Towards linear phononics, quantum information processing and nonlocality tests in ion traps

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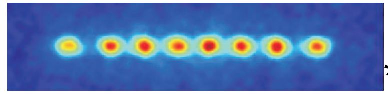
‘Massive’ CV degrees of freedom

Bec.eps NaNo1.eps OpticalLattice.eps  *

Controllable continuous variable degrees of freedom are ubiquitous

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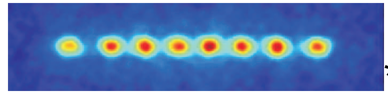
Bec.eps NaNo1.eps OpticalLattice.eps  *

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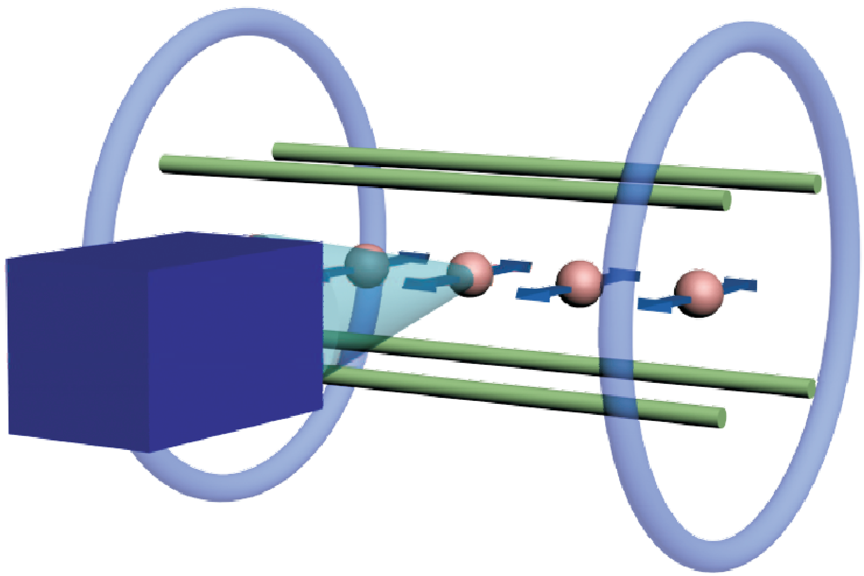
Our pick are the “transverse” (or “radial”) degrees of freedom of trapped ions (Zhu, Monroe, Duan, PRL ’06; C. F. Roos *et al.*, arXiv:0705.0788)



*** courtesy of R. Blatt**

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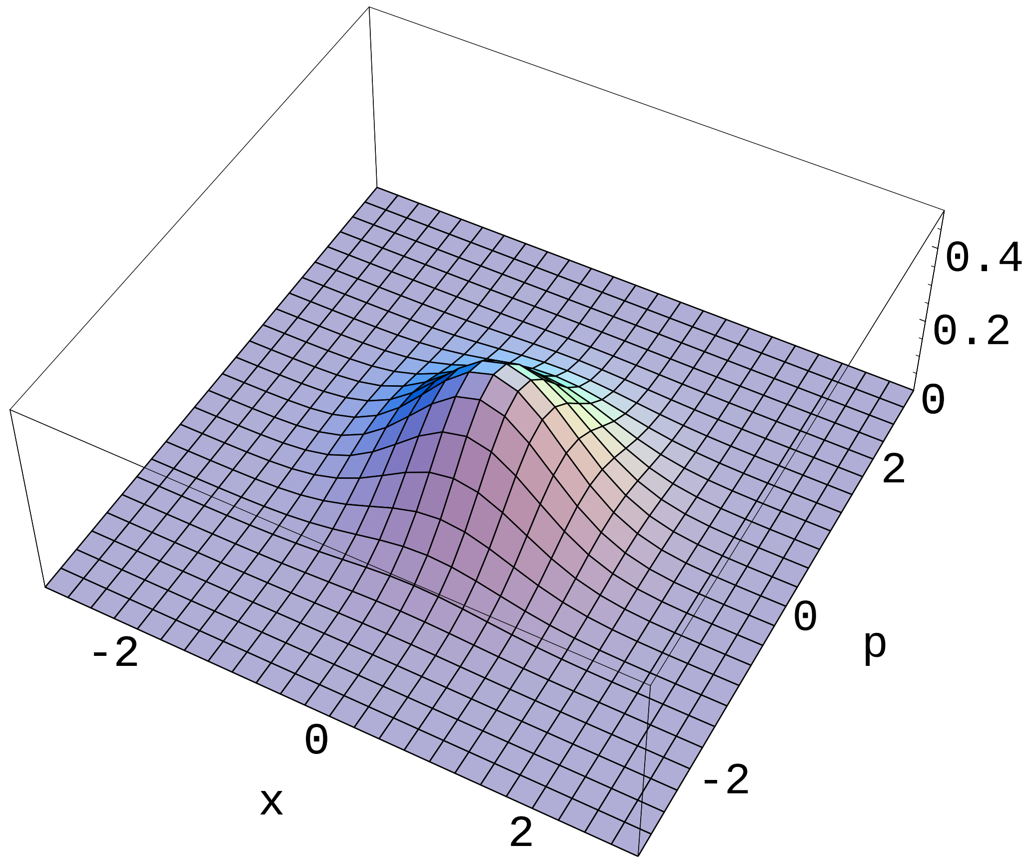

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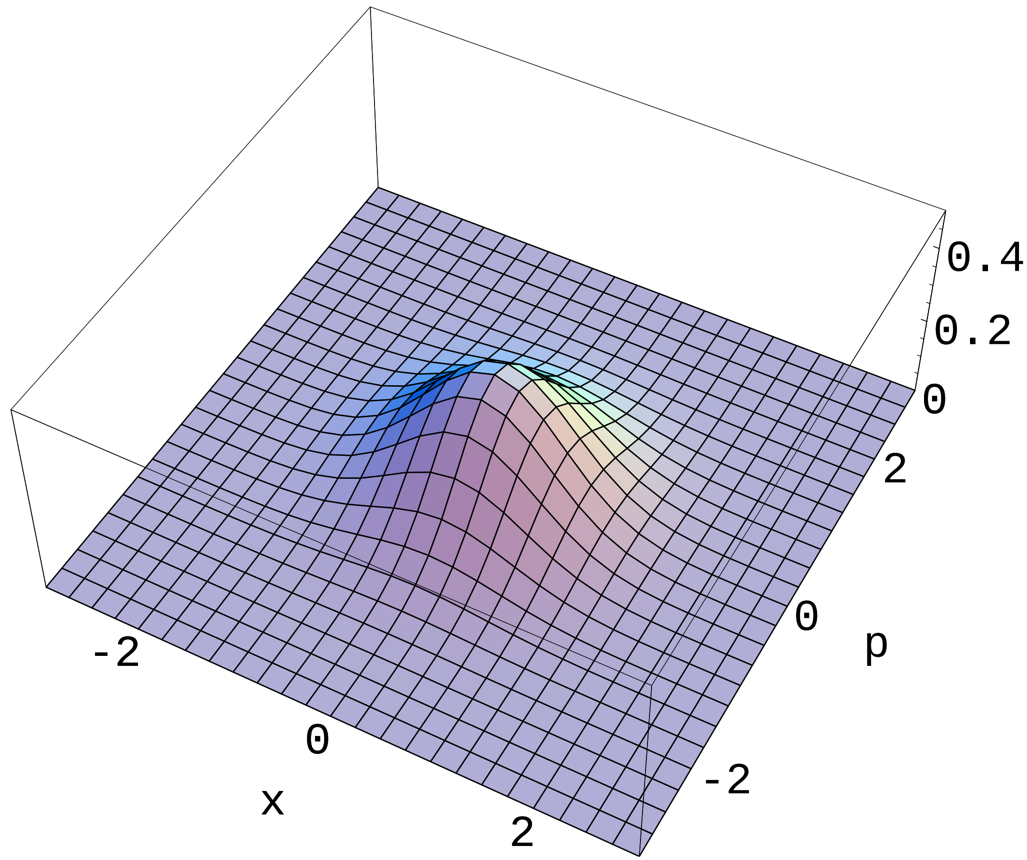

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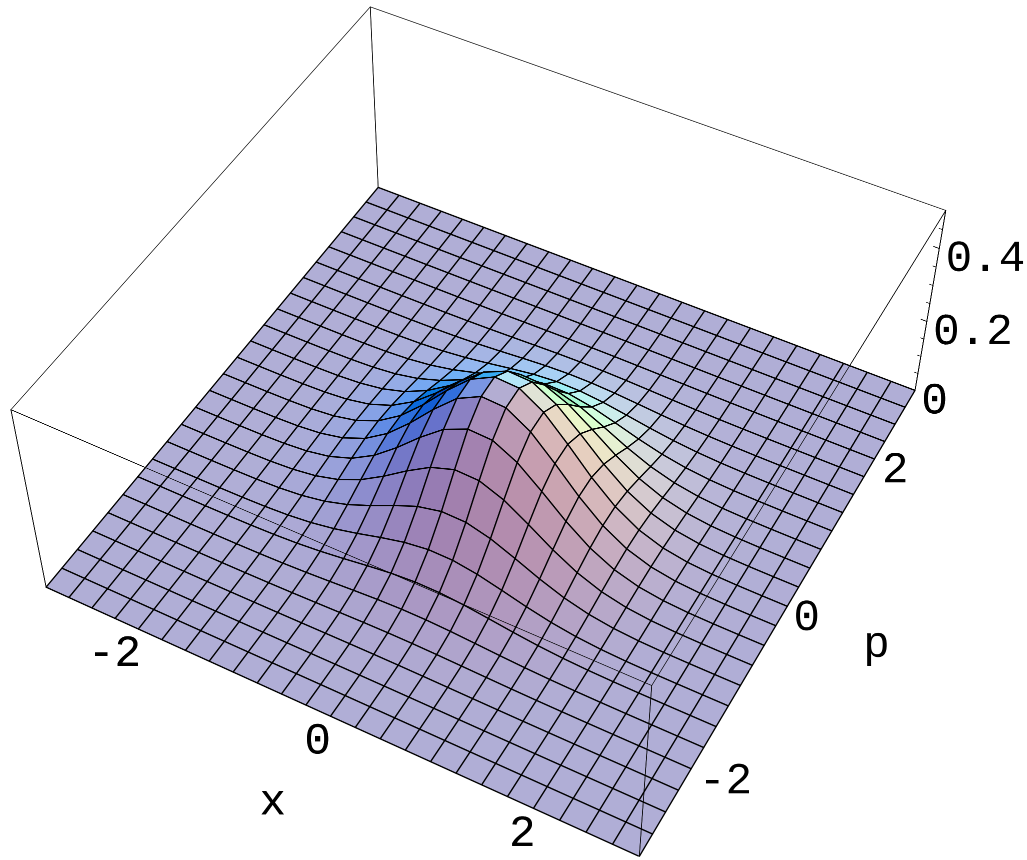

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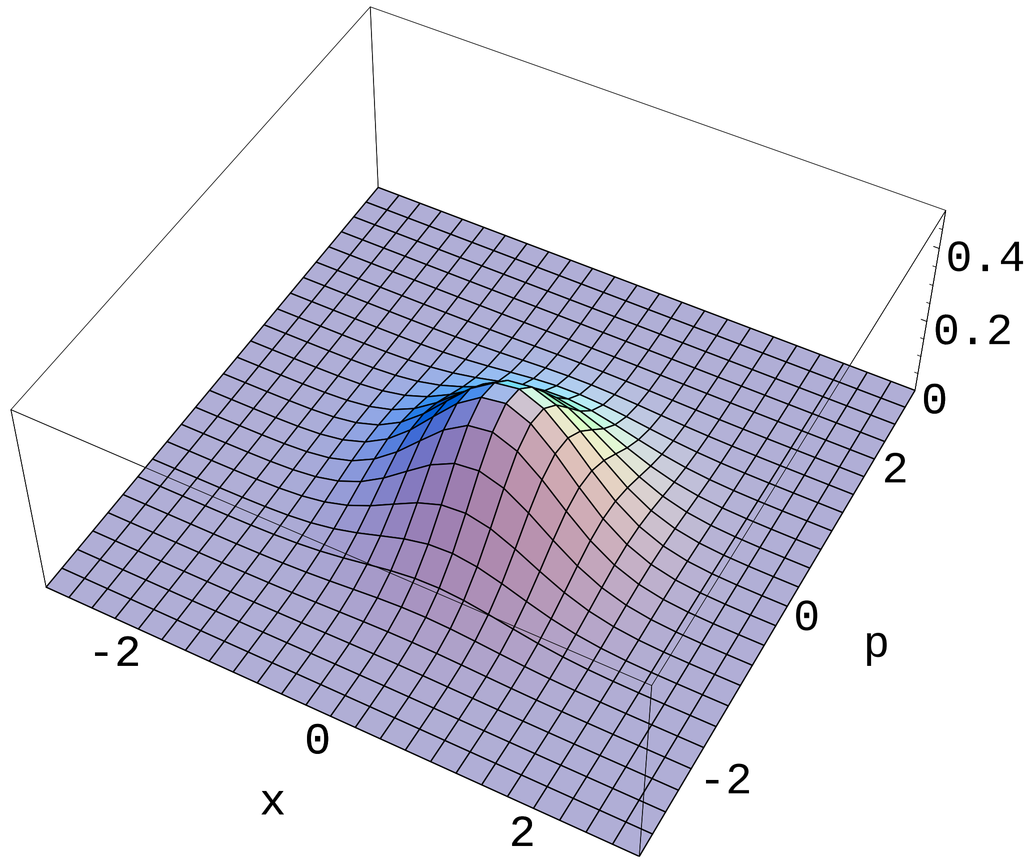

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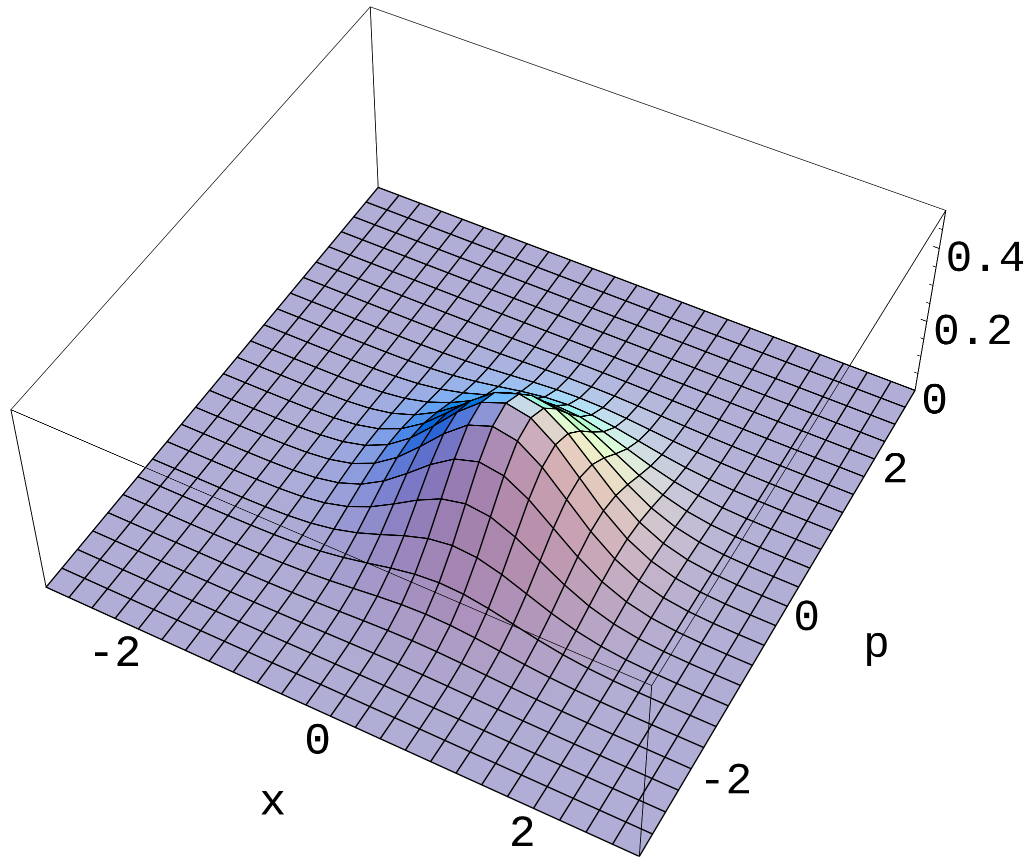

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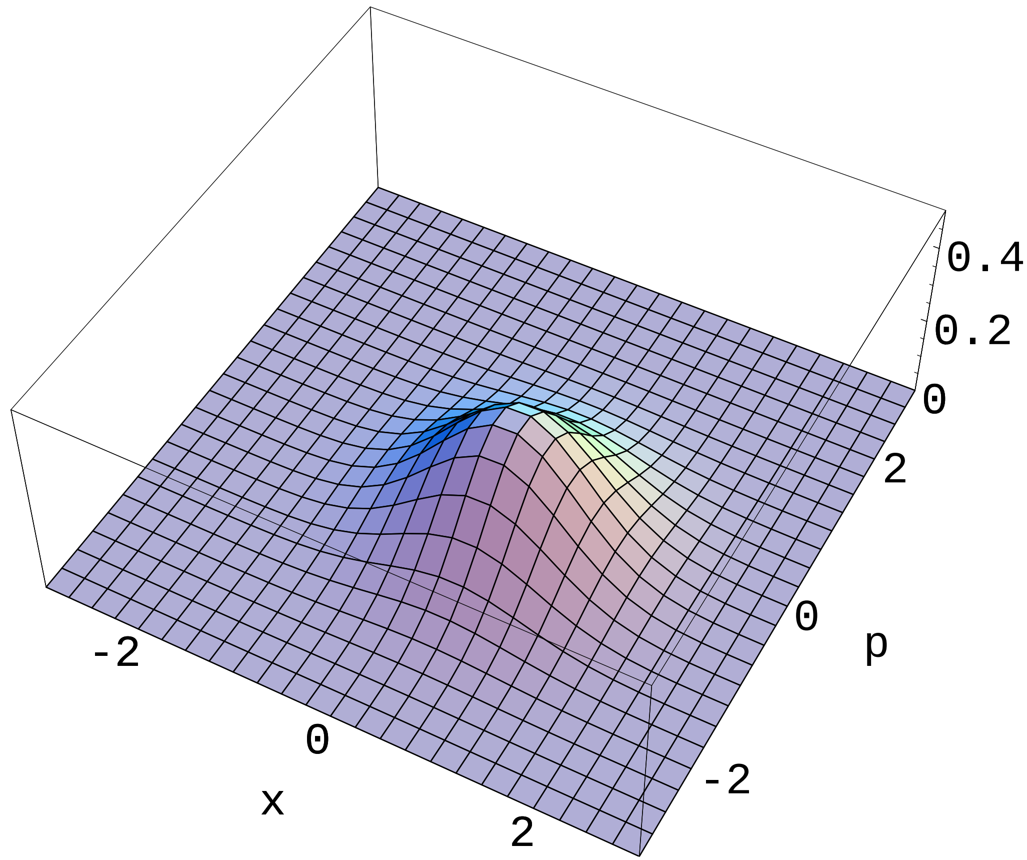

