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Two-Qubit Gate Operation Applied on Nearest Neighboring Neutral Atom Qubits

Elham Hosseini Lapasar

Osaka City University, Japan

In collaboration with

Kenichi Kasamatsu (Kinki Univ.), Síle Nic Chormaic (OIST Graduate Univ.),
Takeji Takui (Osaka City Univ.),Yasushi Kondo (Kinki Univ.),
Mikio Nakahara (Kinki Univ.), Tetsu Ohmi (Kinki Univ.)

Neutral Atom Quantum Computer

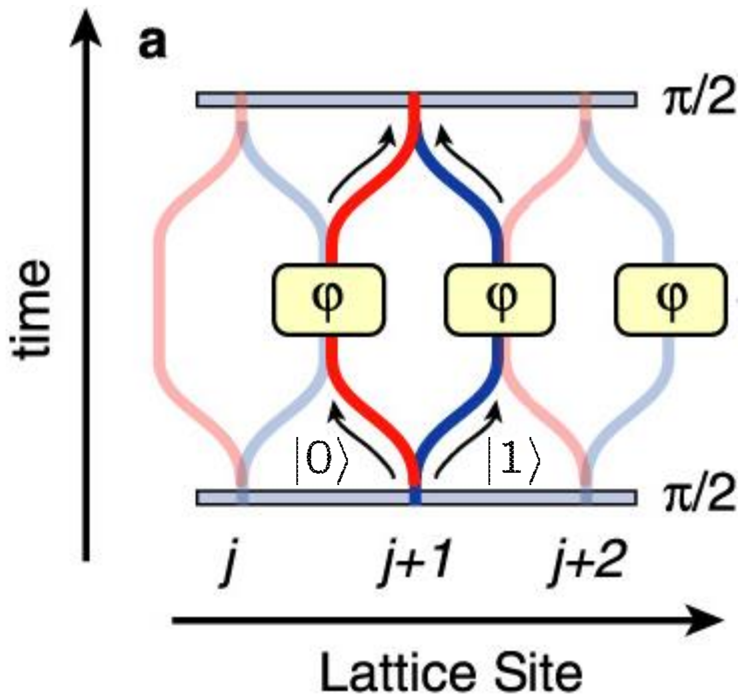
- Alkali atoms are used as qubits.
- Two hyperfine states of an atom span a qubit Hilbert space.
- Neutral atom is robust against external disturbance.
- A large number of atoms can be confined in an optical lattice (1-d, 2-d, and 3-d).

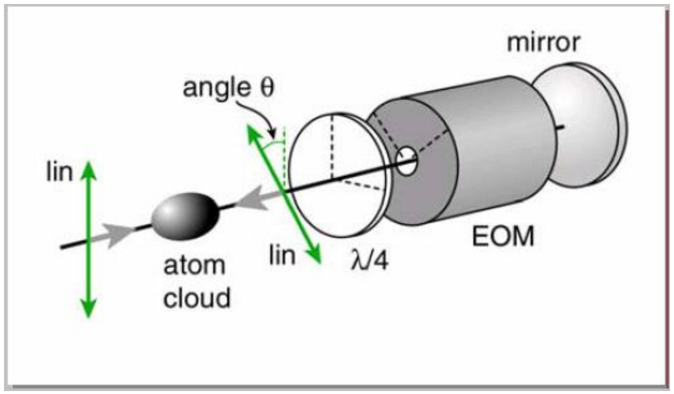
- One-qubit gate operation is implemented by Rabi oscillation and two-photon Raman transition.

- Two-qubit gate in a selective manner is the subject of this talk.
  J. Phys. Soc. Jpn. 83, 044005 (2014)

Two-Qubit Gate:
Mandel *et al.'s* Experiment

Mandel *et al.*, Nature 425, 937 (2003)
Controlled collisions for multi-particle entanglement of optically trapped atoms



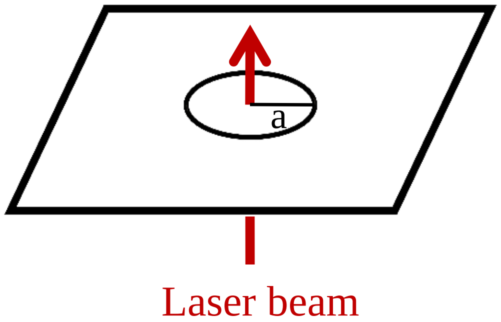


Electro-Optic Modulator

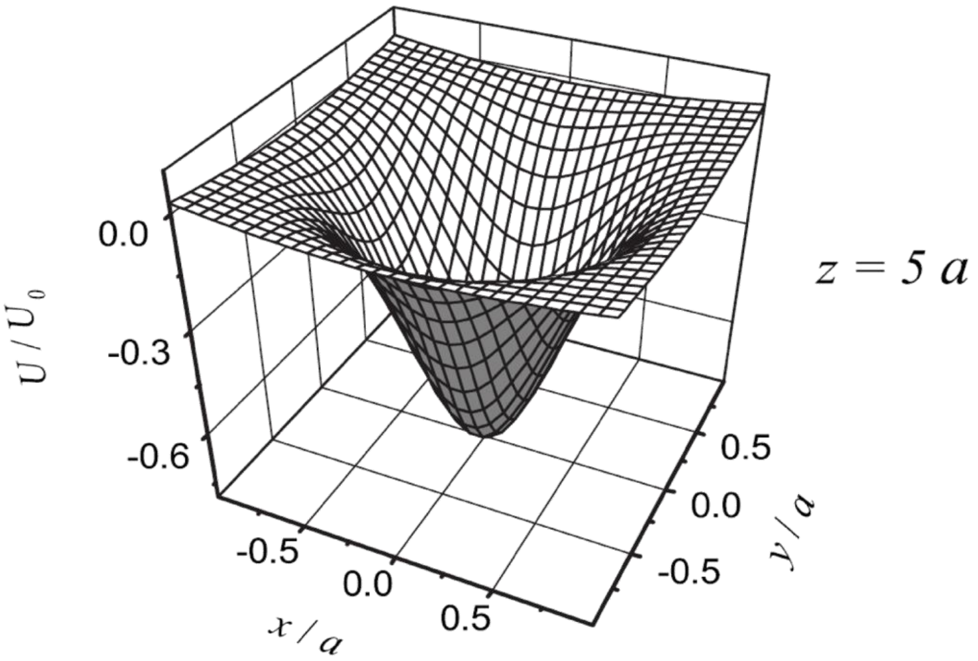
Near-Field Fresnel Trap

Bandi *et al*., PRA 78, 013410 (2008)

