the interactions among atoms
  (Calajo, Ciccarello, Chang, Rabl, PRA 2016)
  (Notice: interaction is waveguide-mediated; slow light)

- moving atoms in 1D photonic waveguide
  (Calajo, Rabl, PRA 2017)
  (strong coupling, slow light)

- circuit QED with single LC resonator: very strong interactions decouples photon mode and projects qubits into entangled gs
  (Jaako, Xiang, Garcia-Ripoll, Rabl, PRA 2016) (ultra-strong coupling)

- effective photon-photon interactions in waveguide-QED
  (Zheng, Gauthier, Baranger, PRL 2013)

- atomic degrees of freedom
  (Paulisch, Kimble, Gonzalez-Tudela, NJP 2016)

- Probing vacuum with artificial atom in front of mirror
  (Hoi1, Kockum, Tornberg, Pourkabirian, Johansson, Delsing, Wilson Nat. Phys. 2015)

comments

- “toy” models: simple(r) physical theories that are able to capture the most salient features of the physics in question
- Q. simulators are sometimes able to realize physical models that are “unreal” (believed not to be found in Nature)
- real-time dynamics and non-perturbative regimes
- one is left to wonder about the meaning of “simulation”

comment on interdisciplinarity

Quantum Technologies blend different physical disciplines (in this case high-energy physics, QED, gauge theories vs solid state, low energy, circuit QED, optics)

Maxwell was a religious person. I wonder whether after this momentous discovery he had in his prayers asked for God’s forgiveness for revealing one of His greatest secrets.

Chen Ning Yang
about gauge invariance, Physics Today 2014