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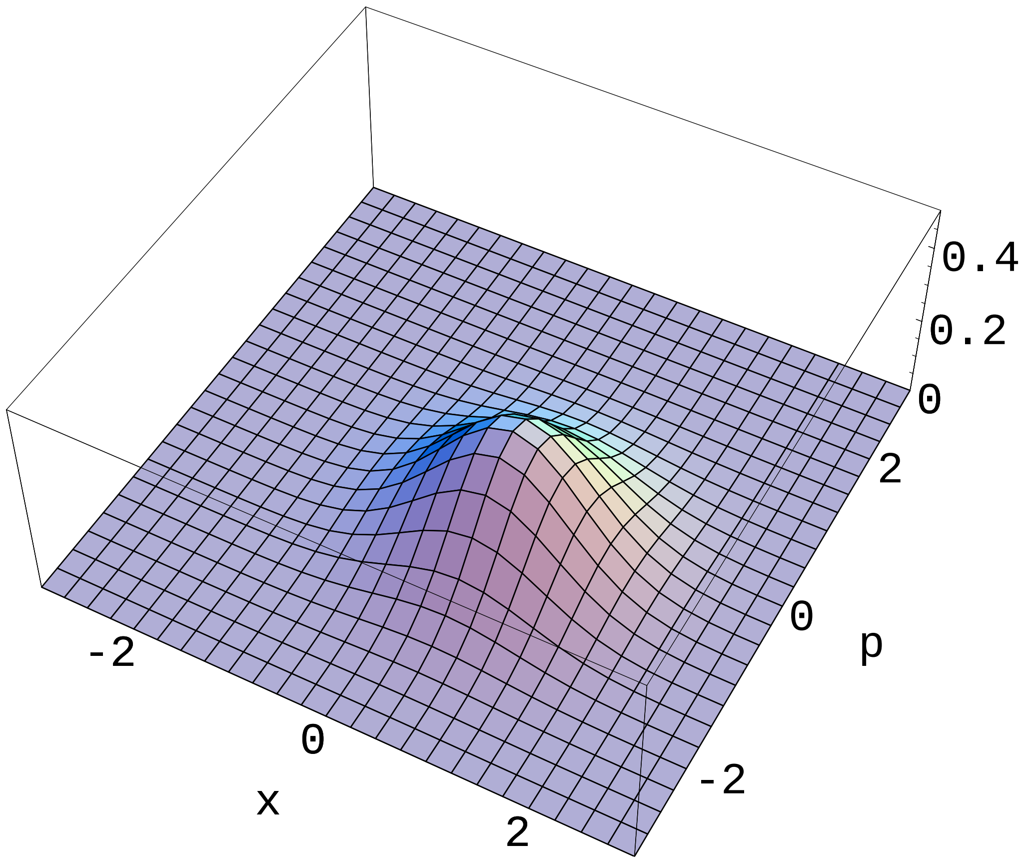

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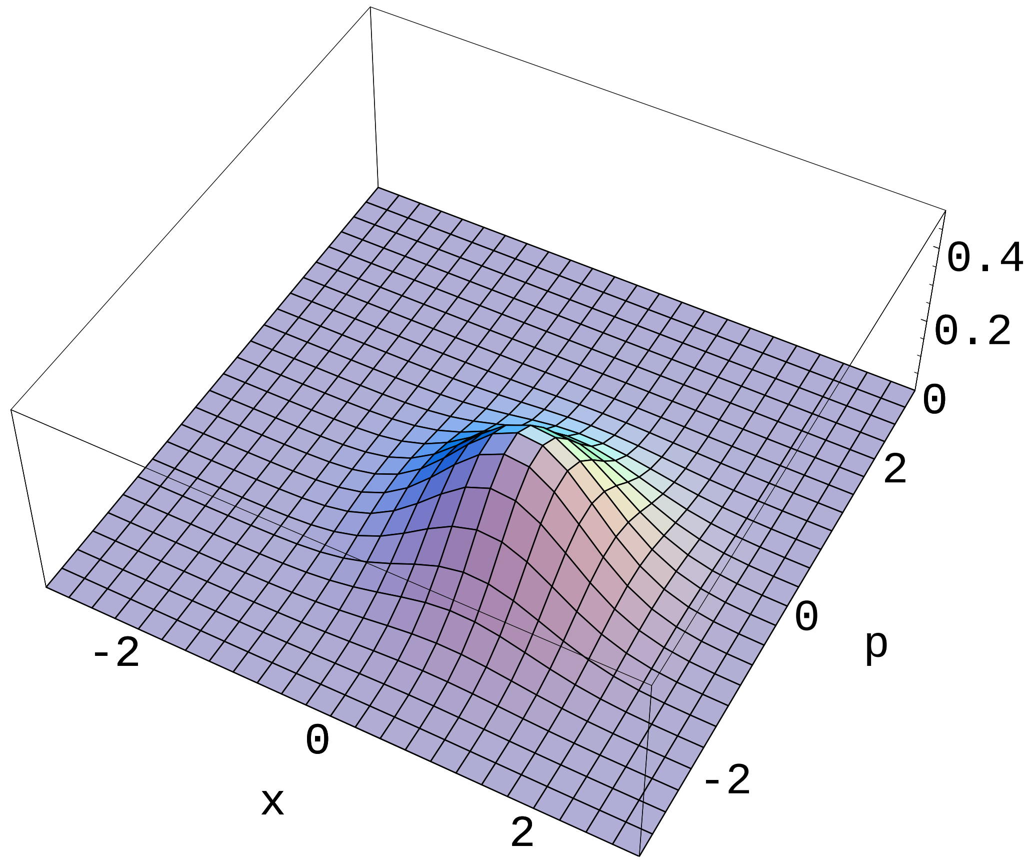

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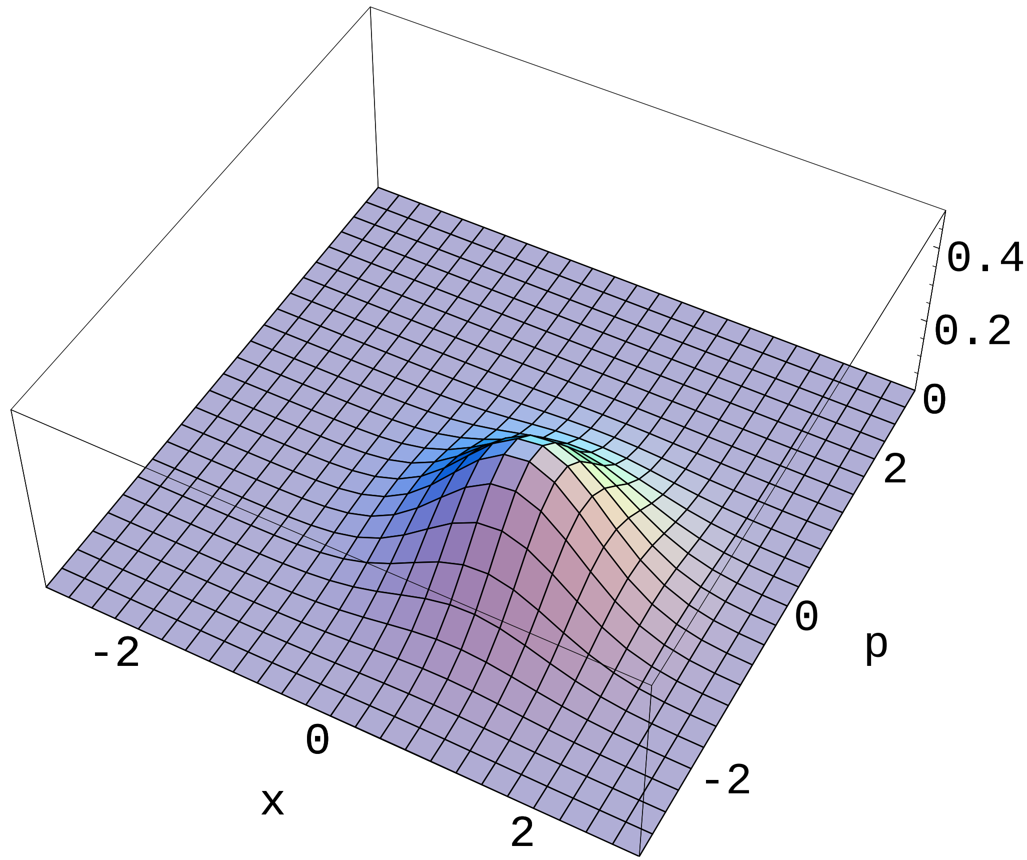

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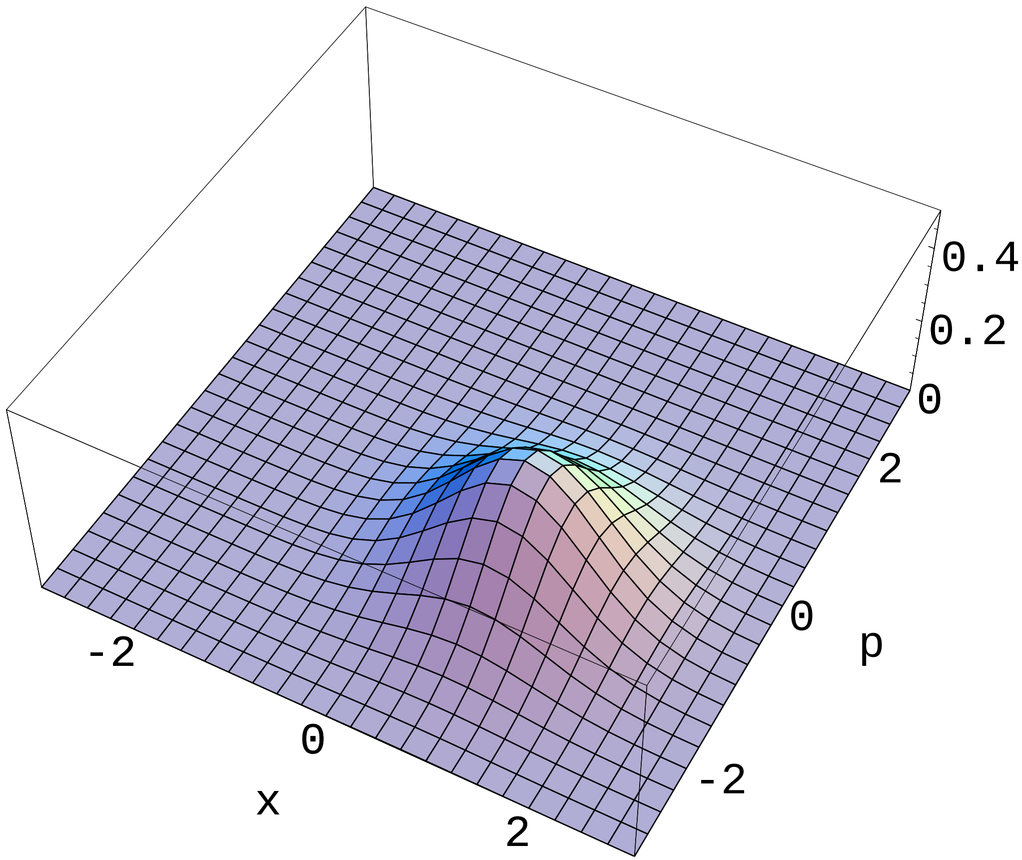

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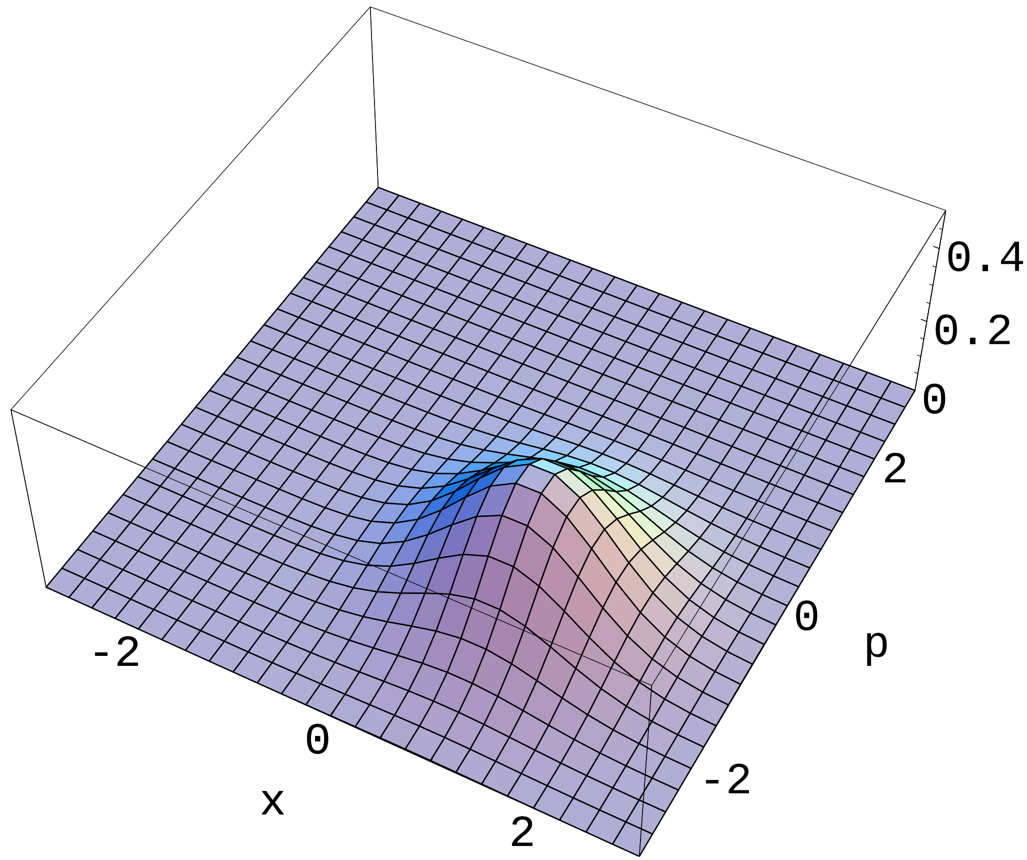

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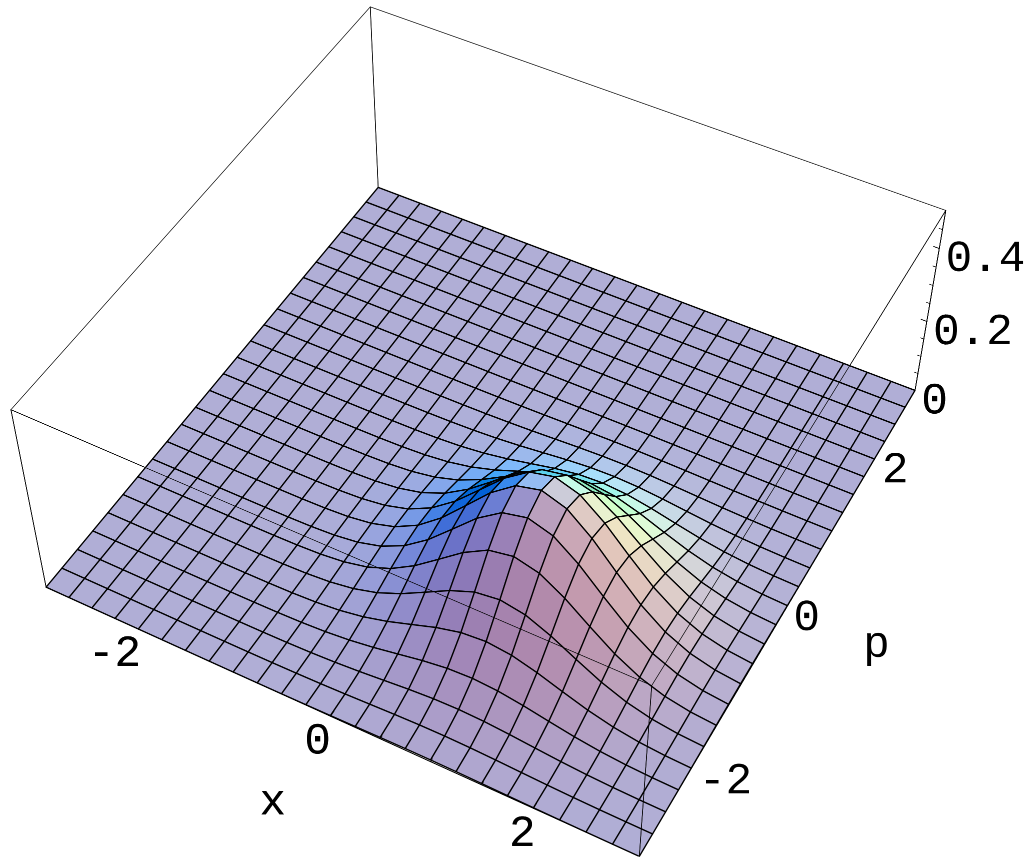