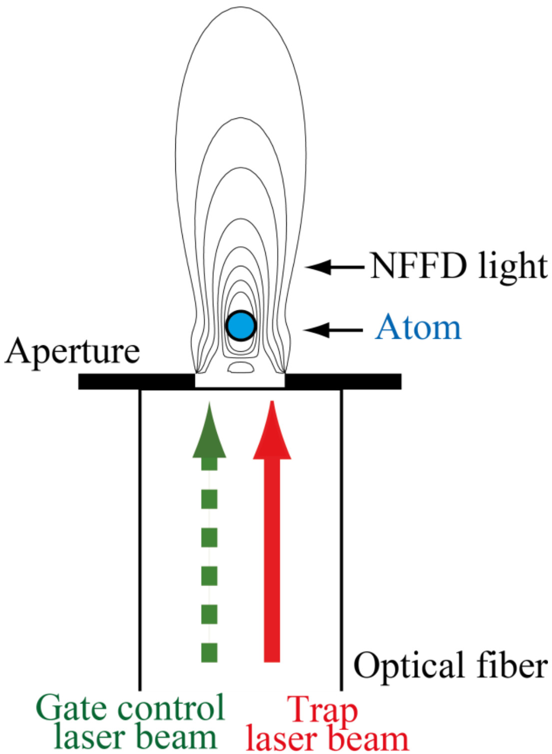
It is possible to confine an atom by a laser beam passing through a small hole whose diameter is comparable to the wave-length of the laser.



