Nonclassical correlation in a multipartite quantum system reconsidered

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Background

Definitions of nonclassical correlation other than entanglement

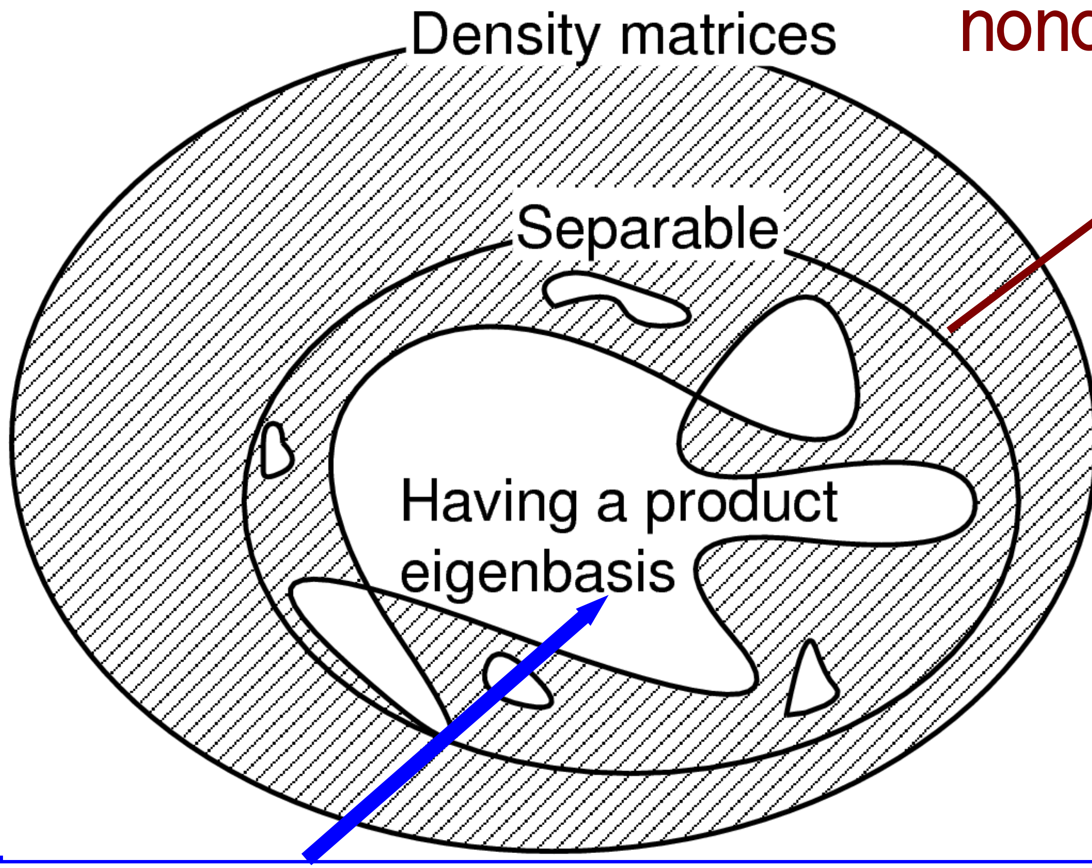
Bennett et al. 1999: A certain nonlocality about locally nonmeasureable separable states.

Ollivier and Zurek 2002: Quantum discord (discrepancy of two expressions of a mutual information that should be equivalent to each other in a classical information theory).

Oppenheim and the Horodecki family 2002: Quantum deficit (discrepancy of CLOCC-localizable information and the total information of a system).

Groisman et al. 2007: o Distance between the state and the nearest classical state. o Distance between the state $\rho^{[A,B]}$ and the state by picking up only diagonal elements under the basis diagonalizing $\mathrm{Tr}_{\mathrm{B}}\rho^{[A,B]}\otimes\mathrm{Tr}_{\mathrm{A}}\rho^{[A,B]}$.

The set of multipartite states with nonclassical correlation



A measure of nonclassical correlation should vanish for a state with a product eigenbasis.

