OPTIMAL MANIPULATIONS WITH QUANTUM INFORMATION: UNIVERSAL QUANTUM MACHINES

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1. Introduction

One of the cornerstones of our present understanding of the Nature is quantum physics. This theory has "nonlocal" characteristics [1-3], due to quantum entanglement - let us just mention the EPR Einstein-Podolsky-Rosen paradox [4] and the violation of the Bell's inequalities [5, 6] which have been experimentally confirmed in several experiments, see e.g. Refs. [7, 8] (more on quantum-mechanical non-locality non-locality see also Refs.[9-18].

The most remarkable property of the *non-relativistic* quantum mechanics (which is inherently *nonlocal*) is that it peacefully coexists with the special theory of relativity in a sense that one cannot exploit quantum-mechanical entanglement between two space-like separated parties for communication of classical messages faster than light [19-28].

It turns out that quantum correlations first discussed in their seminal paper by Einstein, Podolsky, and Rosen [4] result in the measured probabilities which satisfy the *causal communication constraint* [18] [1]. This means that the probability of a particular measurement outcome on any one part of the system should be independent of measurement performed on the other parts. This requirement should guarantee the absence of faster-than-light signals [19] that is usually called as the *no-signaling*. [2].

This fundamental feature of quantum theory, that is that quantum-mechanical correlations (entanglement) entanglement cannot be used for superluminal communications has been challenged in 1982 by Nick Herbert [36] in his proposal

[1]This "signal locality" [25, 29], is also referred to in the literature as the "simple locality" [30, 31], the "parameter independence" [32], or "physical locality" [33].

[2]Probably I should note here that the signal locality can be formulated independently of quantum theory [25, 30, 31, 34, 35]

A.S. Shumovsky and V.I. Rupasov (eds.), Quantum Communication and Information Technologies, 47–84.

of the FLASH - the superluminal communicator [3]. The key idea of the FLASH relies on the possibility to copy (clone) unknown states of quantum systems. Herbert has shown that if the quantum cloning cloning would be possible, then we would be able to enjoy a comfort of super-luminal communication mediated by entangled quantum systems [4].

Herbert's idea has been criticized instantly. Namely, Dieks [37] and Wootters and Zurek [38] have shown that perfect cloning cloning of unknown quantum states is impossible (see Section III). This seemingly closed the whole issue. Years later Mark Hillery and myself [39] have asked a question: "Even though the perfect copying of unknown states is impossible, how well we can clone quantum states?" This question then has triggered a whole series of papers devoted to investigation of universal quantum machines (UQM).

UQM's are quantum mechanical devices that take a certain number of quantum systems (e.g. qubits) as an input and produce an output that is as close as possible to some ideal target state (in general target state depends on input). A typical example is a process of universal cloning. The universal cloning machine produces copies of an input qubit in an *unknown* state, such that the fidelity fidelity of the copies does not depend on the input state [39-59]. In fact exactly this type of machine has been implicitly assumed by Herbert [36]. It has been recently shown [60-64] that for the simplest case (namely a cloner producing 2 copies from 1 qubit) a bound on the quality of the clones could be derived from the no-signaling condition is identical to the bounds derived from quantum mechanics. This means that if the clones were only a little bit better than allowed by quantum mechanics, one could immediately use them for superluminal communication.

So in my lectures, I am going to use the intriguing connection between cloning cloning of quantum states and faster-than-light signaling [65-71] to describe several interesting problems related to some aspects of optimal manipulations with quantum information. To make my presentation selfcontent in Section 2 I will discuss in some detail physical origin of Herbert's proposal. Section 3 will be devoted to the no-cloning theorem, while in Section 4 I will describe universal quantum cloners. In Section 5 I will show how bounds on cloning can be derived from the no-signaling condition and in Section 6 I will

[3] FLASH is an acronym for the *First Laser-Amplified Superluminal Hookup*.

[4] At the *IV Adriatico Conference on Quantum Interferometry* (12 – 15 March 2002, Trieste, Italy) in general discussion Asher Peres has admitted that he was one of the referees of Herbert's paper [36] and even though he found it wrong he had recommend it for publication. According to Peres, the paper was so fundamentally wrong that it deserved the publication. Interestingly enough, Giancarlo Ghirardy who was at the conference as well, has responded to the confession of Asher Peres, that he was the second referee of the same Herbert's paper. Unlike Peres, he has recommend the paper to be rejected and in his referee report he has proved the no-cloning theorem which has been proved independently and published by William Wootters and Wojciech Zurek [38] (for more details see Section III).

discuss the connection between cloning, signaling and generalized measurements. The rest of the notes are devoted to universal NOT gate that "flips" a qubit in an unknown state. t In Section 7 I will describe the U-NOT gate and in Section 8 I will present experimental realization of this gate. I conclude my notes in Section 9.

2. Bell Telephone & FLASH

Let us start an overview of the signaling based on quantum non-locality non-locality with a description of the "Bell telephone" and then will continue with a more detailed description of Herbert's idea for superluminal communication based on amplification (cloning) of individual events in the measurement of entangled particles.

2.1 Bell telephone

The Bell telephone is supposed to be a physical device which "uses" for communication quantum non-locality. In order to operate such device first maximally entangled particles (let these be qubits) have to be distributed to two parties (Alice and Bob). This can be achieved by using polarization states of two-photons. The polarization-entangled pairs of photons are experimentally generated in a parametric down-conversion process in a nonlinear crystal [72]. These entangled photons are generated in a singlet state

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B), \tag{1}$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ describe two polarization states of the photon in a given basis (e.g. horizontal/vertical polarization). The singlet state (1) exhibits perfect quantum correlation (entanglement) for polarization measurements along parallel but *arbitrary* axes. This means that the state (1) is equivalent under *local* transformations to the state

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|\leftarrow\rangle_A|\rightarrow\rangle_B - |\rightarrow\rangle_A|\leftarrow\rangle_B), \tag{2}$$

where $|\substack{\leftarrow \\ \rightarrow}\rangle = (|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}$. Here I stress that even though results of measurements are perfectly correlated the actual outcome of an individual measurement on each of the particles is inherently random.