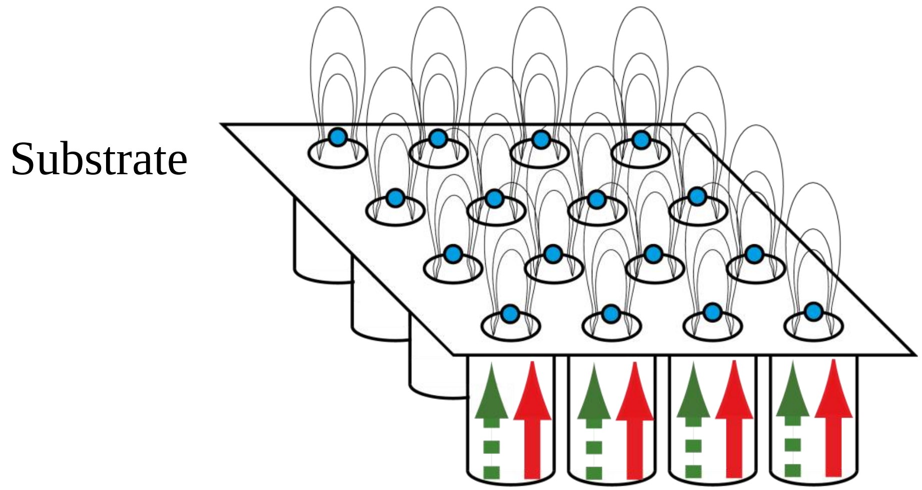
$$a \geq \lambda_f$$



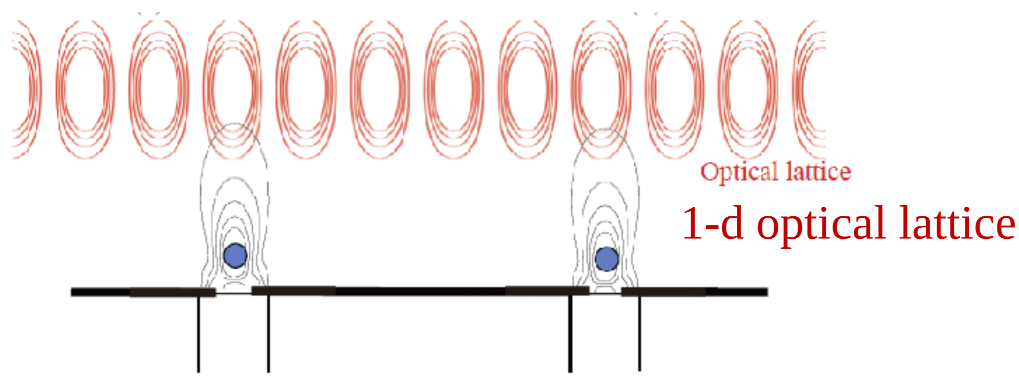
Proposal



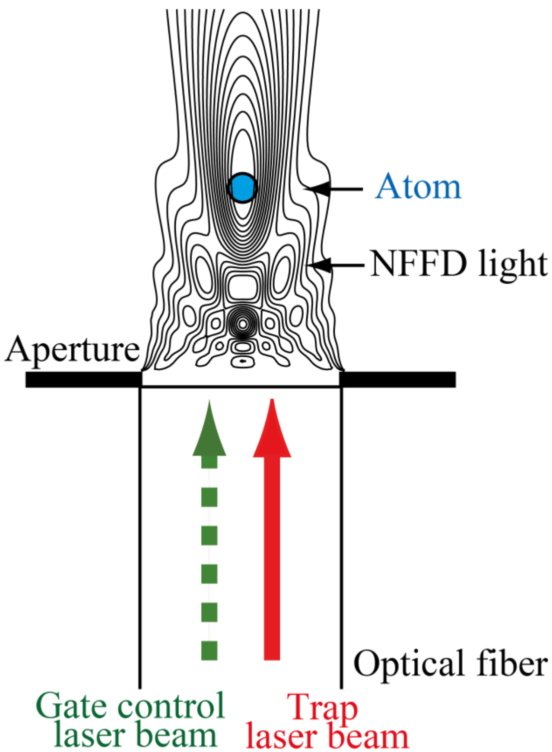
Near-Field Fresnel Trap



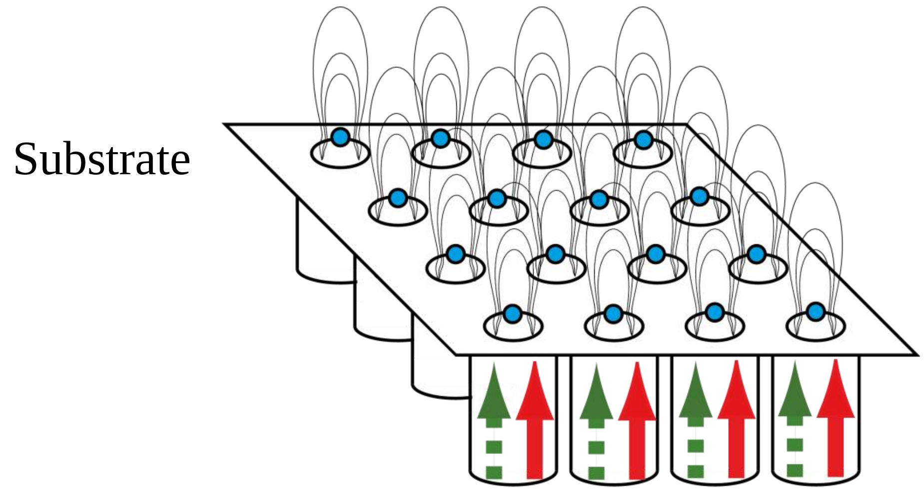
An array of Fresnel traps



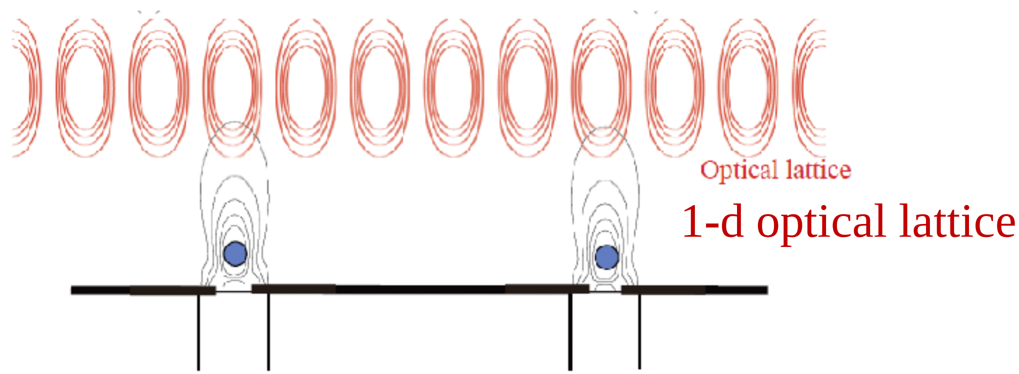
Proposal



Near-Field Fresnel Trap



An array of Fresnel traps



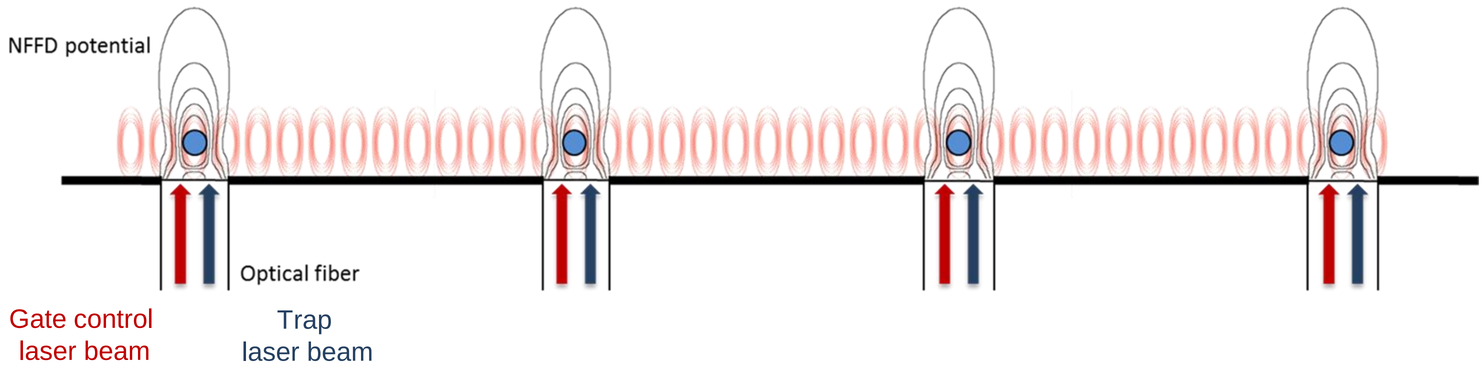
(Spatial Light Modulator with Liquid Crystal on Silicon Technology) and
(Micro-Electro-Mechanical System Technology)