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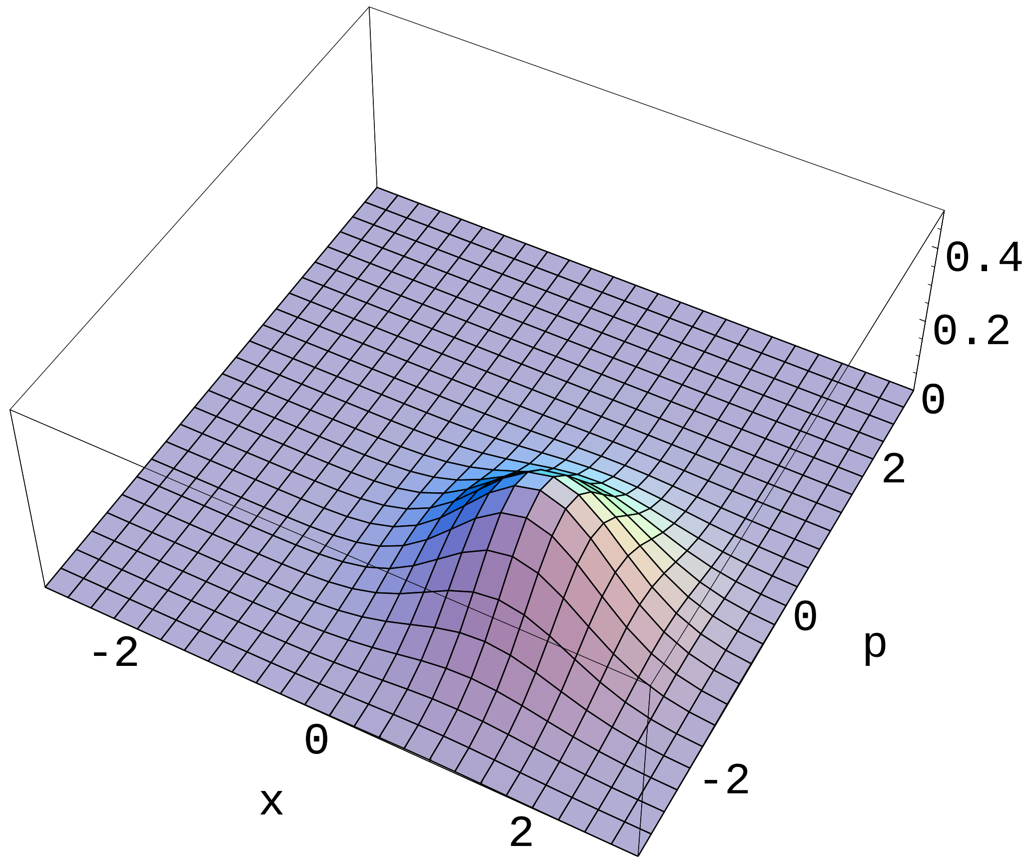

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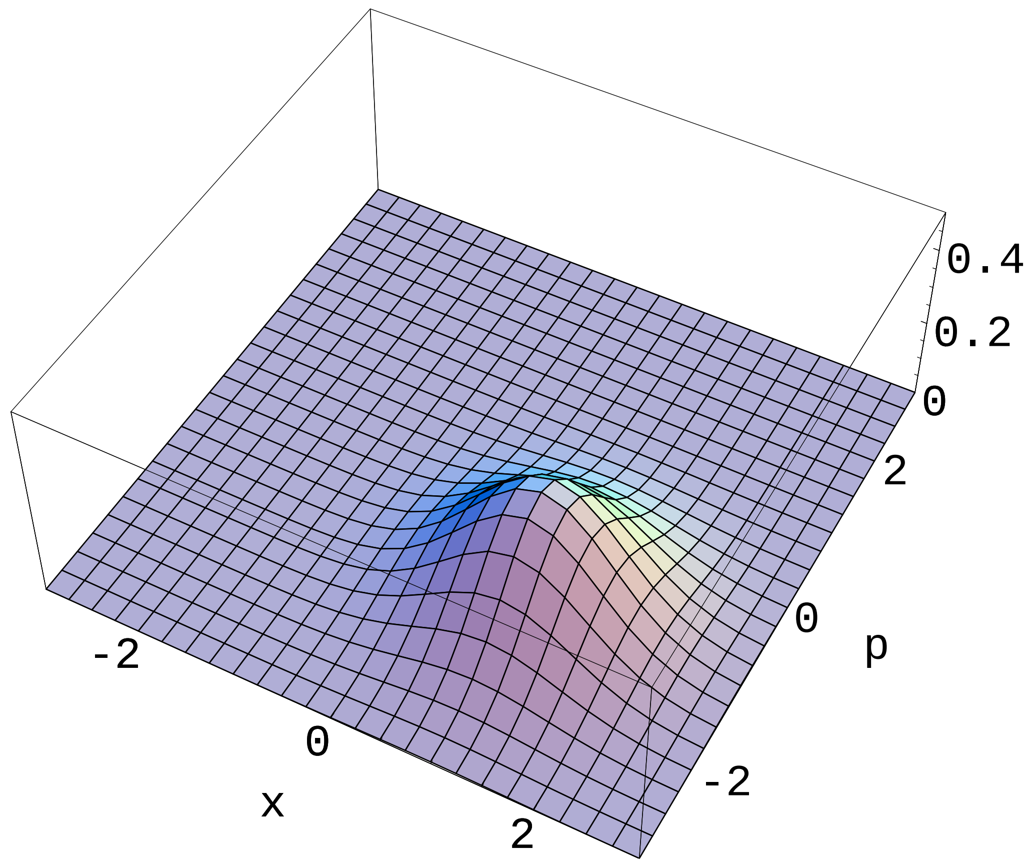

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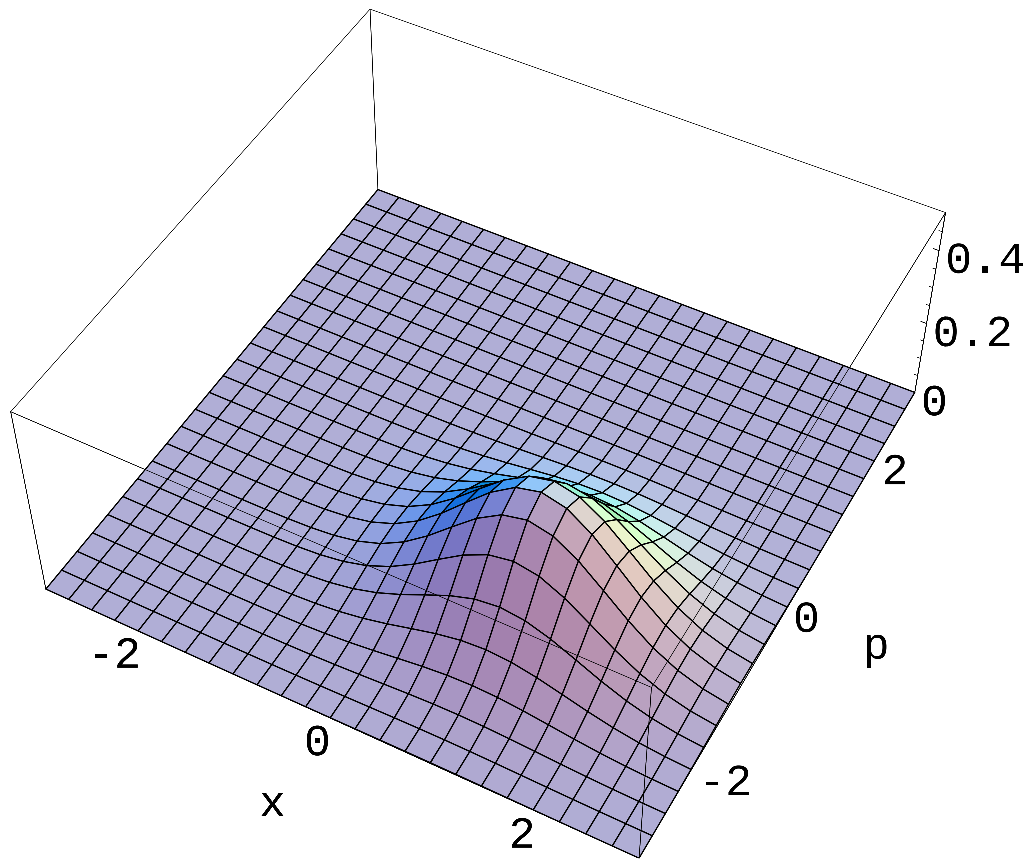

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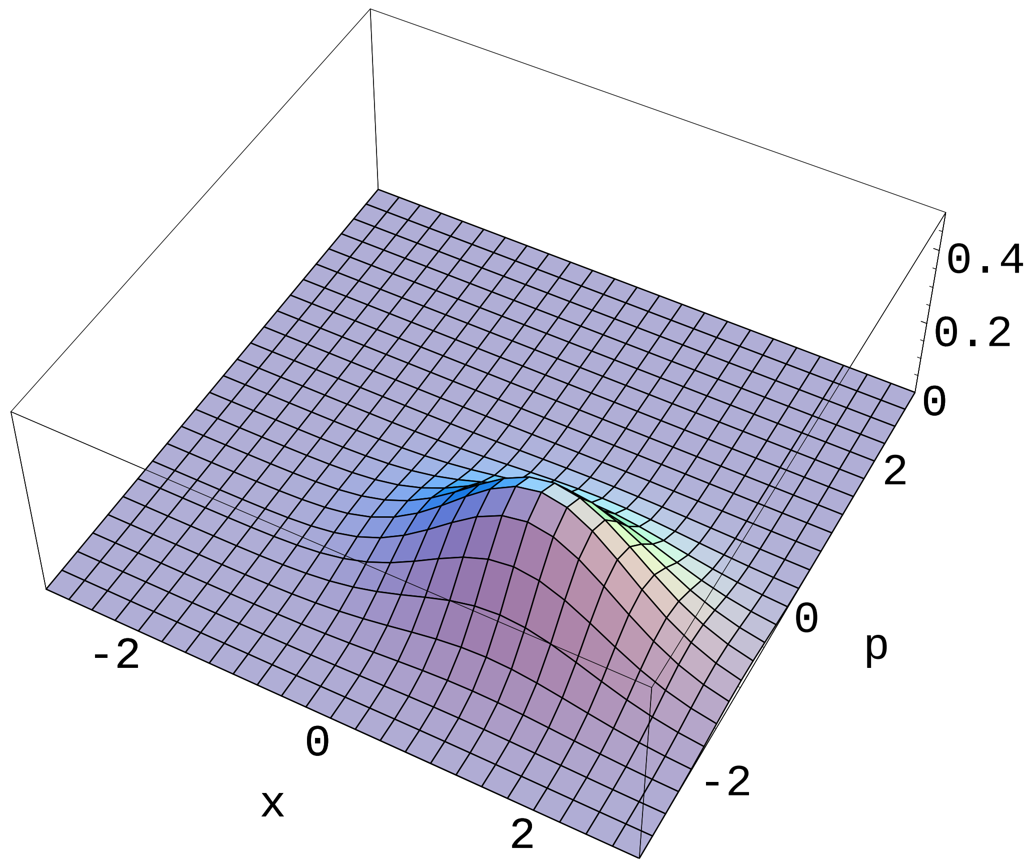

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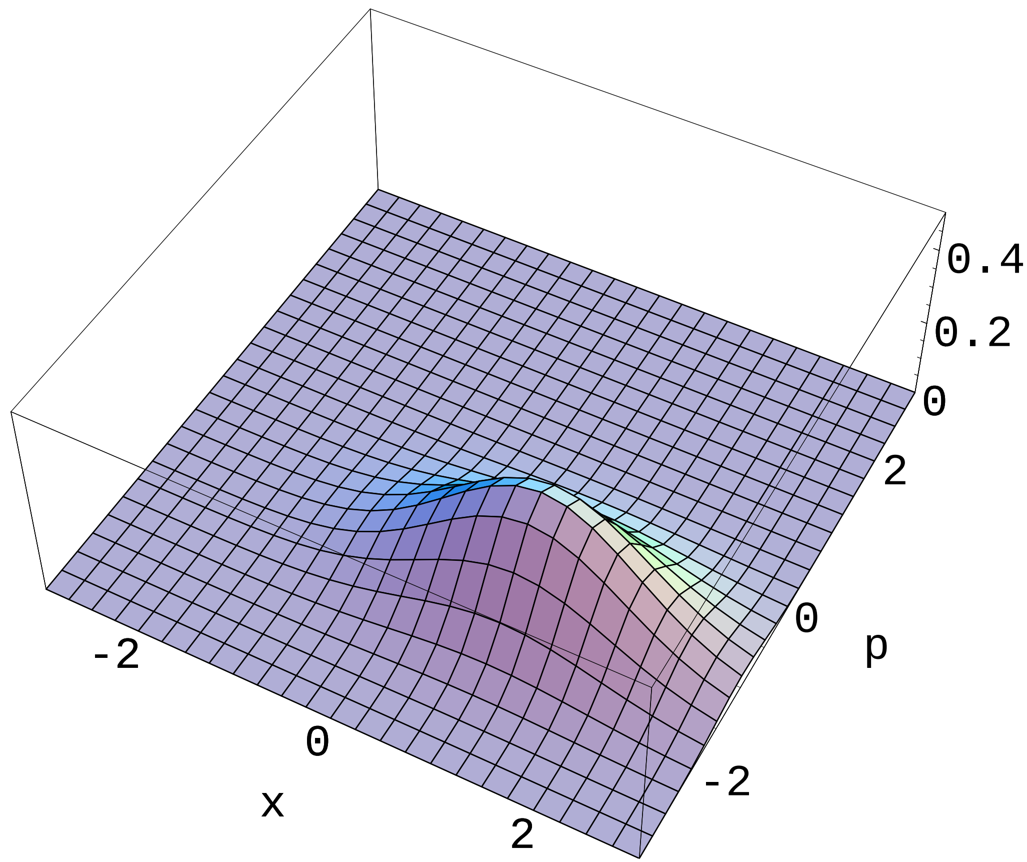

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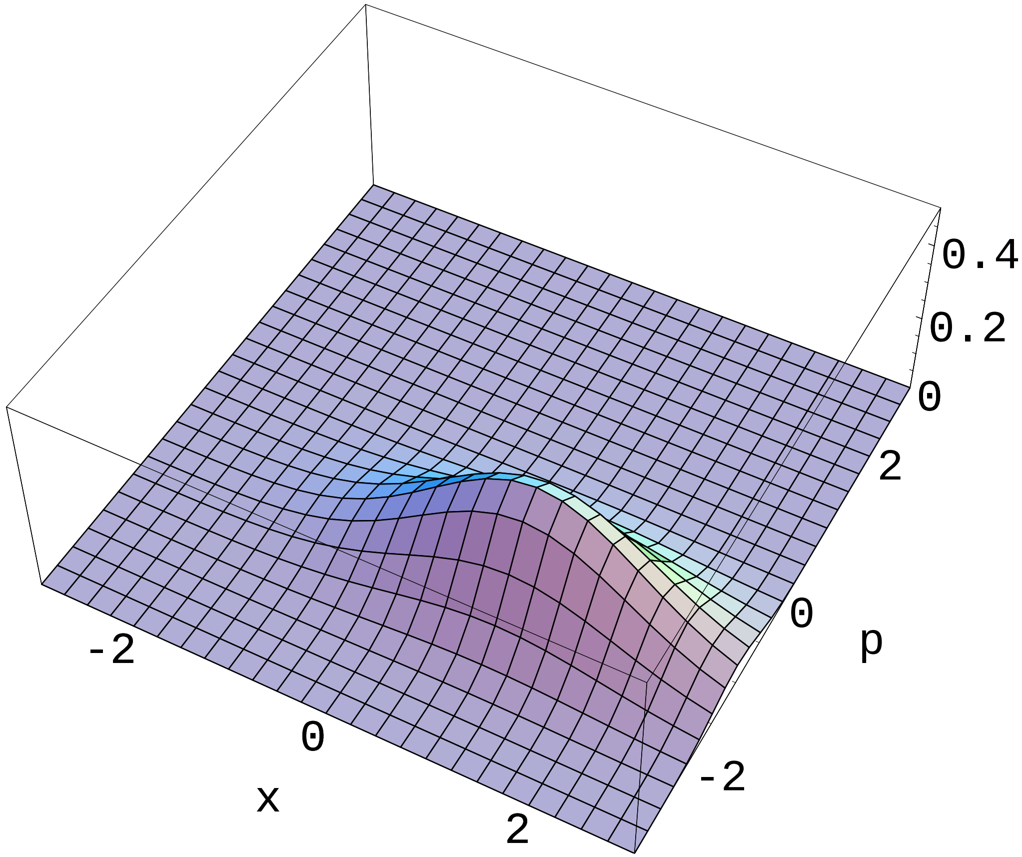