The set of classically correlated states [Oppenheim et al.02]: (m-partite)

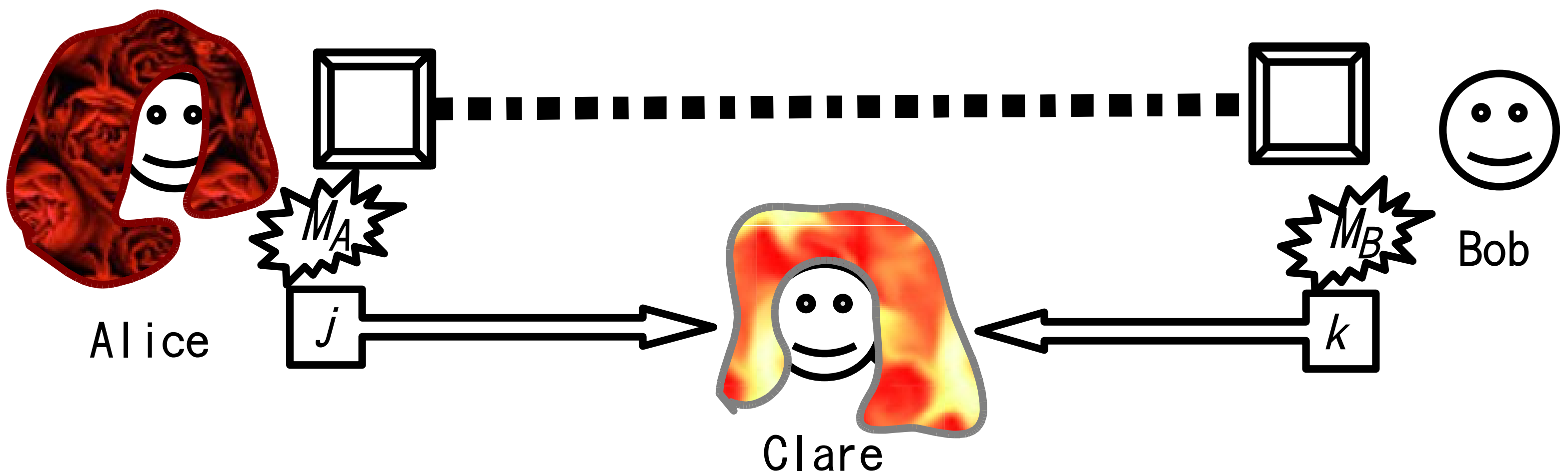
$$\rho_C = \sum_{jk\cdots x} c_{jk\cdots x} |e_j^1\rangle\langle e_j^1| \otimes |e_k^2\rangle\langle e_k^2| \otimes \cdots \otimes |e_x^m\rangle\langle e_x^m|$$

$\{|e_s^i\rangle\}_s$: Eigenbasis of the i th part

Can be diagonalized by local unitary operations.

Measure of nonclassical correlation I

Bipartite Case



1. Alice and Bob have subsystems of a system and they are distant to each other.

2. They send reports to Clare. Alice/Bob can choose a complete orthonormal basis of her/his subsystem (basis $\{|e_j^{[A]}\rangle\}_j$ for Alice and basis $\{|e_k^{[B]}\rangle\}_k$ for Bob) for local measurements.

3. Suppose that Alice and Bob use the observables $M_A = \sum_j j|e_j^{[A]}\rangle\langle e_j^{[A]}|$ and $M_B = \sum_k k|e_k^{[B]}\rangle\langle e_k^{[B]}|$, respectively, and report outcomes j and k to Clare.

The probability that Clare receives j from Alice and k from Bob is $p_{jk} = \langle e_j^{[A]}|\langle e_k^{[B]}|\rho^{[A,B]}|e_j^{[A]}\rangle|e_k^{[B]}\rangle$.