Two-Qubit Gate Operation

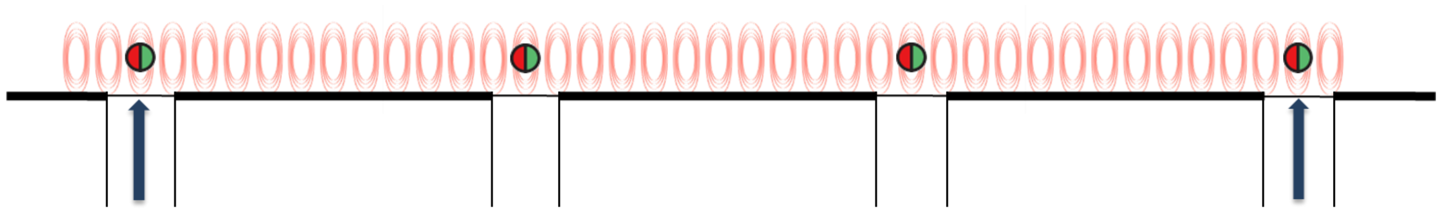
Initialization

$$|0\rangle \quad |1\rangle$$

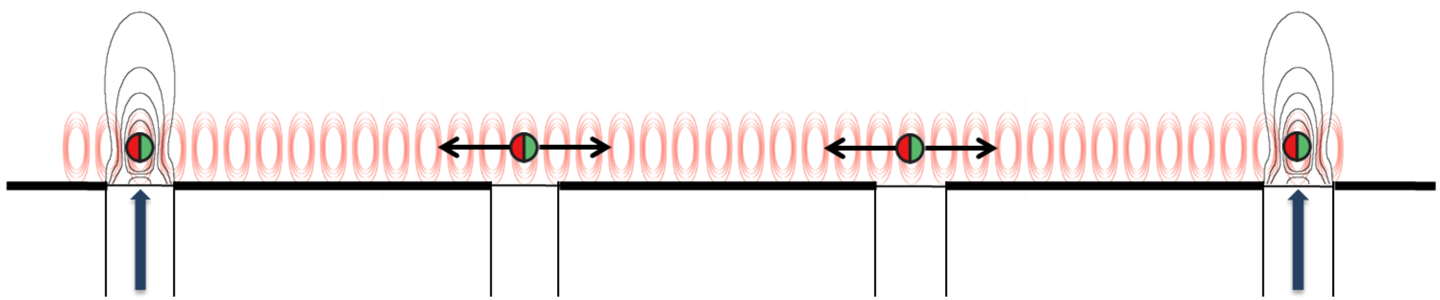
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



❑Step 1



❑Step 2



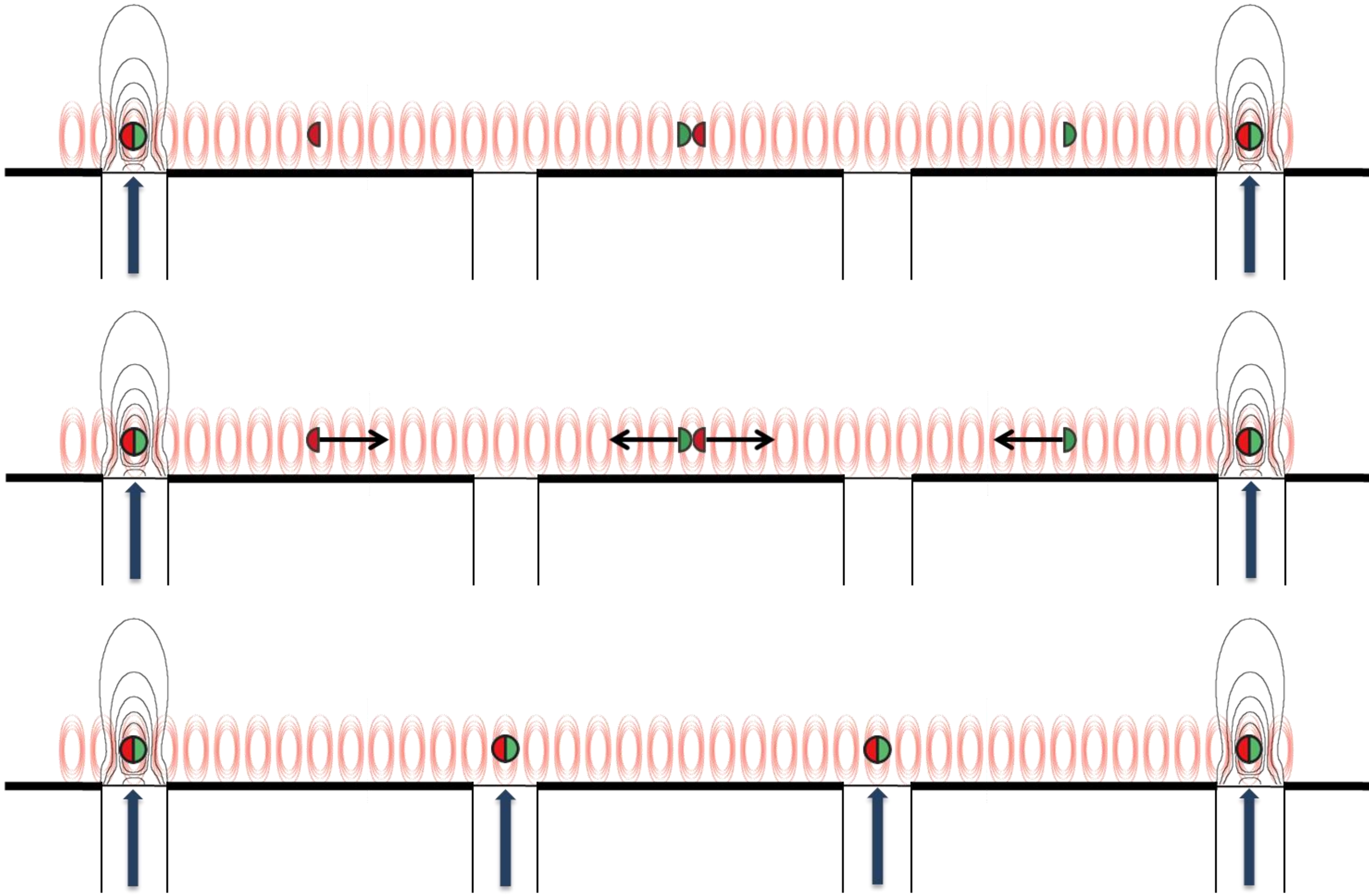
$|0\rangle$ $|1\rangle$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

❑Step 3

❑Step 4

❑Step 5



Numerical Analysis

- ^{87}Rb

- $|0\rangle = |F=1, m_F=1\rangle,$
  $|1\rangle = |F=2, m_F=1\rangle.$

  Compatible with two-photon Raman transition

- Operations must be done adiabatically in the shortest possible time.