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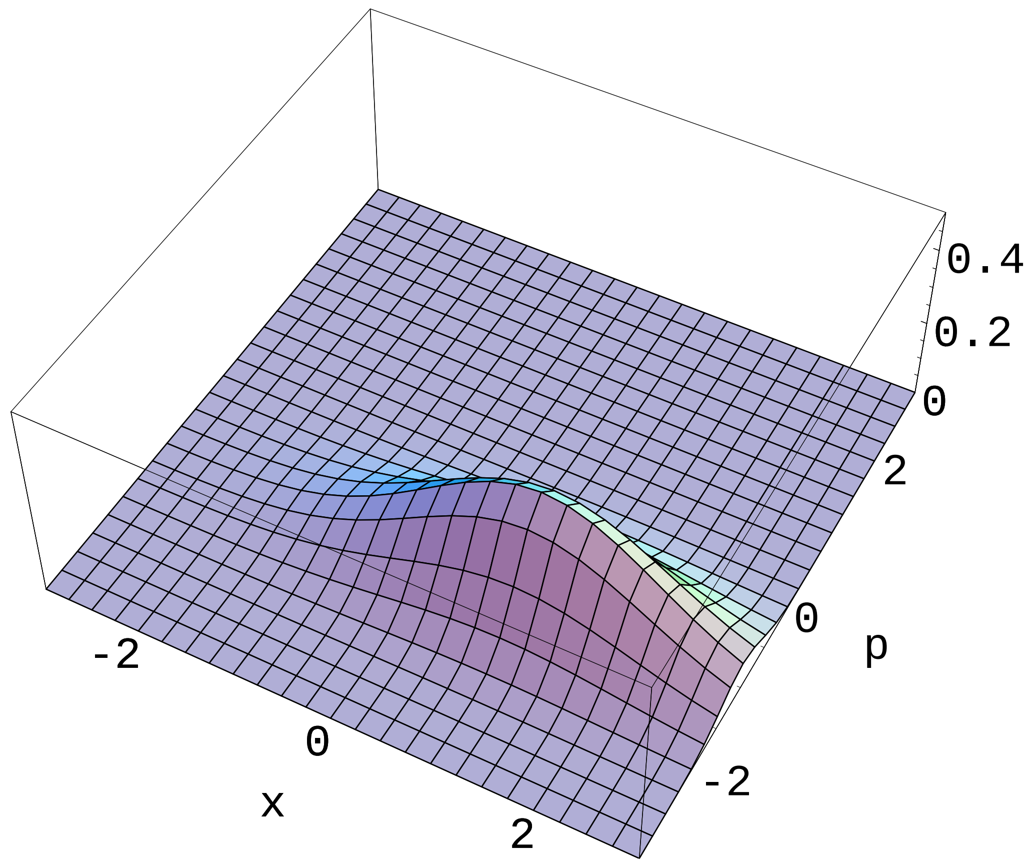

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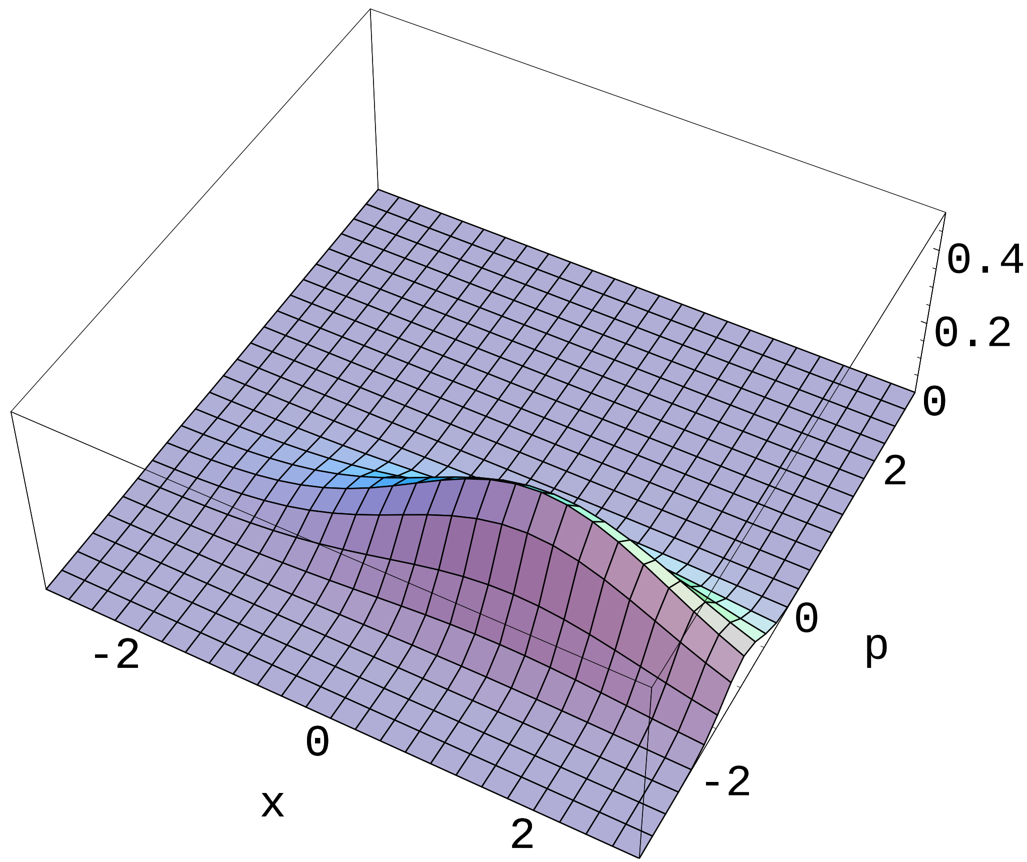

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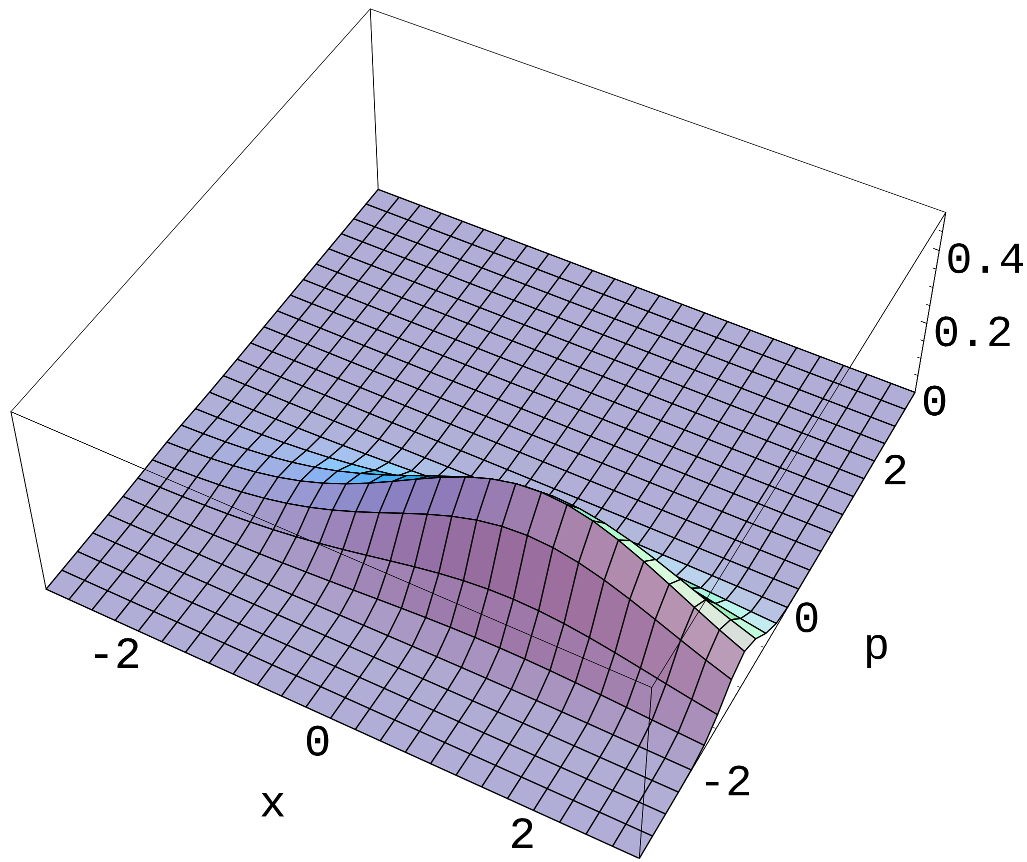

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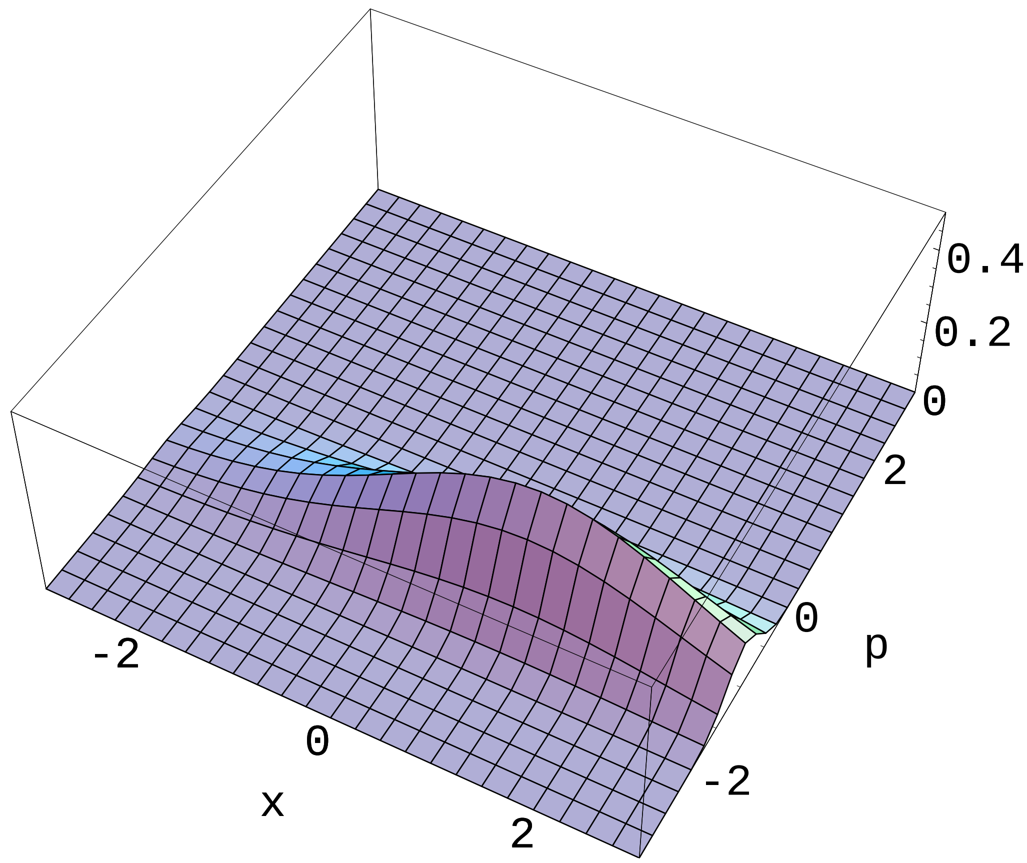

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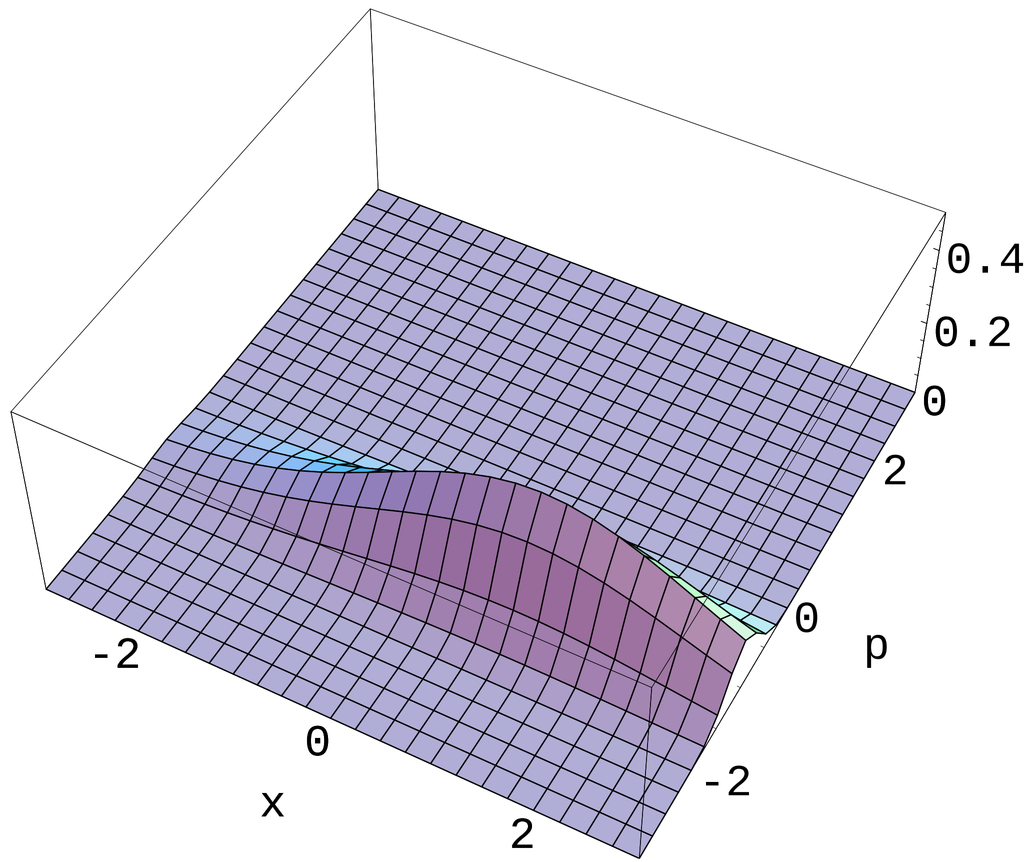

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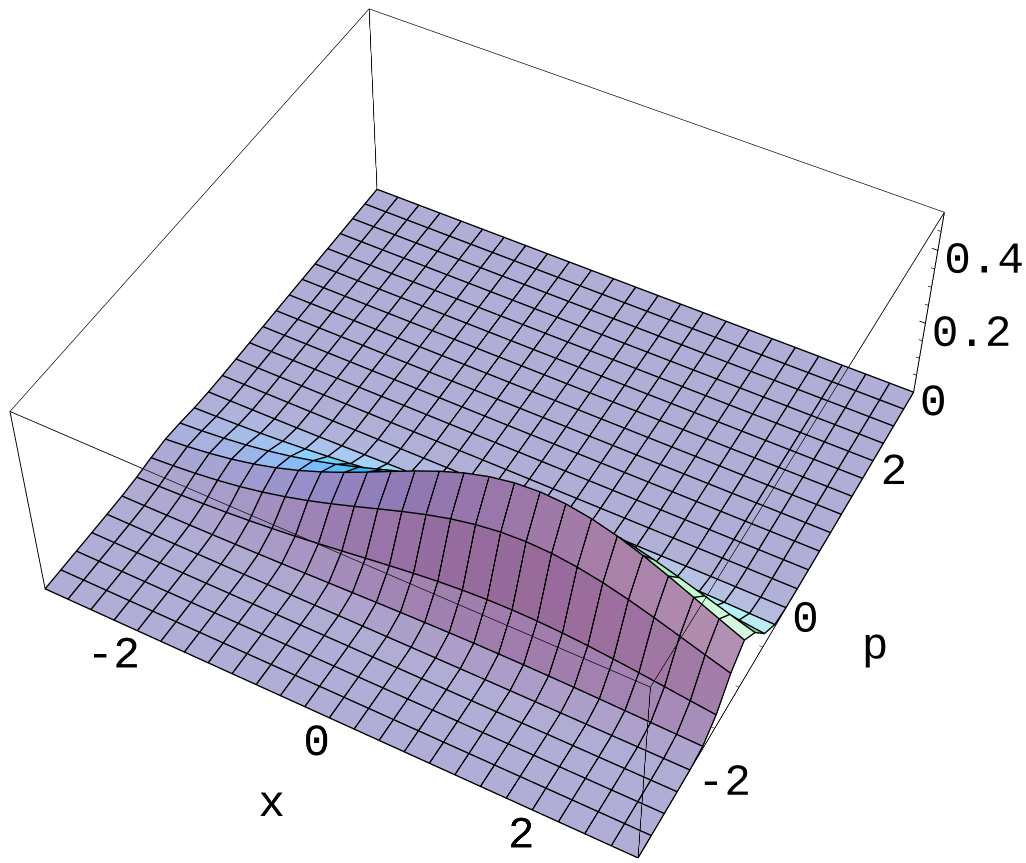

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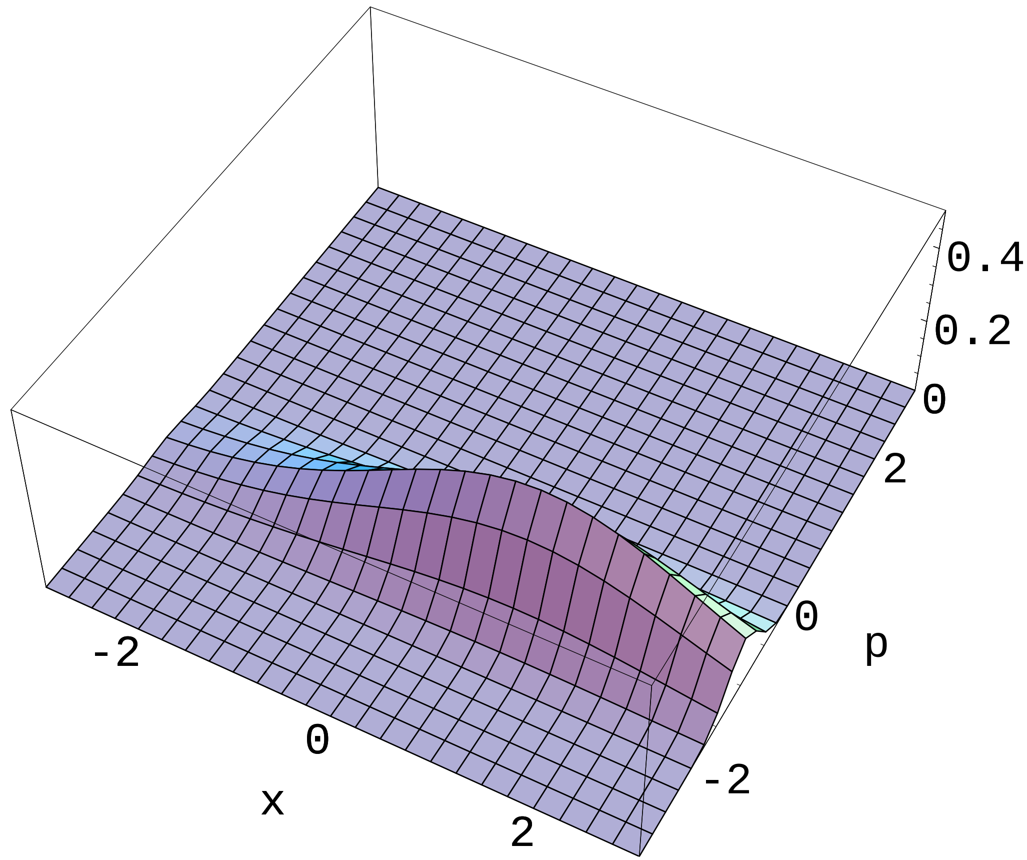