Then, minimum uncertainty (over all possible choices of local bases) that Clare has about $\rho^{[A,B]}$ after listening to their reports is

$$D(\rho^{[A,B]}) = \min_{\text{local basis}} \left(-\sum_{jk} p_{jk} \log_2 p_{jk} \right) - S_{\mathrm{vN}}(\rho^{[A,B]}).$$

As is well-known, *the von Neumann entropy is the smallest possible among all the mixing entropies*[1]. $D(\rho^{[A,B]})$ is a discrepancy between the smallest possible among all the local-mixing entropies and that among all the global-mixing entropies.

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[1] I. Bengtsson and K. Życzkowski, *Geometry of quantum states, an introduction to quantum entanglement*, (Cambridge University Press, New York, 2006).

m - partite Case

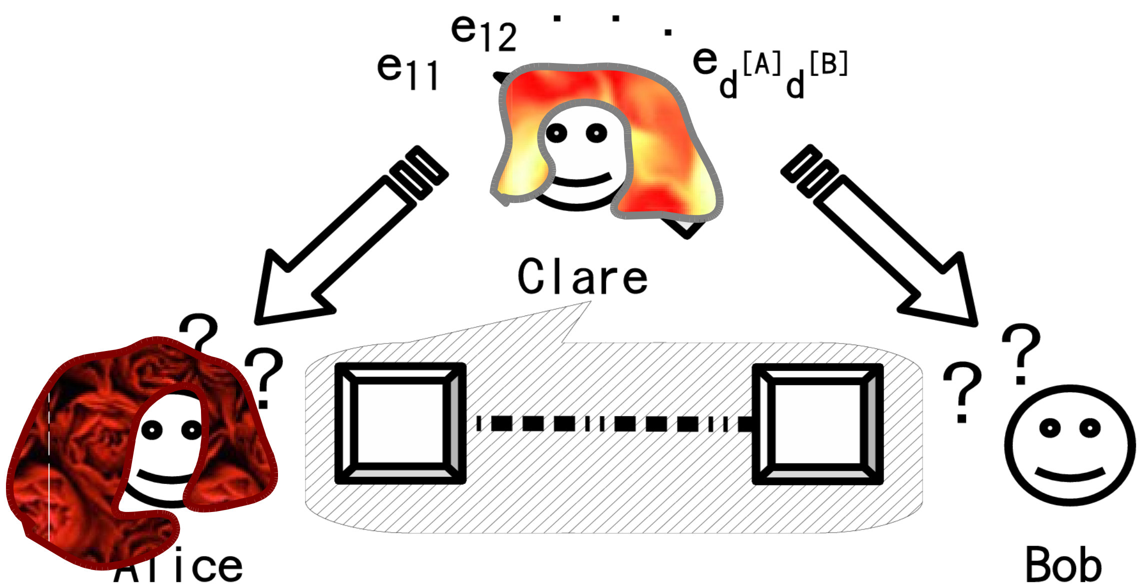
$$p_{jk\cdots x} = \sum_{jk\cdots x} \langle e_j^1 | \langle e_k^2 | \cdots \langle e_x^m | \; \rho^{[1,2,\cdots m]} \; | e_x^m \rangle \cdots | e_k^2 \rangle | e_j^1 \rangle$$

$$D(\rho^{[1,2,\cdots m]}) = \min_{\text{local bases}} \left(-p_{jk\cdots x} \log_2 p_{jk\cdots x} \right) - S_{\mathrm{vN}}(\rho^{[1,2,\cdots m]}).$$

This measure is fully additive: $D(\sigma \otimes \tau) = D(\sigma) + D(\tau)$.

Measure of nonclassical correlation II

Clare knows all eigenvalues of $\rho^{[A,B]}$



Alice wants to know the eigenvalues $\{e_j\}_{j=1}^{d^{[A]}}$ of the reduced density matrix of A.

Alice partitions the $d^{[A]} \times d^{[B]}$ eigenvalues into $d^{[A]}$ sets and makes mimic eigenvalues

$$\tilde{e}_j = \sum_{k=1}^{d^{[B]}} e_{jk}$$ (for the *j* th set).