- Individual process fidelity = $|\langle\psi_0|\psi(t=T)\rangle|$
  Overlap between the ground state of the final potential and the time-evolved wave function
  = 0.99

Parameters

Parameters	Numerical values
Radius of an aperture	$a = 1.5\ \lambda_F \approx 1.2$ µm
Wavelength of the trap laser beam	λ_F= 795.118 nm
Laser intensity	$I = 2.5 \times 10^5$ W/cm^2
Wavelength of the optical lattice	λ_{OL}= 785 nm
D_1 transition of ^{87}Rb	λ_0= 794.979 nm
Depth of trapping potential at t=0	$U_0 = h \times 1.03 \times 10^6$ Hz
Depth of the optical lattice potential at t=0	$V_0 = h \times 1.47 \times 10^5$ Hz
Minimum of NFFD potential	$z_m \approx 1.7$ µm

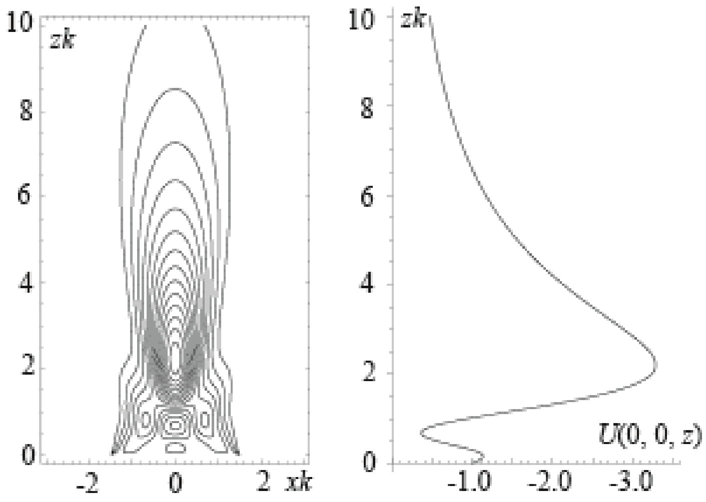
Fresnel Trap Potential

Rayleigh-Sommerfeld formula

$$U_{\mathsf{F}}(x,y,z) = -U_0 \frac{|\epsilon(x,y,z)|^2}{E_0^2}$$

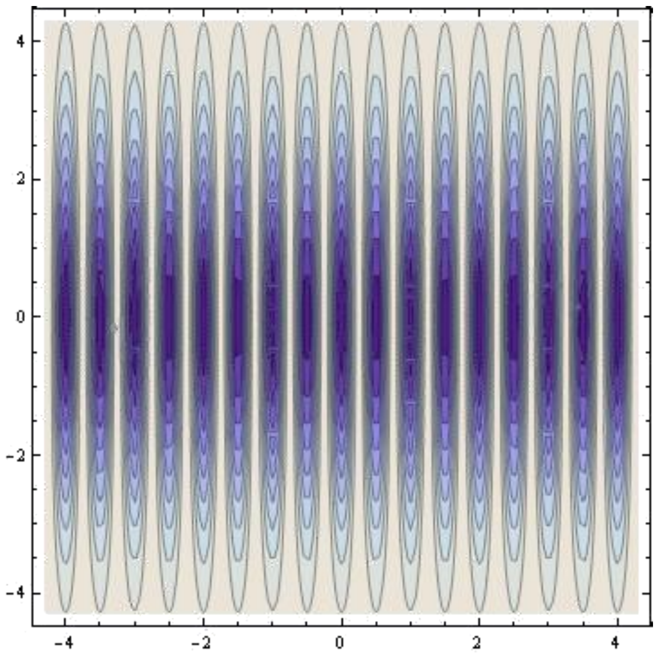
$$U_0 = \frac{3}{8} \frac{\gamma}{|\delta|} \frac{E_0^2}{k^3}$$

$$\epsilon(x,y,z) = \frac{E_0}{2\pi} \iint \frac{e^{ikr}}{r} \left(\frac{z}{r}\right) \left(\frac{1}{r} - ik\right) dx' dy'$$



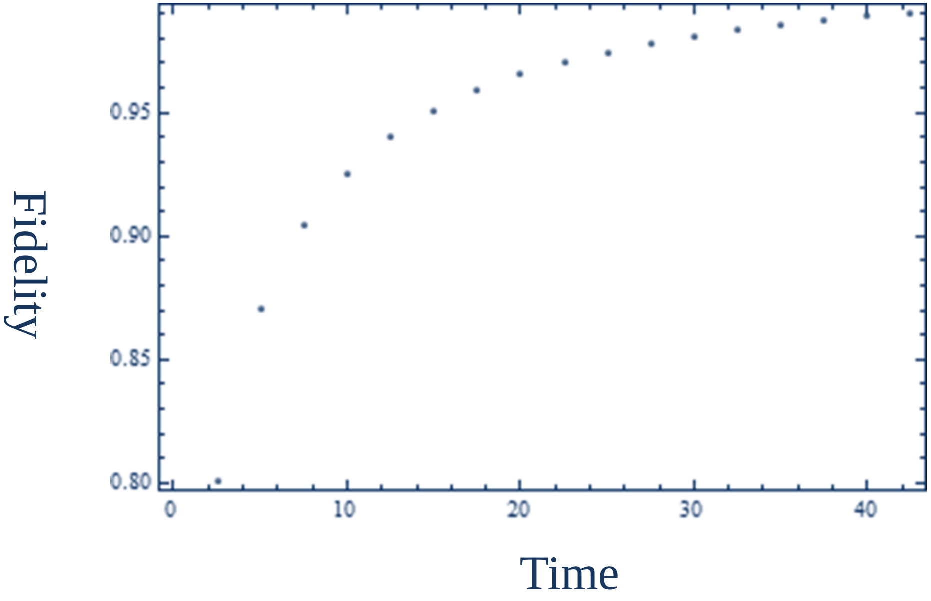
Optical Lattice Potential

$$U_{\mathrm{OL}}(x) = -V_0 \cos^2(2k_{\mathrm{OL}}x)e^{-2(y^2+(z-z_m)^2)/w^2}$$



Step1

$$U_F(\boldsymbol{x},t) = \cos^2\left(\frac{\pi}{2T_1}t\right)U_F(\boldsymbol{x})$$