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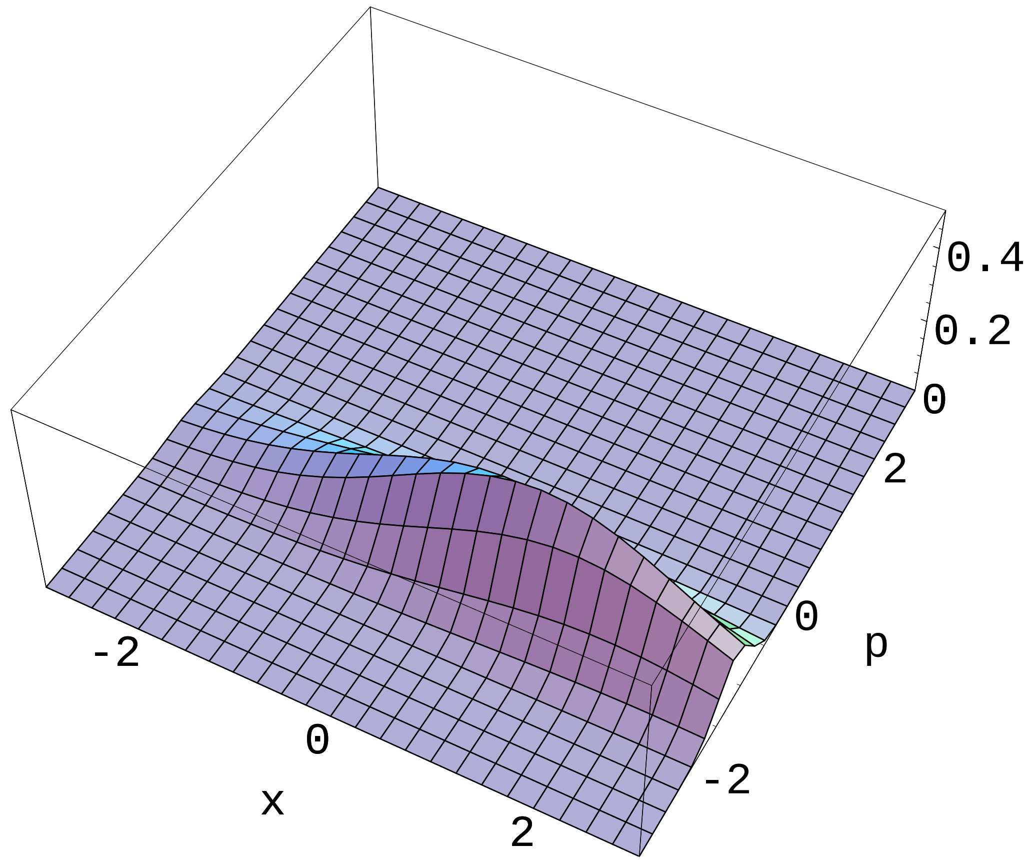

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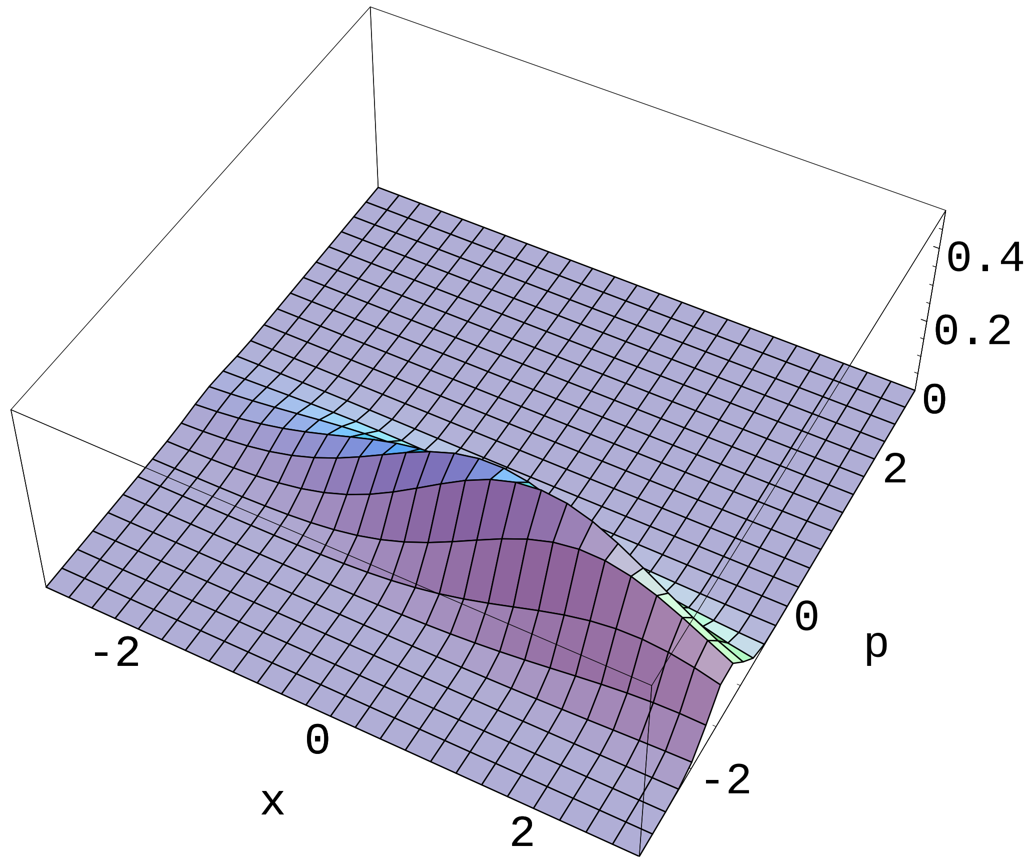

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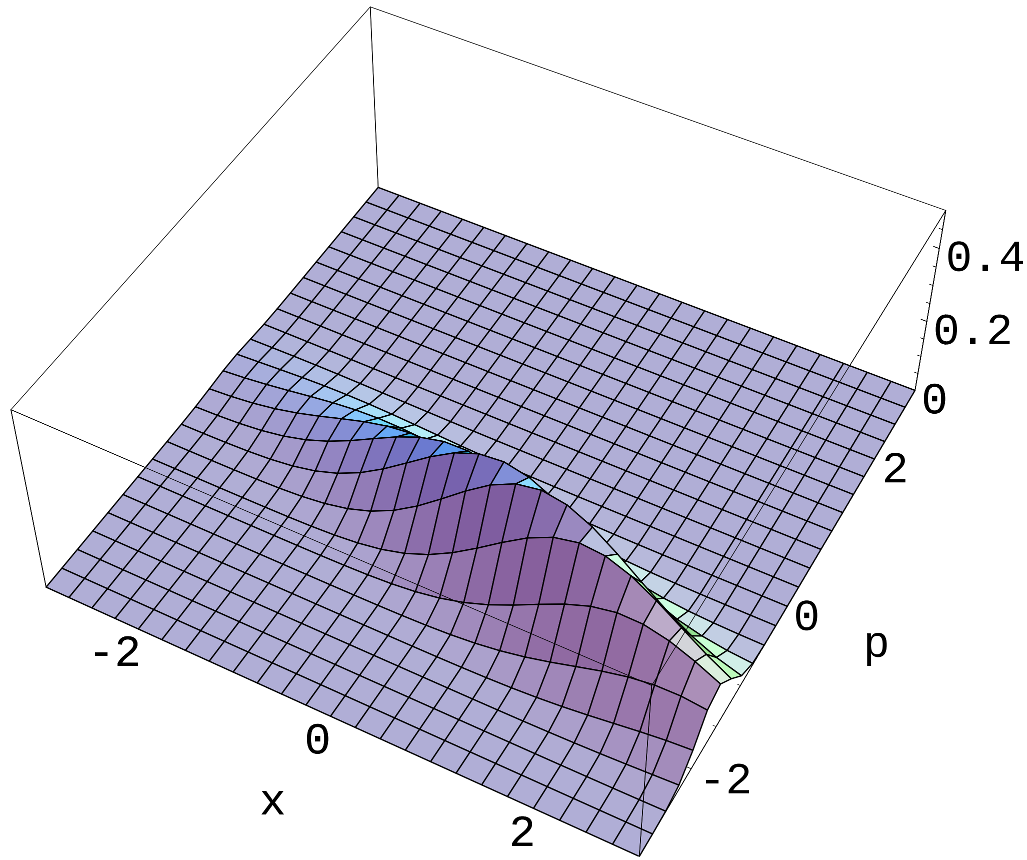

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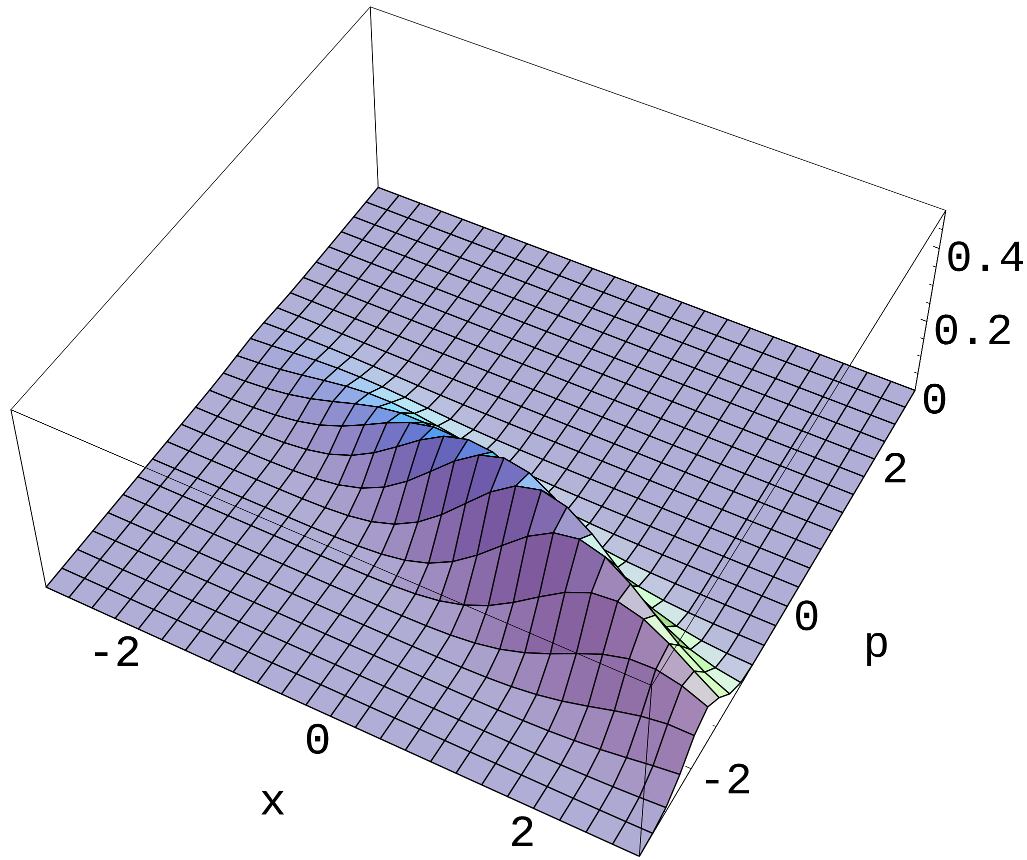

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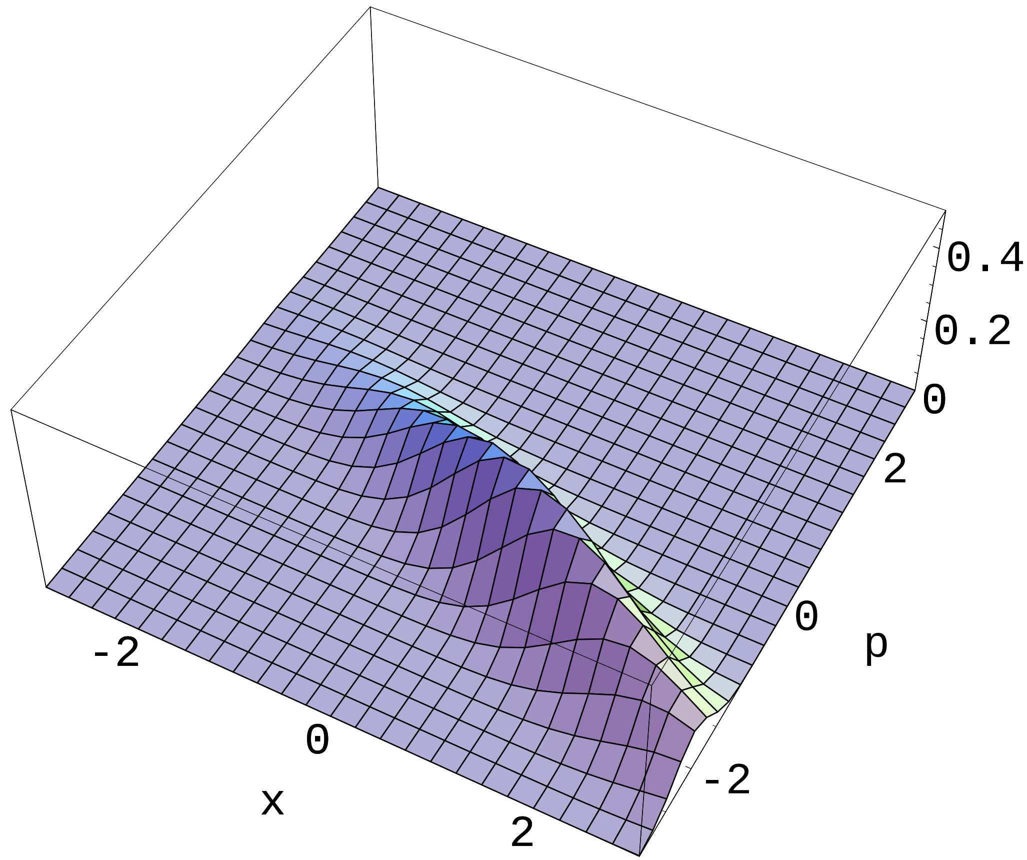

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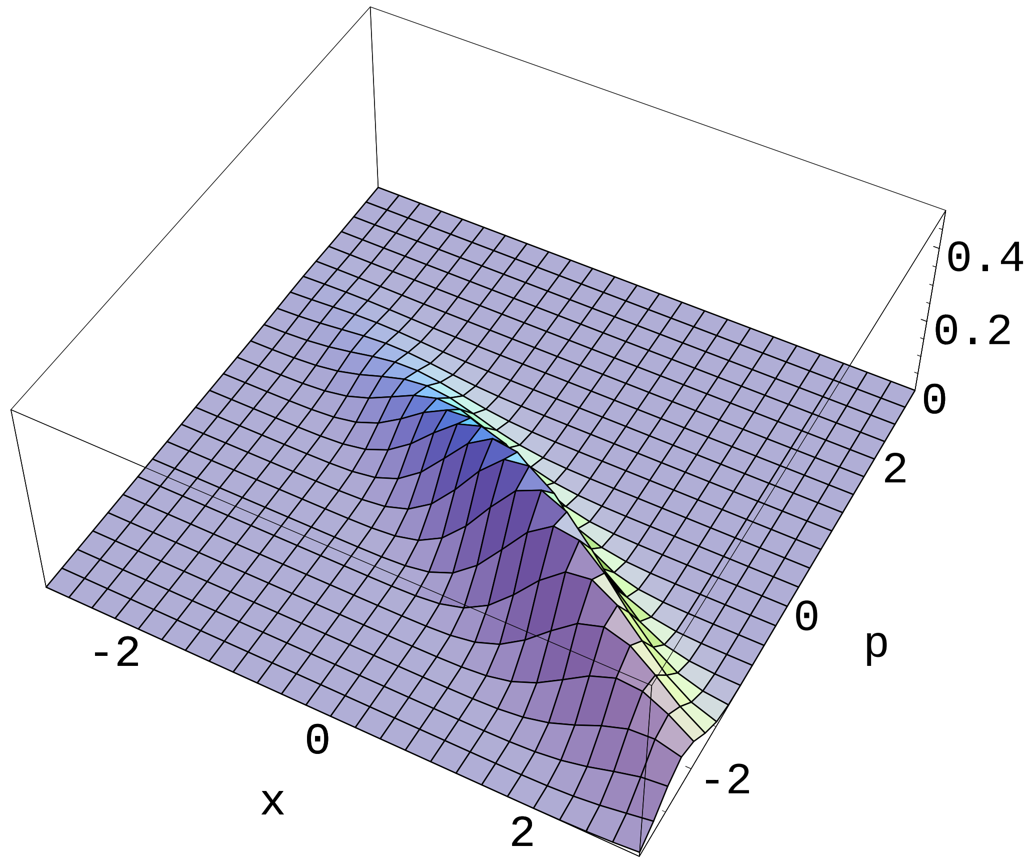