Minimum uncertainty for Alice about her eigenvalues:

$$F_A(\rho^{[A,B]}) = \min_{\text{partitionings}} \left|\sum_j (\tilde{e}_j \log_2 \tilde{e}_j - e_j \log_2 e_j)\right|$$

Consequently, one may write a measure as

$$G(\rho^{[A,B]}) = \max\{F_A(\rho^{[A,B]}), F_B(\rho^{[A,B]})\}.$$

m - partite Case

Kate partitions $\prod_{i=1}^{m} d^{[i]}$ eigenvalues into $d^{[k]}$ sets and makes $d^{[k]}$ mimic eigenvalues $\{\tilde{e}_j^{[k]}\}_{j=1}^{d^{[k]}} = \{\sum_{k=1}^{d^{[1]}\cdots d^{[k-1]} d^{[k+1]}\cdots d^{[m]}} e_{jk}\}_{j=1}^{d^{[k]}}$.

$$F_k(\rho^{[1,\cdots,m]}) = \min_{\text{partitionings}} \left|\sum_j (\tilde{e}_j^{[k]} \log_2 \tilde{e}_j^{[k]} - e_j^{[k]} \log_2 e_j^{[k]})\right|$$

$$G(\rho^{[1,\cdots,m]}) = \max_k\{F_k(\rho^{[1,\cdots,m]})\}.$$

This measure is subadditive: $G(\sigma \otimes \tau) \leq G(\sigma) + G(\tau)$.

Numerical computation of nonclassical correlation

- Measure I should be numerically estimated in general.

Repeat 1.0 x $10^{4 \sim 6}$ times

(i) Generate a set of local complete orthonormal bases randomly

(ii) Compute the value

$$D \Leftarrow -p_{jk\cdots x} \log_2 p_{jk\cdots x} - S_{\mathrm{vN}}(\rho^{[1,2,\cdots m]}).$$

$$p_{jk\cdots x} = \sum_{jk\cdots x} \langle e_j^1 | \langle e_k^2 | \cdots \langle e_x^m | \ \rho^{[1,2,\cdots m]} \ | e_x^m \rangle \cdots | e_k^2 \rangle | e_j^1 \rangle$$