T_{FT} = Time required to attain fidelity 0.99

= 1.8 ms

Step 2

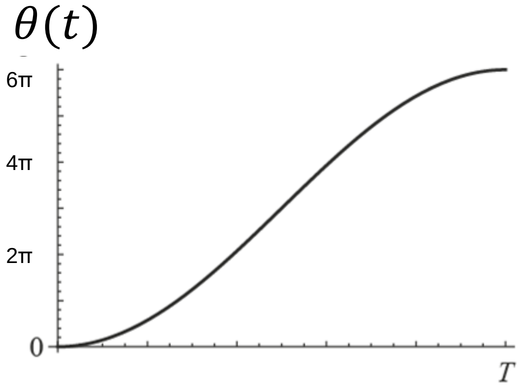
- Optical lattice potential which is produced by a pair of counterpropagating laser beams which have different polarizations.

$$E_+(x) + E_-(x) \propto \sigma^+ \cos(kx - \theta) + \sigma^- \cos(kx + \theta)$$

- Optical potentials

$$V_\pm(x) \propto \cos^2(kx \mp \theta)$$

$$\theta(t) = n\pi \sin^2\left(\frac{\pi}{2T_{OL}} t\right)$$

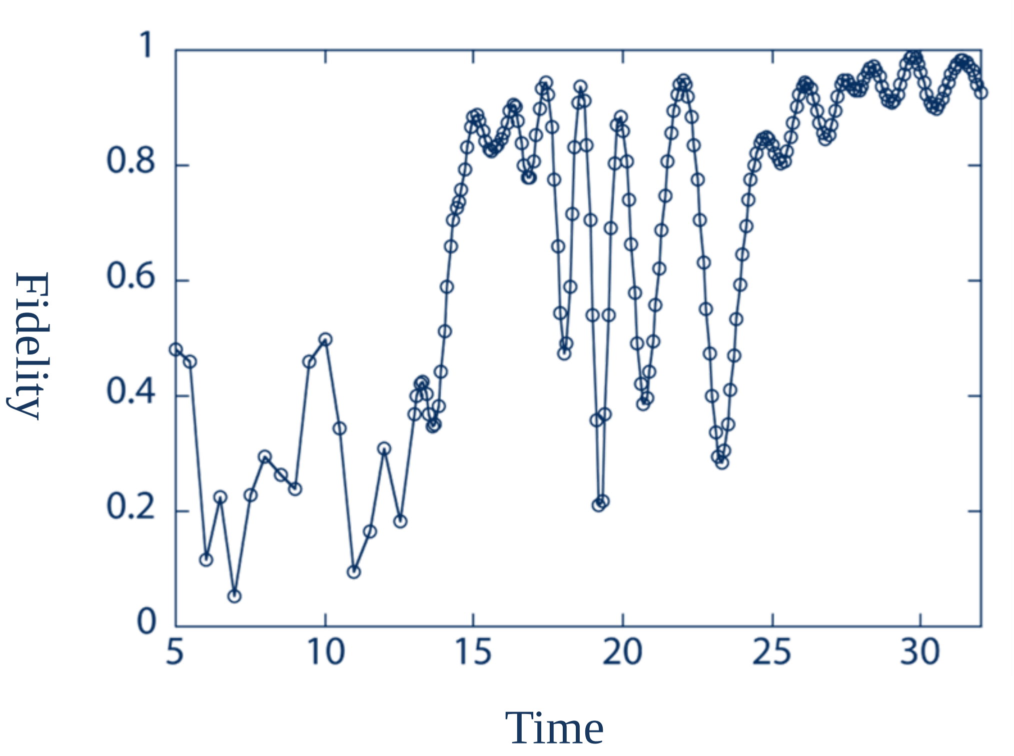


- Effective potential for each hyperfine state

$$V_{|0\rangle}(x) = \frac{1}{4}V_+(x) + \frac{3}{4}V_-(x),$$

$$V_{|1\rangle}(x) = \frac{3}{4}V_+(x) + \frac{1}{4}V_-(x).$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V_{|1\rangle}(t)\psi$$



T_{OL} = Time required to attain fidelity 0.99
= 1.28 ms

Step 3

- Interaction energy

$$U_{\text{int}} = g \int |\psi|^4 dx \qquad g = 4\pi\hbar^2 a/m$$

- $U_{\text{int}} t_{\text{hold}} = \pi$

- Two-qubit gate

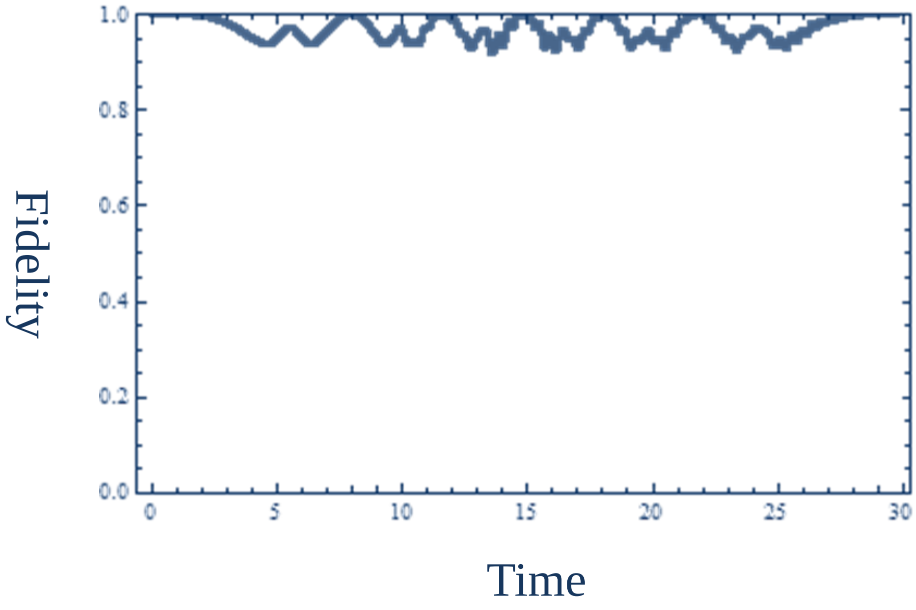
$$U = |00\rangle\langle 00| + e^{-i\pi}|01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|$$
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$T_{\text{Int}} = 2.29$ ms

Step 4 and Step 5

Atoms are returned to their initial positions in the optical lattice (reverse of Step 2) and then NFFD traps are switched on (inverse of Step 1).