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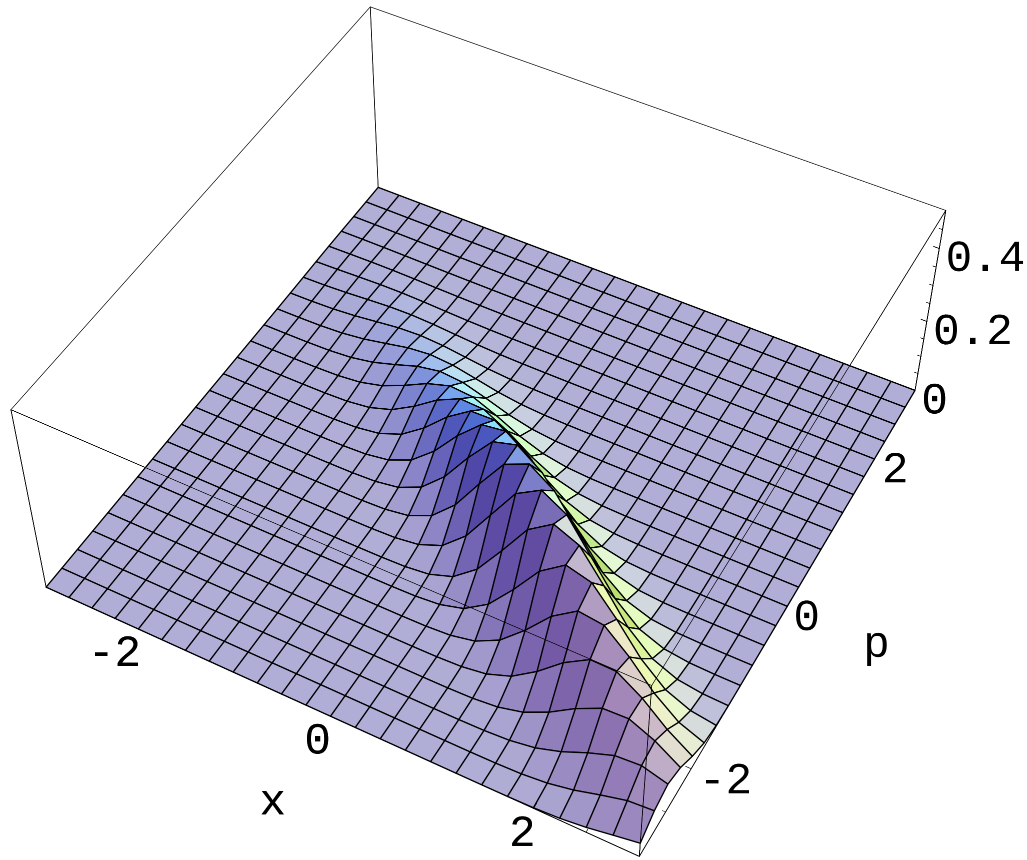

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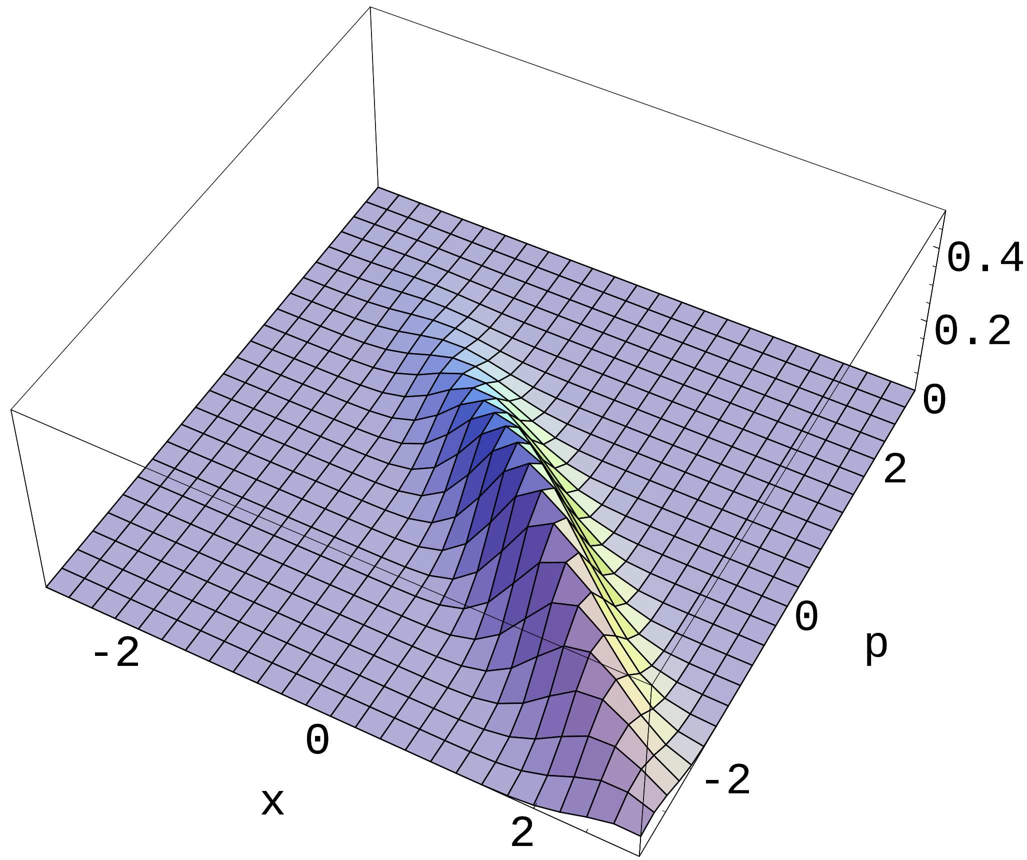

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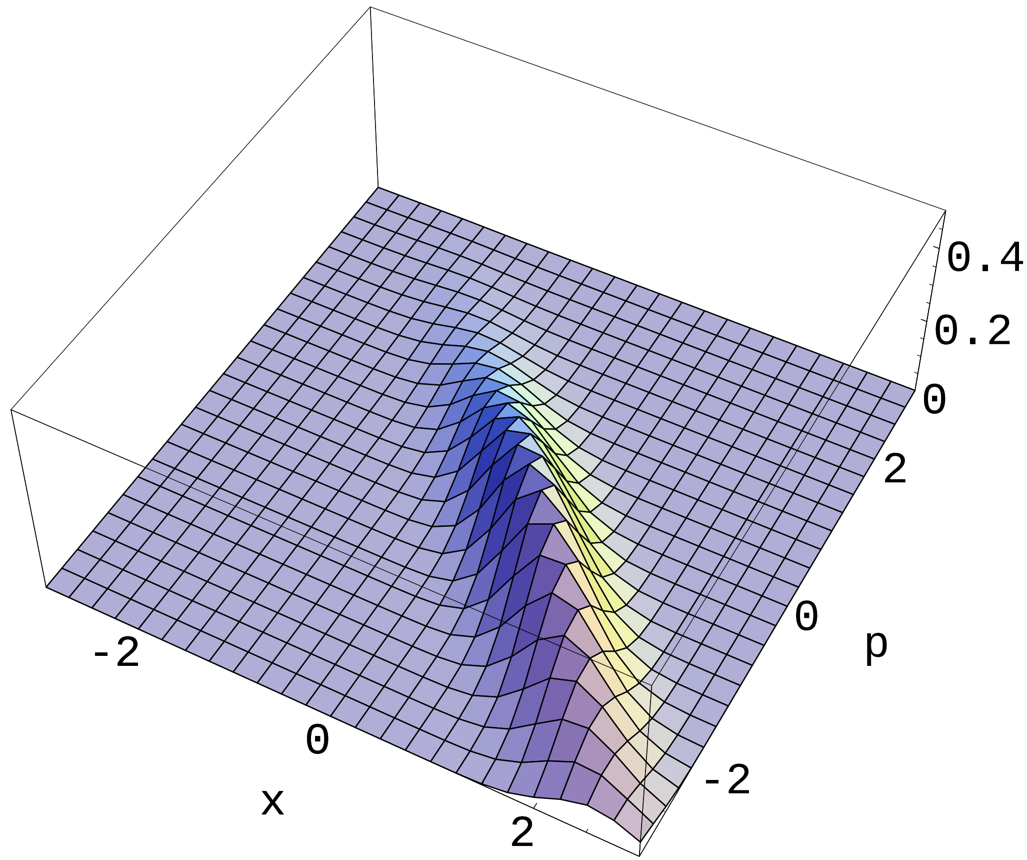

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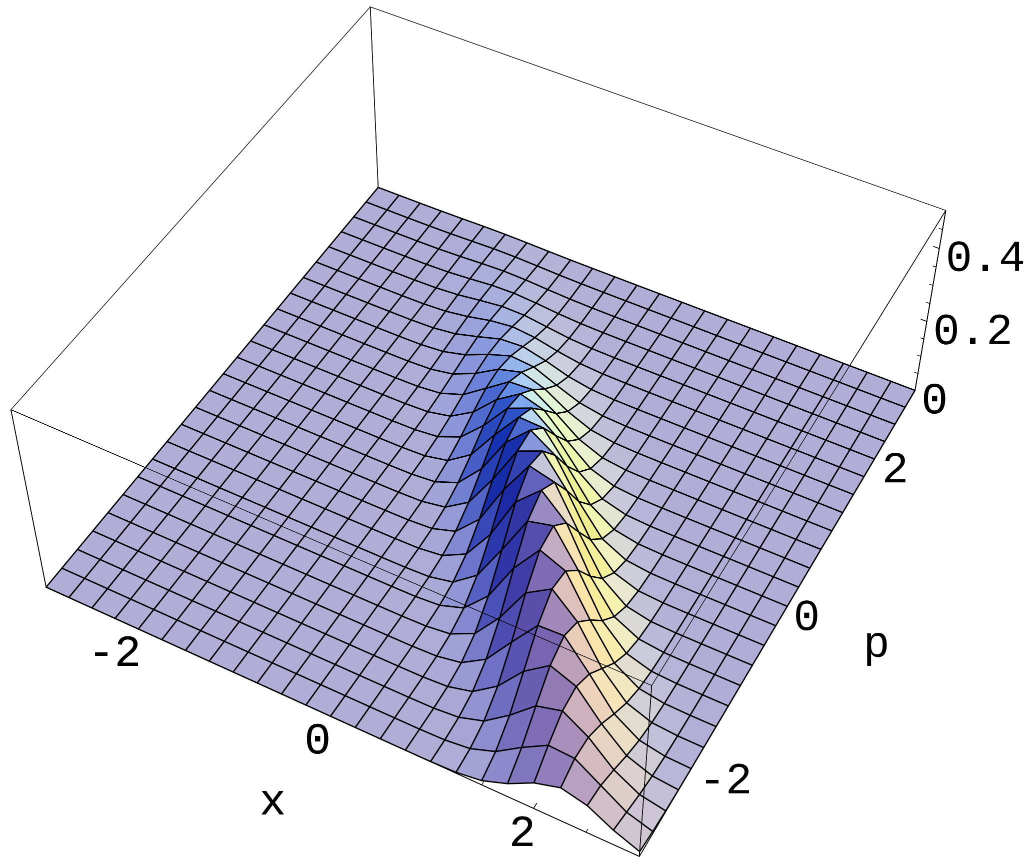

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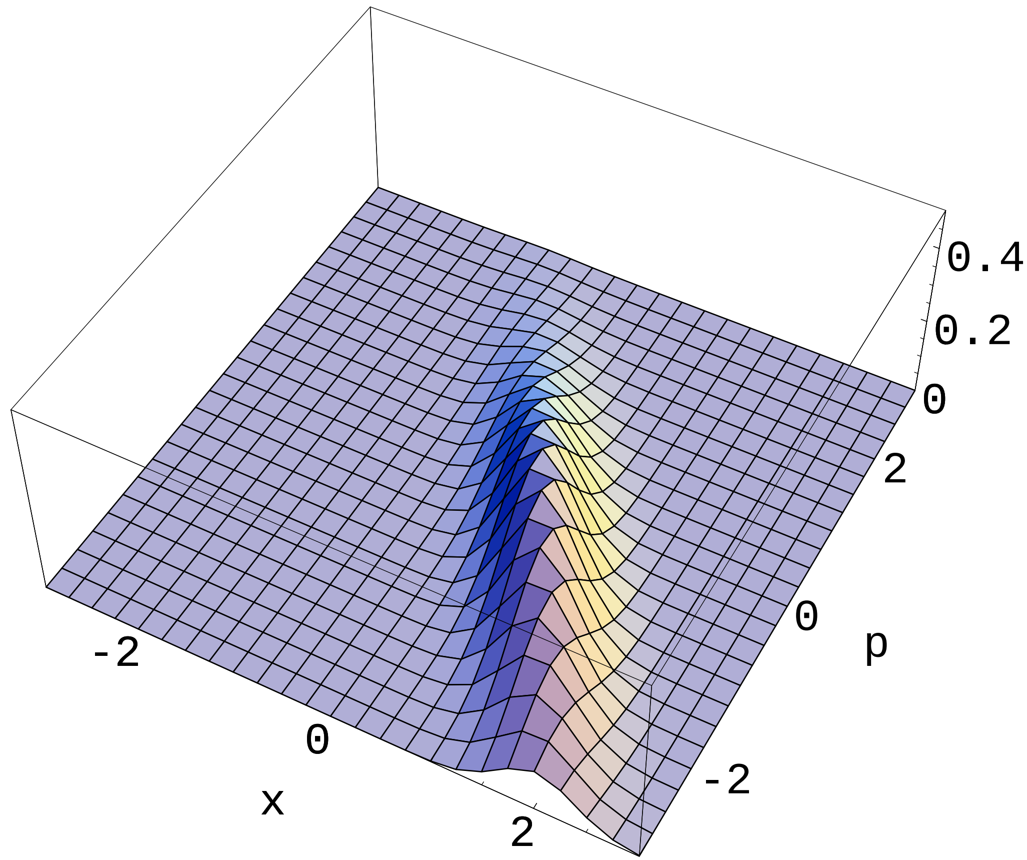

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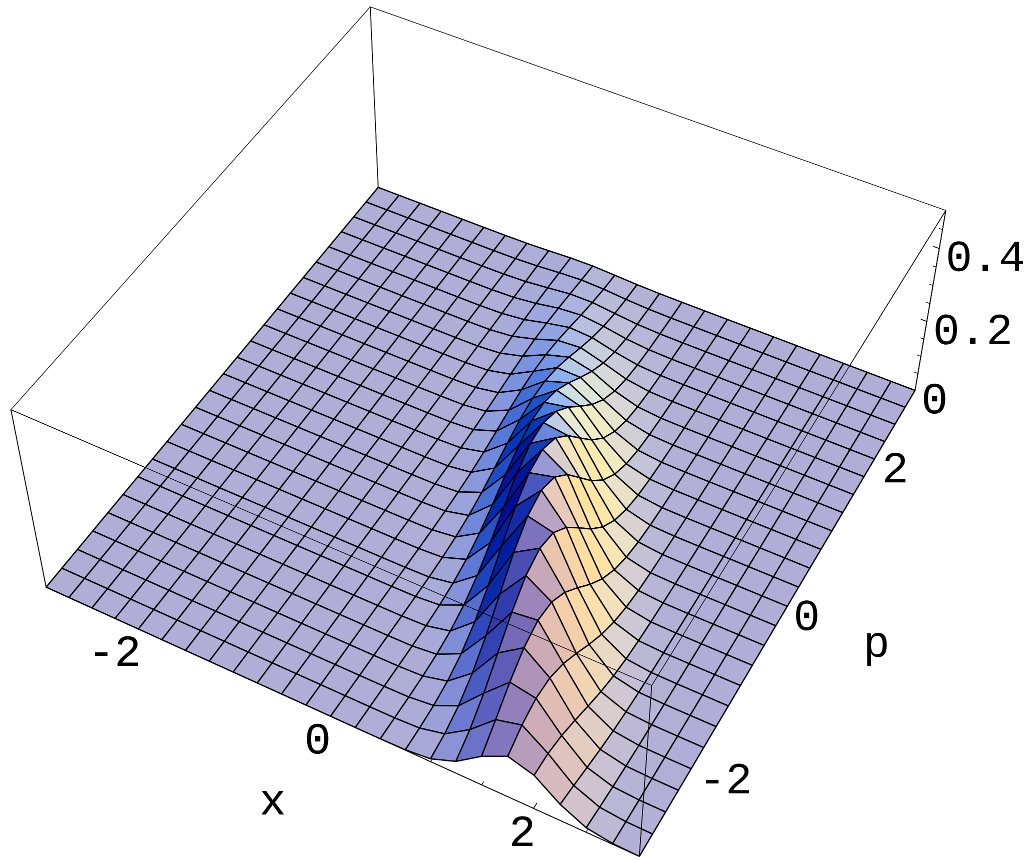

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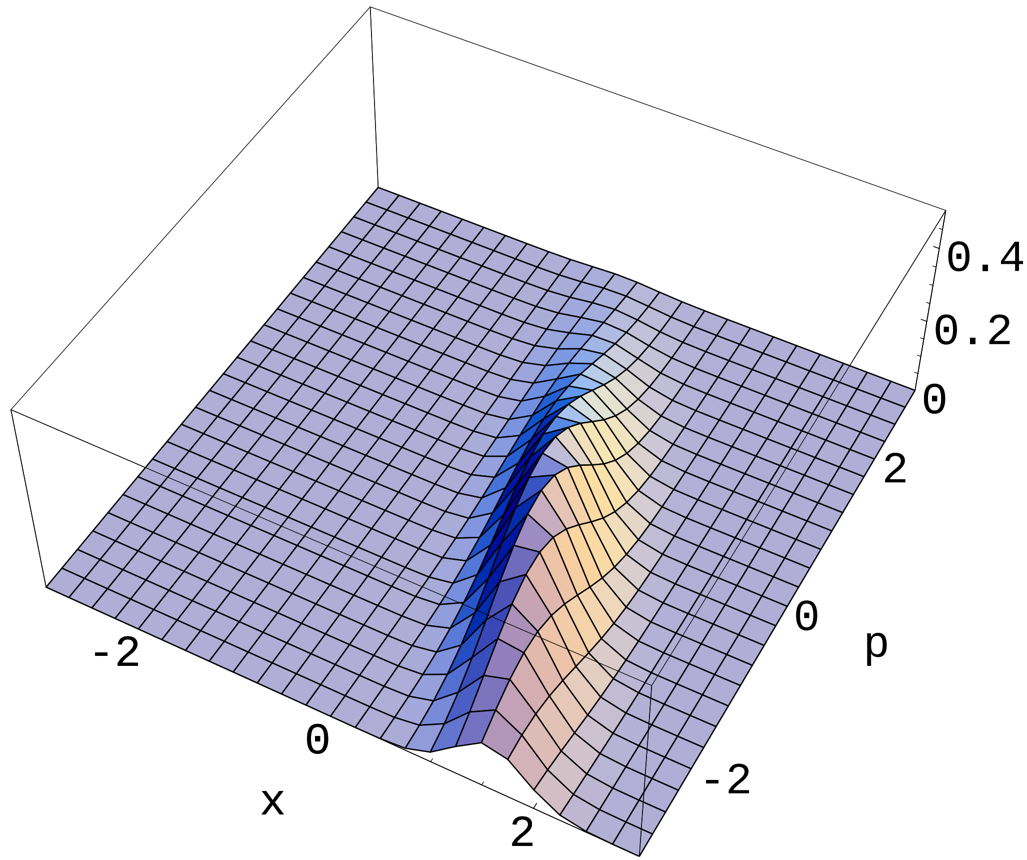