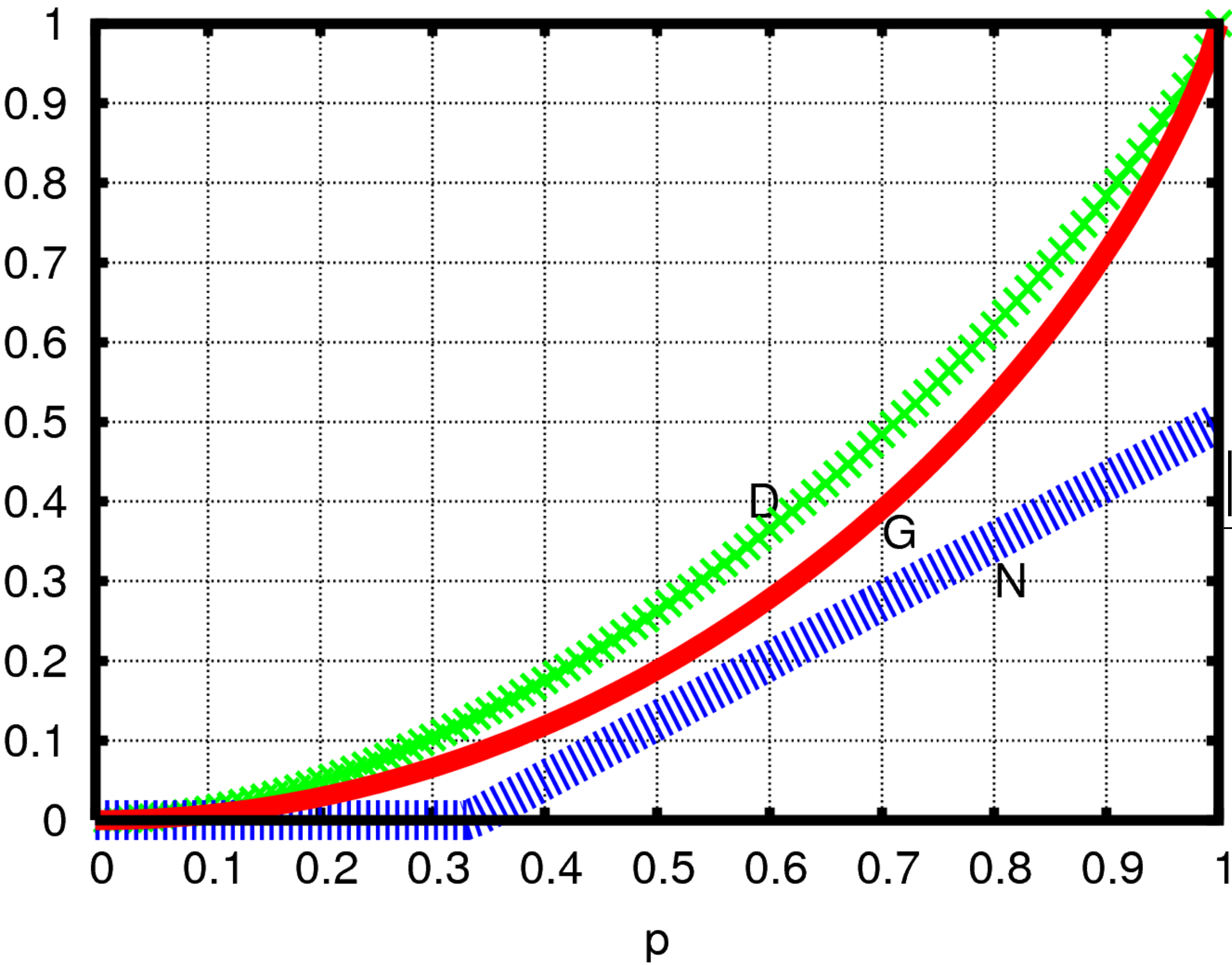
(iii) Keep the minimum ***D*** ever computed.

※Use a good random number generator e.g. Mersenne Twister.

- Measure II can be calculated exactly for a small dimension.

Example 1: Two-qubit pseudo-pure state

$$\rho_{\mathrm{ps}} = p|\psi\rangle\langle\psi| + (1-p)\frac{I}{4} \quad \text{with} \quad |\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$