Fidelity of spectator atom during the two-qubit gate operation



Execution Time and Fidelity

- Overall execution time

$$T_{overall} = 2(T_{FT} + T_{OL}) + T_{int} \approx 8.45 \text{ ms.}$$

- Overall fidelity

$$0.99^8 = 0.923$$

Summary

- Selective two-qubit gate operation is possible if atoms are trapped in an optical lattice generated by near field Fresnel diffraction of light at an aperture of variable size.
- We proposed an experiment towards the demonstration of a selective two-qubit gate operation.
- We have obtained an upper bound of the gate operation time of 8.45 ms with corresponding fidelity of 0.923.
- Reduction of the gate operation time in is possible by increasing the laser intensity.
- The proposal should be feasible within existing technology.

Thank you for your attention

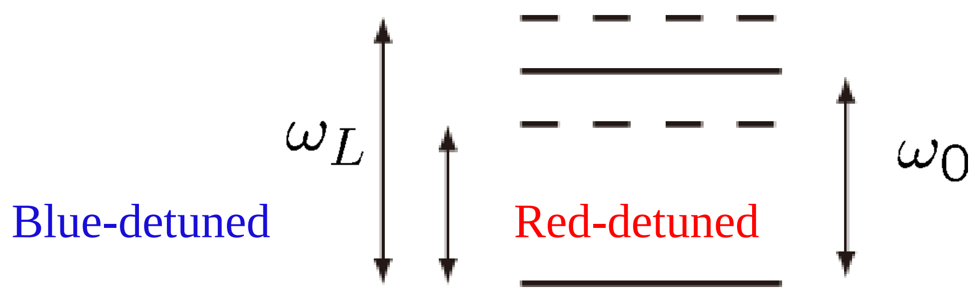
AC Stark Shift (Light Shift)

$$\boldsymbol{E}(\boldsymbol{x},t) = \mathrm{Re}(\boldsymbol{E}_0(\boldsymbol{x})e^{-i\omega_L t})$$

$$H_{\mathrm{i}} = -\frac{1}{2}(\boldsymbol{E}_0 \cdot \boldsymbol{d})(e^{-i\omega_L t} + e^{i\omega_L t}) \qquad |\psi\rangle = c_g|g\rangle + c_e|e\rangle$$

$$V(\boldsymbol{x}) = \frac{\hbar|\Omega_{eg}(\boldsymbol{x})|^2}{4(\Delta_{eg})}$$

$$\Omega_{eg} = \langle e|\boldsymbol{d}|g\rangle \cdot \boldsymbol{E}_0(\boldsymbol{x})/\hbar \qquad \Delta_{eg} = \omega_L - \omega_0$$



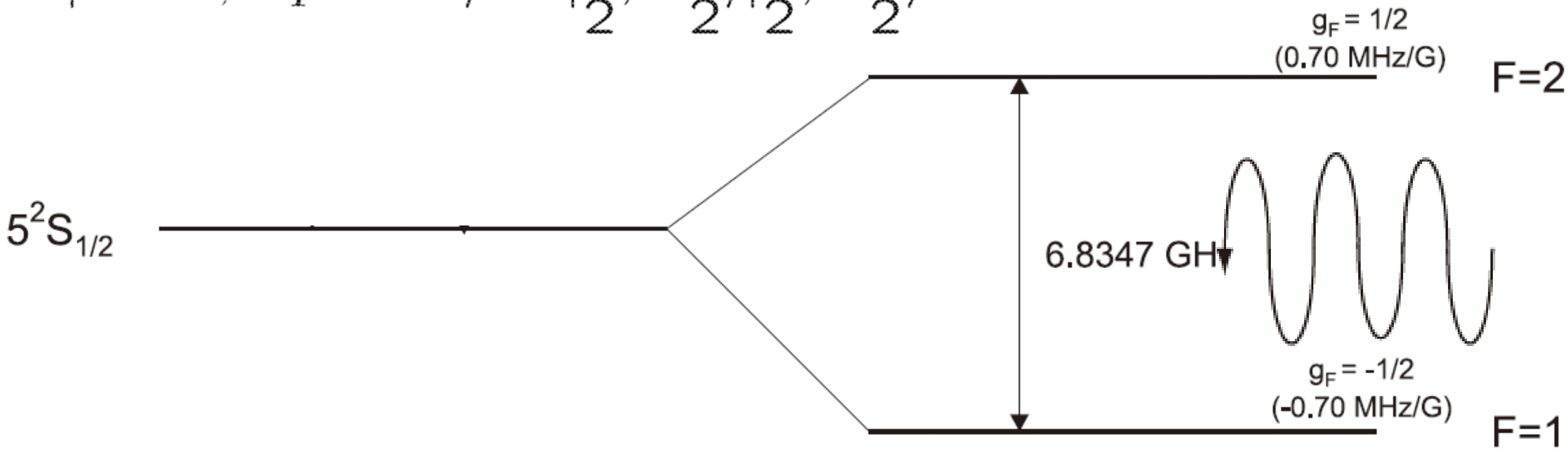
One-Qubit Gate:
Mandel *et al's* Proposal

One-qubit gate implementation by MW pulses

^{87}Rb

$$|0\rangle = |F=1, m_F=-1\rangle = -\frac{\sqrt{3}}{2}\left|\frac{3}{2},-\frac{3}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle + \frac{1}{2}\left|\frac{3}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle,$$

$$|1\rangle = |F=2, m_F=-2\rangle = |\frac{3}{2},-\frac{3}{2}\rangle|\frac{1}{2},-\frac{1}{2}\rangle.$$



One-Qubit Gate Operation

One-qubit operation by two-photon Raman transition
(site selective addressing)

- Detuning $\hbar\Delta = \hbar\omega_L - (E_e - E_0)$.