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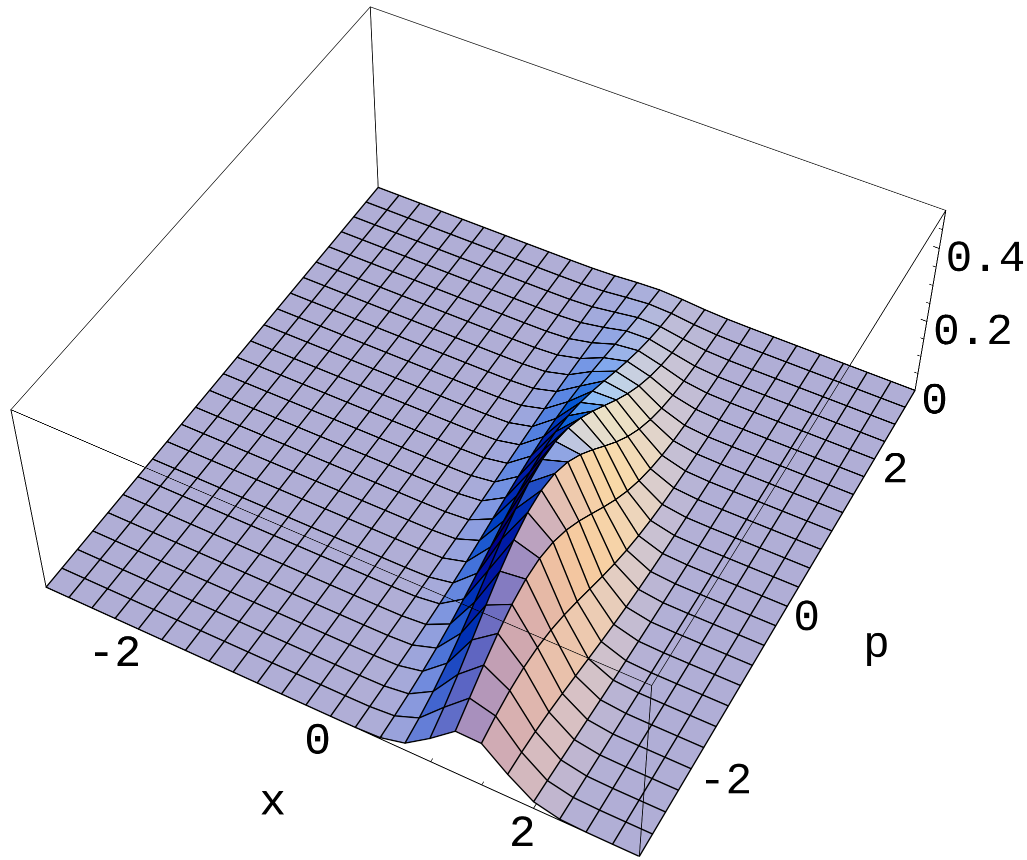

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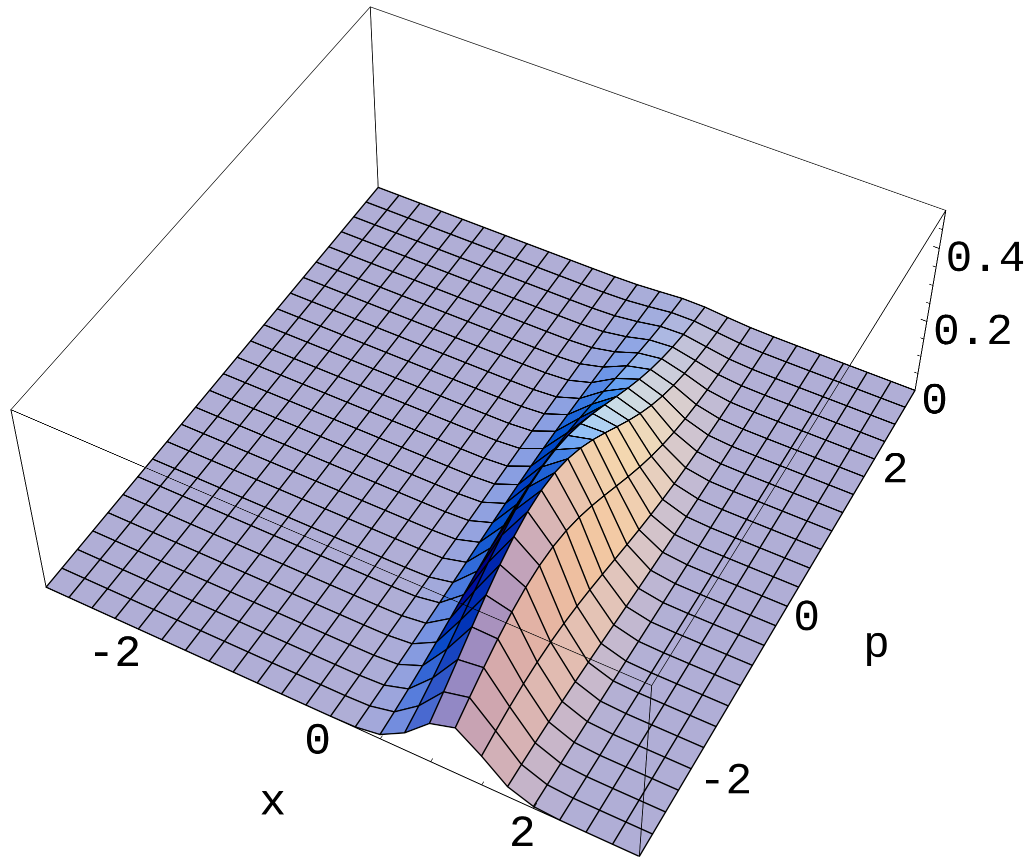

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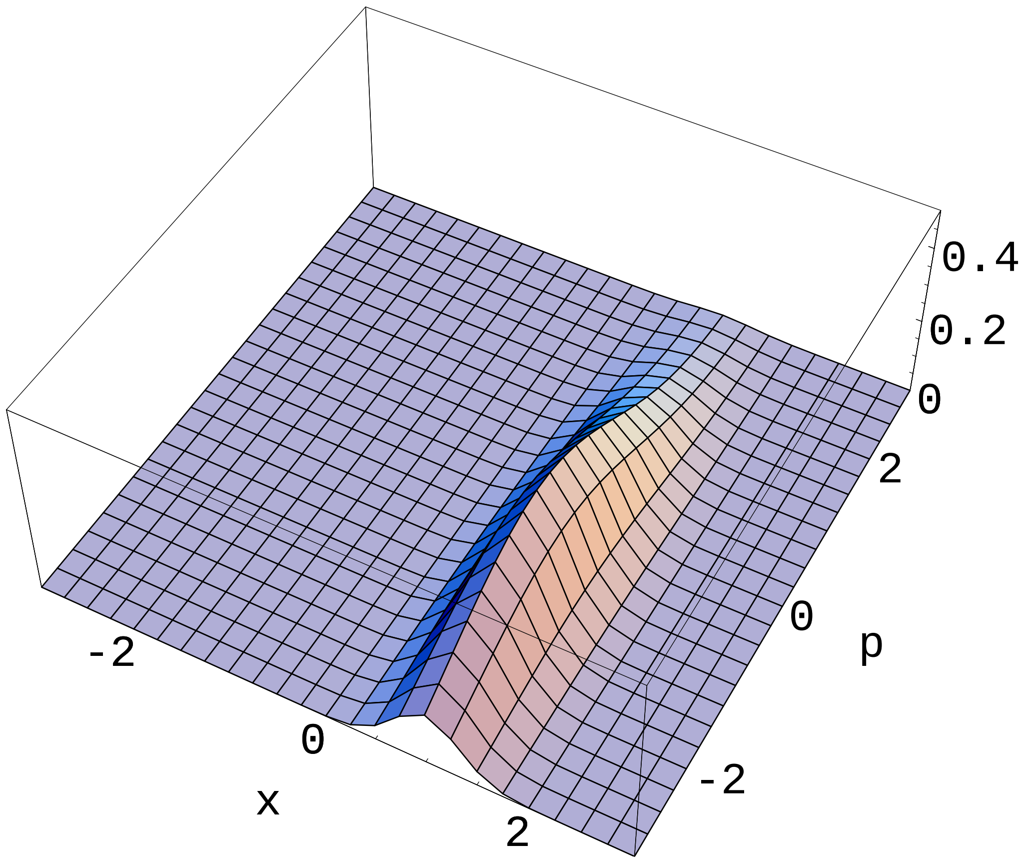

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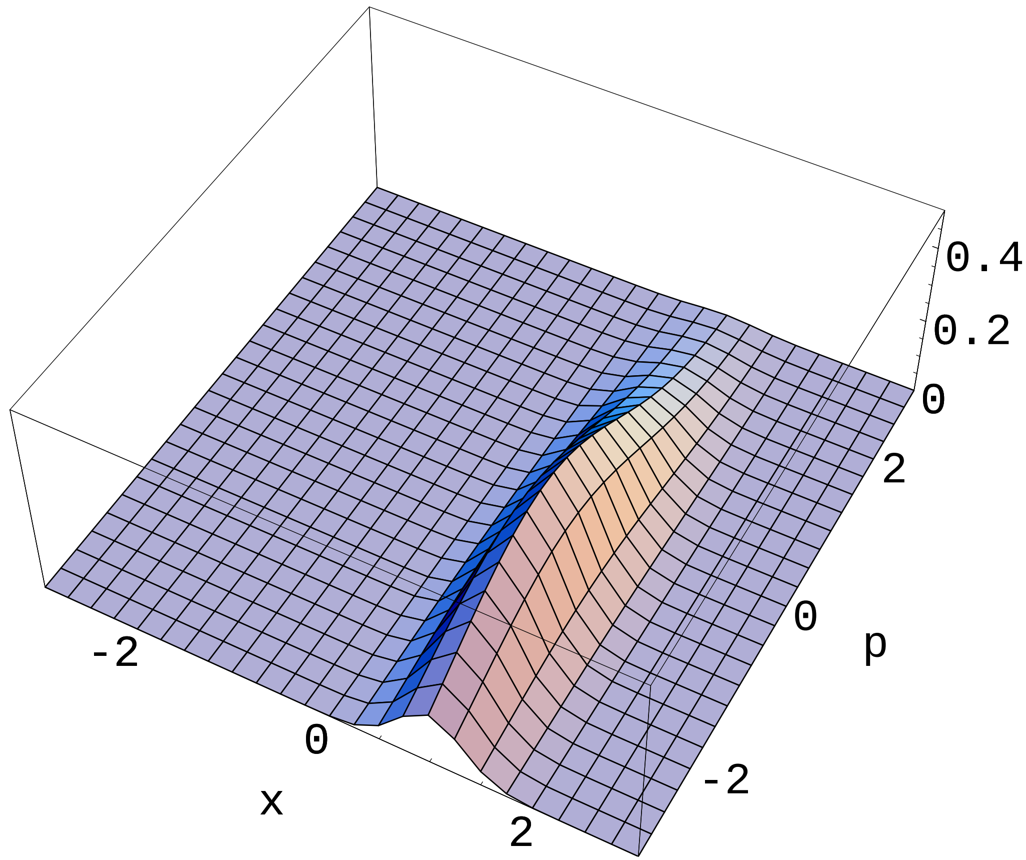

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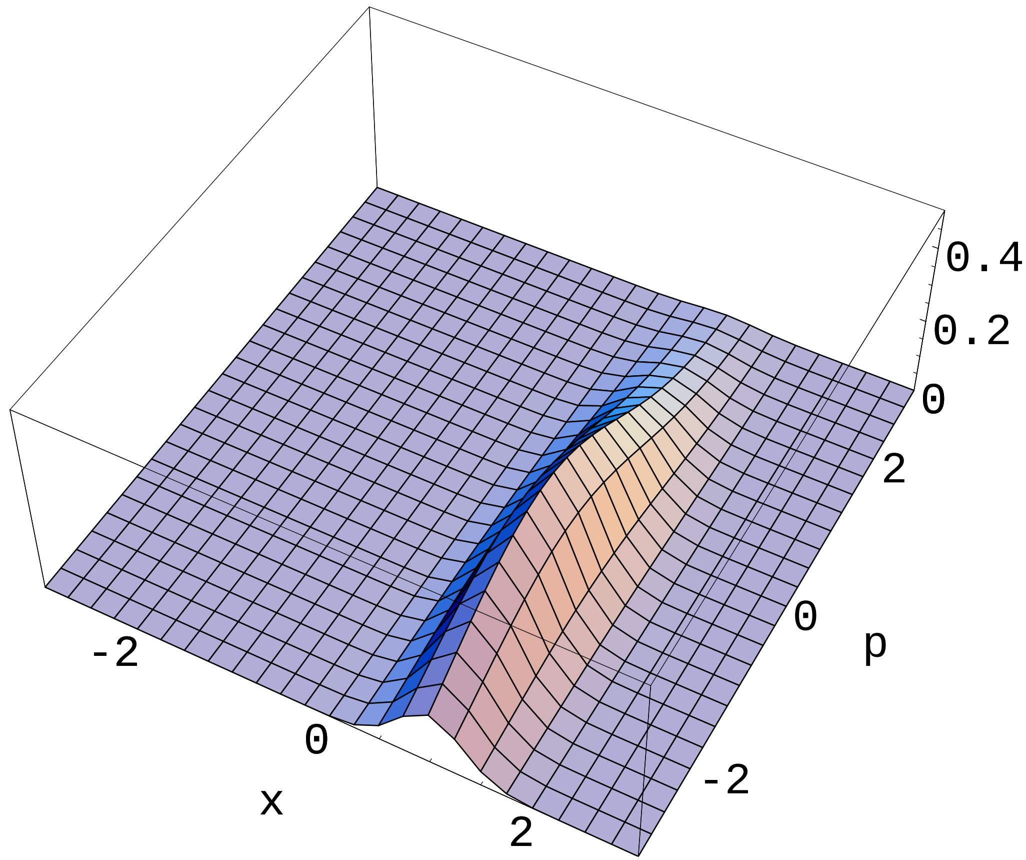

Linear phononics: the lonely ion

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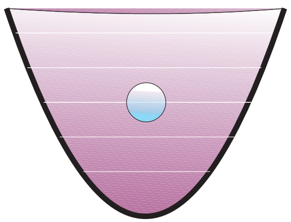

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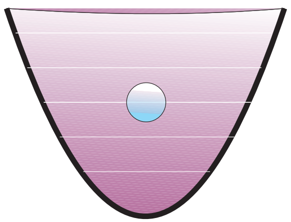

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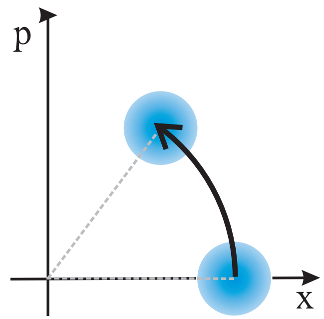

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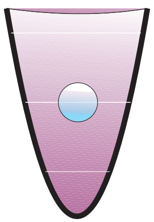

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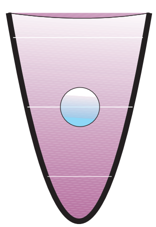

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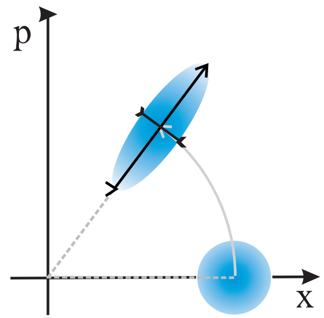

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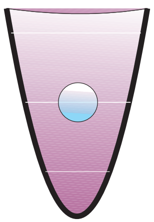

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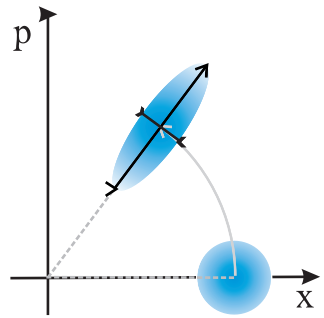

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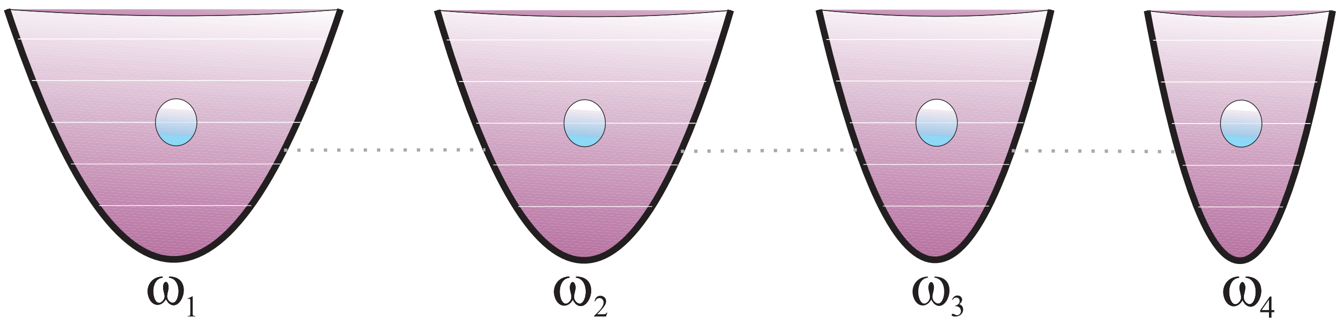