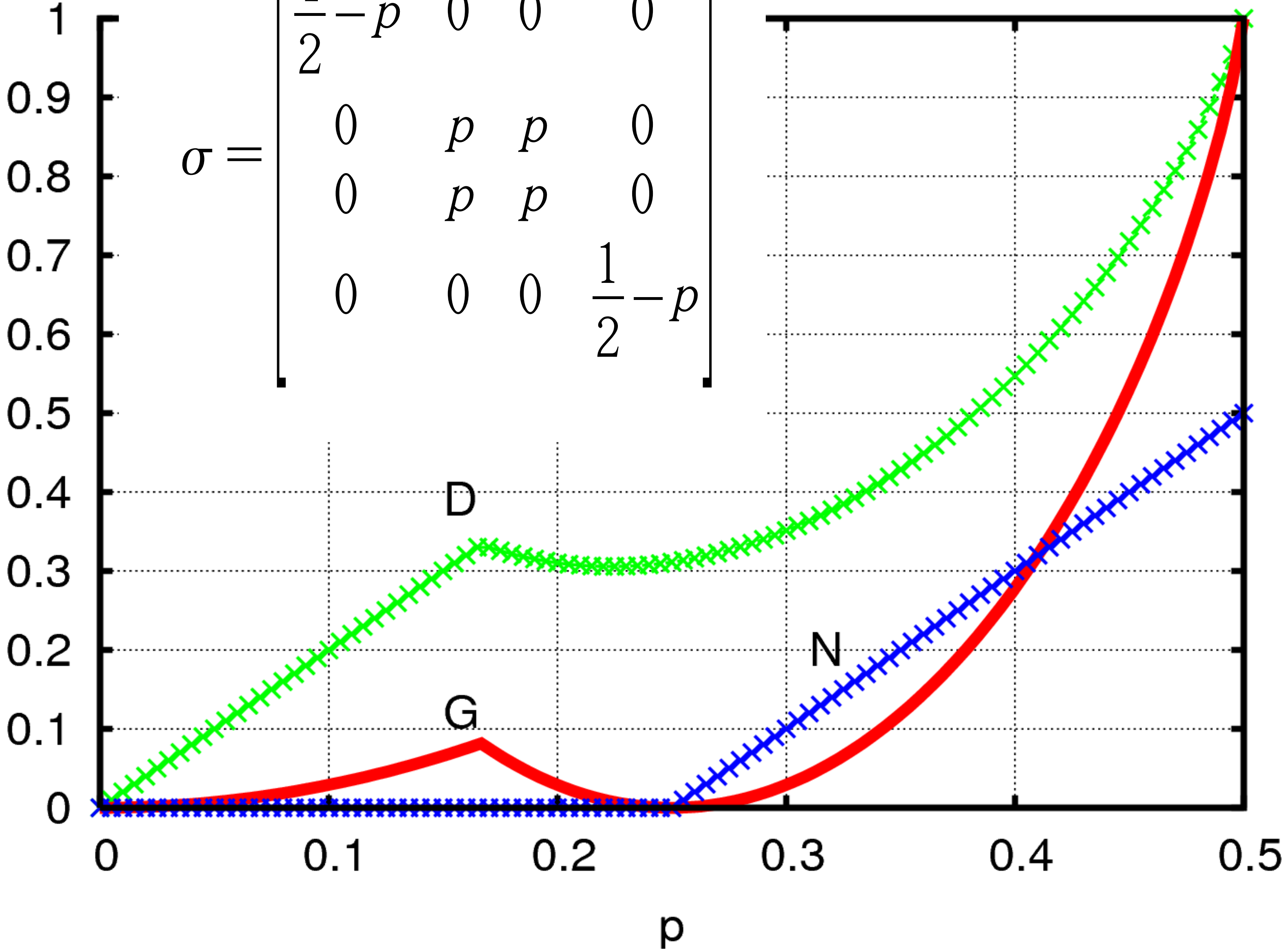
Nonclassical correlation measures

Negativity

$$\frac{\|(I \otimes \Lambda_T)\rho^{[A,B]}\| - 1}{2}$$

Example 2: A mixture of $|00\rangle$, $|11\rangle$, $\frac{|01\rangle+|10\rangle}{\sqrt{2}}$