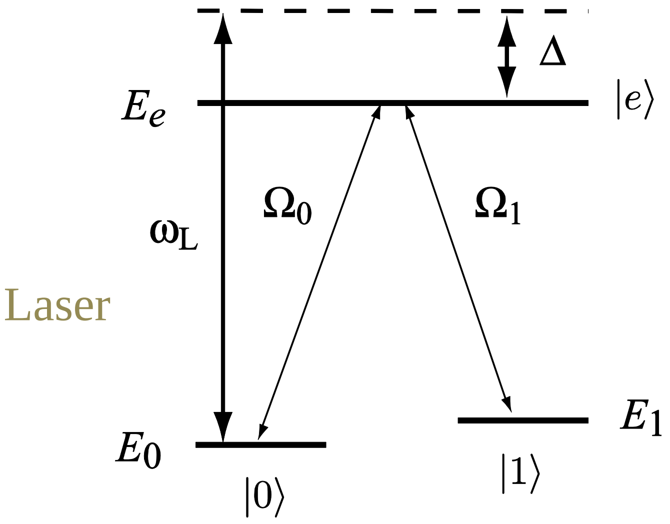
Rabi Osc. $\Omega_i : |i\rangle \leftrightarrow |e\rangle$.

- General state

$|\psi\rangle = c_0|01\rangle + c_1|11\rangle + c_e|e0\rangle$.

$(|ik\rangle = |i\rangle \otimes |\text{photon } \# \ k\rangle)$

$$i\frac{d}{dt}\begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = H \begin{pmatrix} c_0 \\ c_1 \end{pmatrix}$$

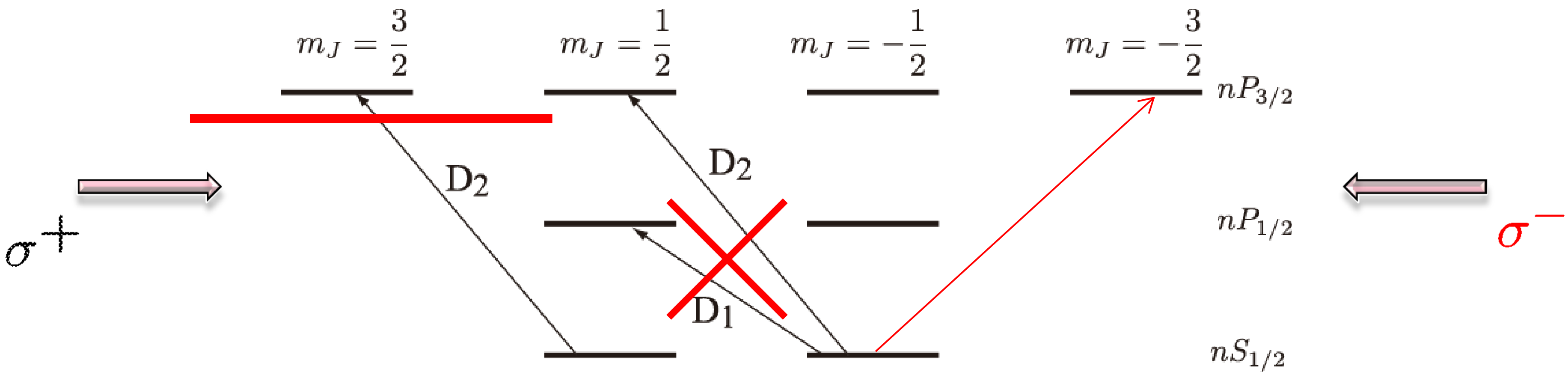


$$H = \frac{1}{2}\epsilon\sigma_z - \frac{\Omega_0\Omega_1}{4\Delta}\sigma_x.$$ Controllable $\epsilon, \Omega_i, \Delta$.

$$\epsilon = E_1 - E_0 + \frac{\Omega_1^2}{4\Delta} - \frac{\Omega_0^2}{4\Delta}$$

$i\sigma_x, i\sigma_z$ are generators of $\mathfrak{su}(2)$.

State Selective Potential



$$\boldsymbol{E}_+ \propto e^{ikx}(\hat{\boldsymbol{z}} \cos\theta + \hat{\boldsymbol{y}} \sin\theta) \; + \; \boldsymbol{E}_- \propto e^{-ikx}(\hat{\boldsymbol{z}} \cos\theta - \hat{\boldsymbol{y}} \sin\theta)$$

$$\boldsymbol{E}_+ + \boldsymbol{E}_- \propto \sigma^+ \cos(kx - \theta) + \sigma^- \cos(kx + \theta).$$

Optical lattice potentials

$$V_{\pm}(x) \propto \cos^2(kx \mp \theta)$$ θ is controllable.

Effective potential for each hyperfine state

$$|0\rangle = |F=1, m_F=1\rangle = \frac{\sqrt{3}}{2}\left|\frac{3}{2},\frac{3}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle - \frac{1}{2}\left|\frac{3}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle,$$

$$|1\rangle = |F=2, m_F=1\rangle = \frac{\sqrt{3}}{2}|\frac{3}{2},\frac{1}{2}\rangle|\frac{1}{2},\frac{1}{2}\rangle + \frac{1}{2}|\frac{3}{2},\frac{3}{2}\rangle|\frac{1}{2},-\frac{1}{2}\rangle.$$

$$\to V_{|0\rangle}(x) = \frac{1}{4}V_+(x) + \frac{3}{4}V_-(x), V_{|1\rangle} = \frac{3}{4}V_+(x) + \frac{1}{4}V_-(x).$$

Electro-Optic Modulator