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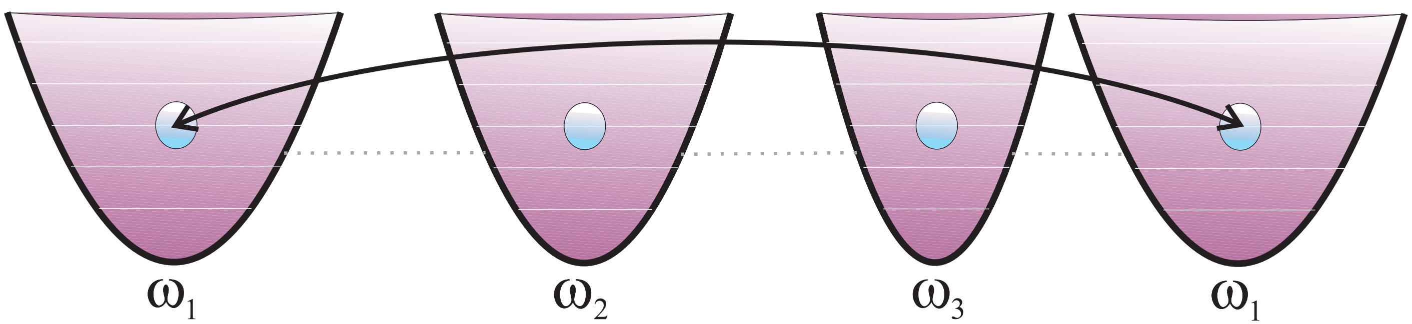

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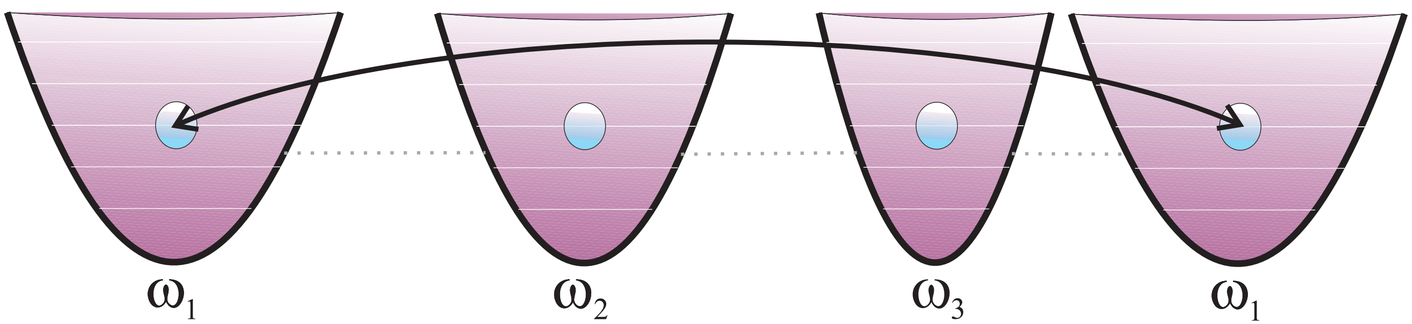

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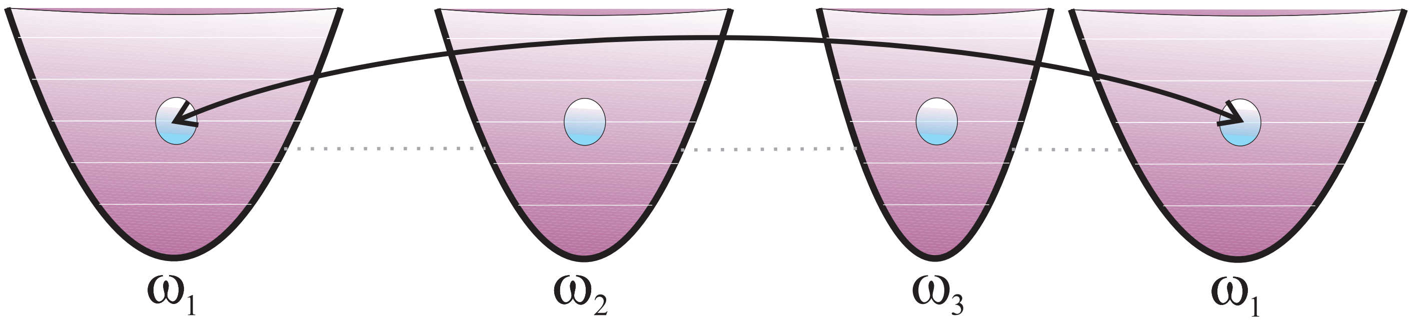

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- Possibility to go beyond Gaussian when anharmonicities kick in

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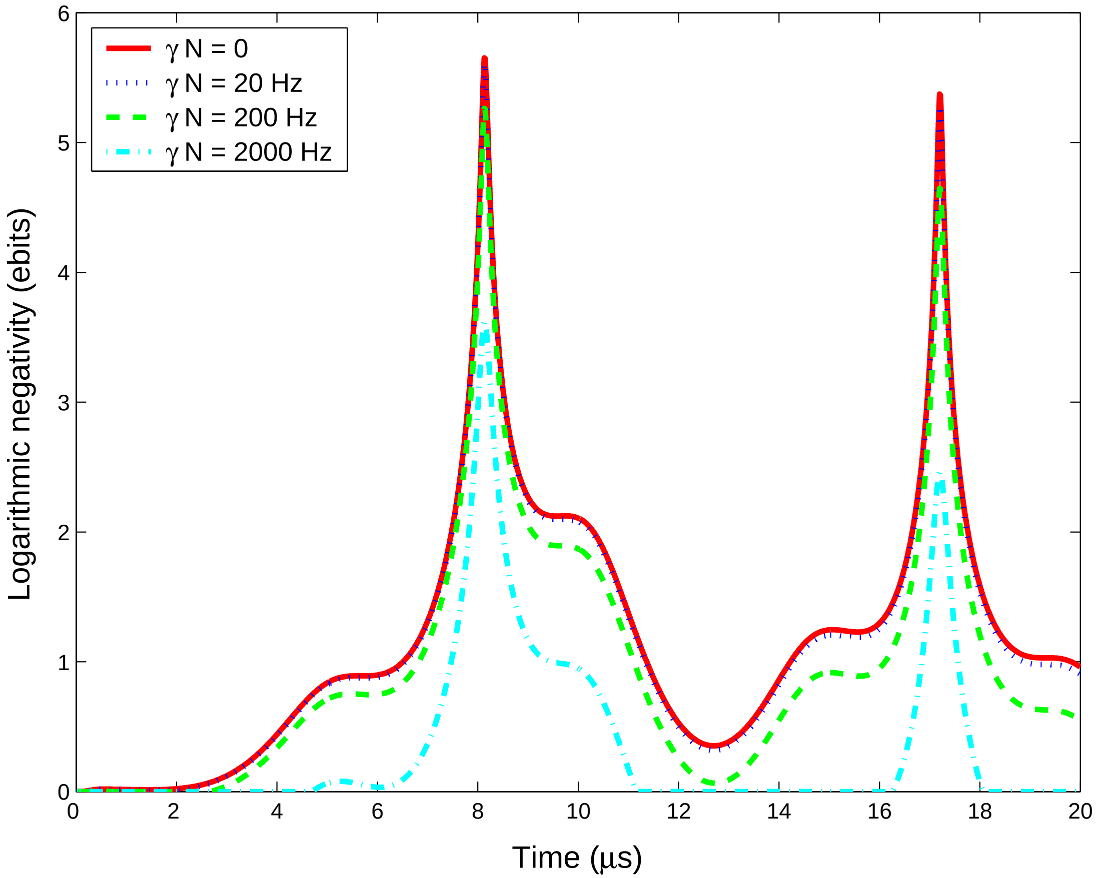
- From now on, only *global* control is assumed: all the trapping frequencies are the same at all times (but can be changed all together)

Entanglement generation

- Three ions: starting from the ground state for trapping frequency $\omega_i = 100\,\mathrm{MHz}$ and evolving with frequency $\omega_f = 2\,\mathrm{MHz}$, $T \simeq 21°C$. (LogNeg between ion 1 and 3)



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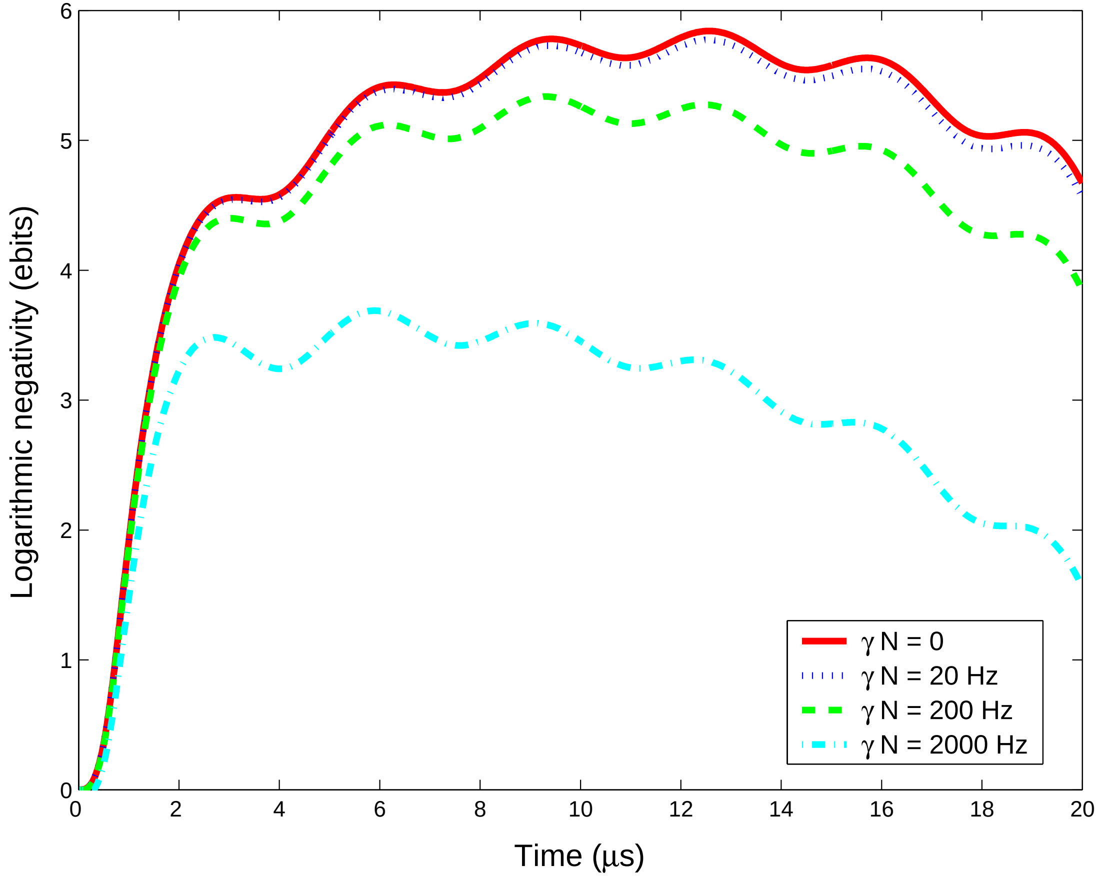

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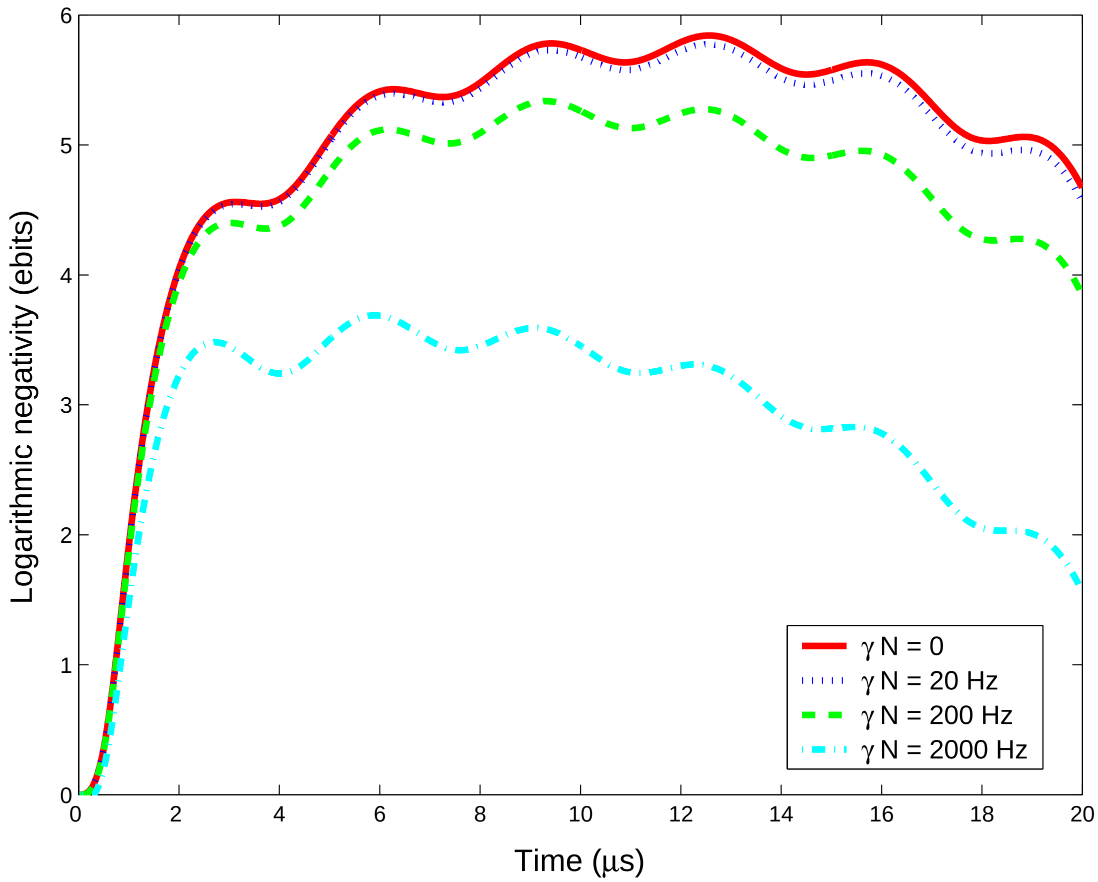

- If swapped to light: entanglement generator for quantum optics

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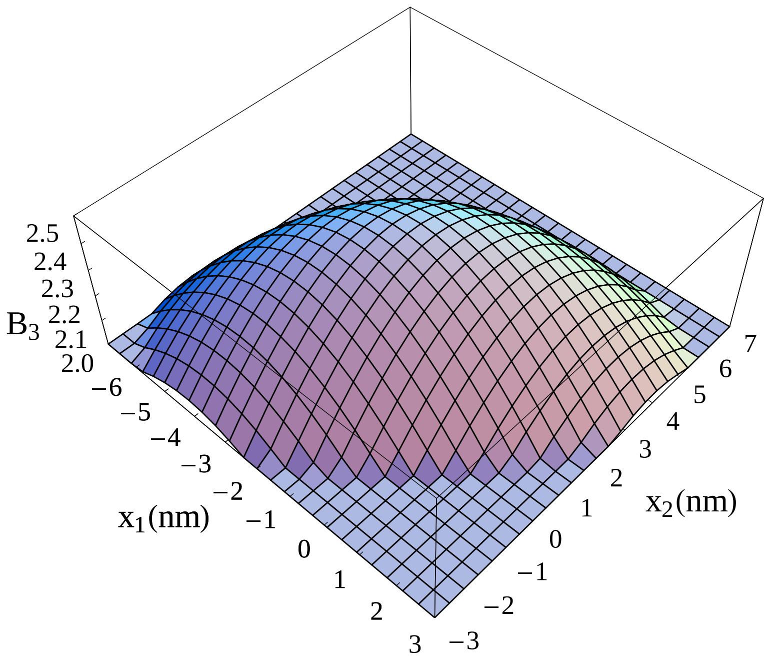

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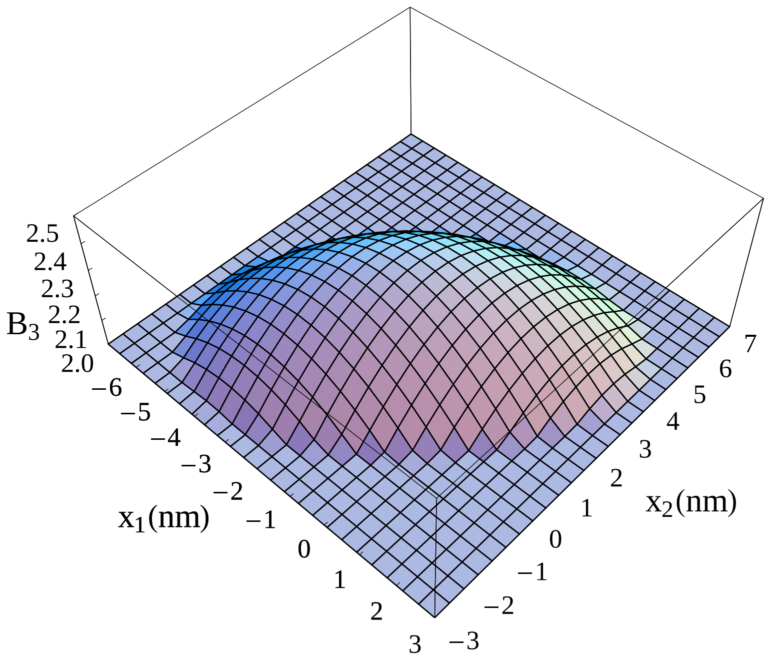

effect of thermal noise: $T \simeq 21°C$, $\gamma N \simeq 200\mathrm{Hz}$

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$\Rightarrow$ **Promising both for quantum information processing and as probes of fundamental physics**