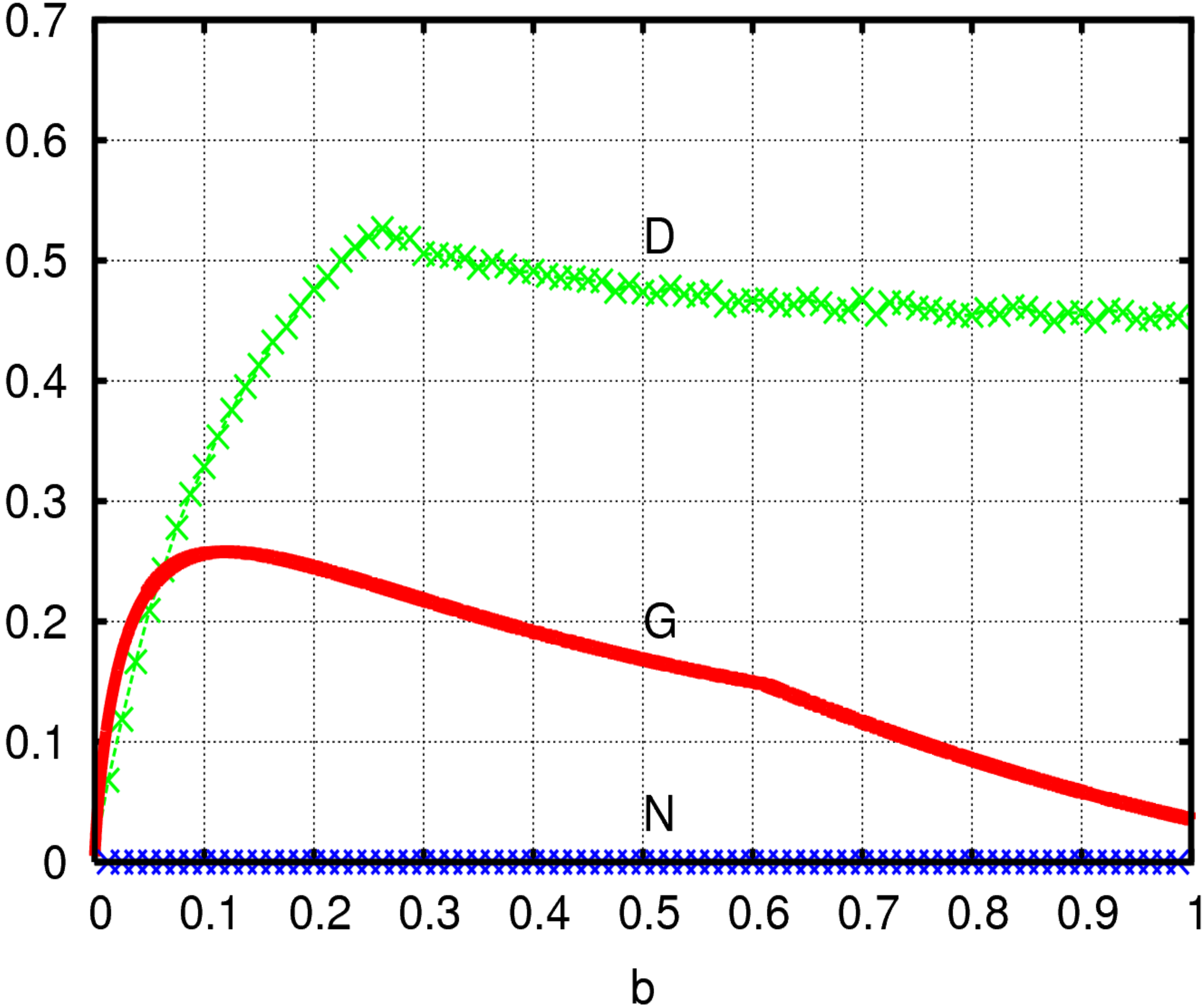
$$\sigma = \begin{bmatrix} \frac{1}{2}-p & 0 & 0 & 0 \\ 0 & p & p & 0 \\ 0 & p & p & 0 \\ 0 & 0 & 0 & \frac{1}{2}-p \end{bmatrix}$$



Example 3: 2 x 4 splitting

$$\sigma_b = \frac{1}{7b+1}\begin{bmatrix} b & 0 & 0 & 0 & 0 & b & 0 & 0 \\ 0 & b & 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & b & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1+b}{2} & 0 & 0 & \frac{\sqrt{1-b^2}}{2} \\ b & 0 & 0 & 0 & 0 & b & 0 & 0 \\ 0 & b & 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & b & 0 & \frac{\sqrt{1-b^2}}{2} & 0 & 0 & \frac{1+b}{2} \end{bmatrix}$$

This matrix was introduced by P. Horodecki, Phys. Lett. A 232, 333 (1997).


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
D
G
N
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
b

Summary and tasks

- Two measures of nonclassical correlation have been introduced.
- Measure ***D*** should be numerically estimated by using a random search of the minimum over product complete orthonormal bases.
- A simulated annealing may be used for computing ***D*** for a density matrix with a large dimension.
- Measure ***G*** can be calculated from eigenvalues of a density matrix.
- Examples for bipartite splitting case have been shown.
- One can define a measure ***M*** of nonclassical correlation in many different ways as far as ***M*** vanishes for a density matrix with a product eigenbasis.
- It is better that ***M*** is additive or subadditive.