Alice and Bob receive their particles well before any communication via quantum channel is performed. It is also assumed that the singlet state does not decohere under the influence of the environment. Once the entangled particle are distributed Alice might like to send a message to Bob. To do so she decides to perform a measurement on her particle. She can rotate her measurement device arbitrarily, but in order to make our discussion simple let us assume that she is going to perform a measurement in one of the two bases ($\{|\uparrow\rangle, |\downarrow\rangle\}$ and $\{|\leftarrow\rangle, |\rightarrow\rangle\}$). After Alice performs her measurement in the given basis (let say, $\{|\uparrow\rangle, |\downarrow\rangle\}$) then she can predict with certainty what Bob's result would be if he performs the measurement in the *same* basis. Hence if Bob can attain somehow the information about the measurement basis he would then have a possibility of receiving signals at the superluminal speed.

Even though Alice can determine which basis she will use she definitely is not able to predetermine the outcome of her measurement to be $|\uparrow\rangle$ or $|\downarrow\rangle$ since both these results materialize with probability a $1/2$. Therefore the message can best be defined to be Alice's choice in which basis she is going to perform a measurement on her qubit. That is, which observable she is going to measure (we will not consider an option that she choose not to measure at all). In this situation Bob's task is to determine the state of a qubit on his side of the Bell telephone via a single measurement on his qubit.

Formally, the signal from Alice is encoded into a *binary* alphabet with each of the two letters corresponding to a specific choice of Alice's basis, i.e. the measurement procedures $\mathcal{A}^{(s)}$: $(s = 0, 1)$,

$$\begin{aligned} \mathcal{A}^{(0)} &: \{|\uparrow\rangle_A\langle\uparrow| \otimes \mathbf{1}_B; |\downarrow\rangle_A\langle\downarrow| \otimes \mathbf{1}_B\}, \\ \mathcal{A}^{(1)} &: \{|\leftarrow\rangle_A\langle\leftarrow| \otimes \mathbf{1}_B; |\rightarrow\rangle_A\langle\rightarrow| \otimes \mathbf{1}_B\}, \end{aligned} \tag{3}$$

respectively. This means that if Alice wants to send a logical "0" ("1") then she performs a measurement $\mathcal{A}^{(0)}$ ($\mathcal{A}^{(1)}$). There is no way to say *a priori* what the outcome of her measurement will be. So after measuring e.g. $\mathcal{A}^{(0)}$ Bob's particle will be either in the state $\rho_{\uparrow}^{(0)} = |\uparrow\rangle\langle\uparrow|$ or in the state $\rho_{\downarrow}^{(0)} = |\downarrow\rangle\langle\downarrow|$. These outcomes are realized with the same probability. Analogous situation takes place when Alice sends a logical "1", i.e. performs the measurement $\mathcal{A}^{(1)}$.

The main task of Bob is to determine which state his particle is in. To do so he performs a measurement on his particle. For simplicity, let us assume that he performs a projective measurement $\mathcal{B} = \{O_r\}$, such that $O_r = |\psi_r\rangle\langle\psi_r|$ and $\sum_r O_r = \mathbf{1}$. Ideally, Bob wants to perform a measurement such that $\mathrm{Tr}[O_r\rho_{j(r)}^{(s)}] = 1$ for just one of the "input" states $\rho_{j(r)}^{(s)}$ while for all others the trace is equal to zero. But this requires the signal (input) states corresponding to different outcomes of Alice measurements to be orthogonal, i.e. $\mathrm{Tr}[\rho_i^{(s)}, \rho_j^{(s')}] = 0$. Which obviously, is not the case in the present scheme. Therefore no projection measurement exists which would yield a *reliable* signal analysis from *individual* outcomes. If many entangled pairs of qubits have been used for communication using the above scheme, then depending which of the two measurements

$$\begin{aligned} \mathcal{B}^{(0)} &: \{\mathbf{1}_A \otimes |\uparrow\rangle_B\langle\uparrow|;\ \mathbf{1}_A \otimes |\downarrow\rangle_B\langle\downarrow|\}, \\ \mathcal{B}^{(1)} &: \{\mathbf{1}_A \otimes |\leftarrow\rangle_B\langle\leftarrow|;\ \mathbf{1}_A \otimes |\rightarrow\rangle_B\langle\rightarrow|\}, \end{aligned} \tag{4}$$

Bob performs his resulting ensembles of qubits are represented by two density operators

$$\begin{aligned} \rho_{\uparrow\downarrow}^{(B)} &= \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|) = \frac{1}{2}\mathbf{1}; \\ \rho_{\overleftarrow{\rightarrow}}^{(B)} &= \frac{1}{2}(|\leftarrow\rangle\langle\leftarrow| + |\rightarrow\rangle\langle\rightarrow|) = \frac{1}{2}\mathbf{1}, \end{aligned} \tag{5}$$

which are identical. Consequently, within the linear quantum mechanics (for further discussion see Section 4) these two ensembles are indistinguishable and no information can be signalled this way. In other words, Bob is not able to find what decision has been made by Alice. Here I stress once again, that even though the two ensembles $\rho_{\uparrow\downarrow}^{(B)}$ and $\rho_{\overleftarrow{\rightarrow}}^{(B)}$ are indistinguishable, the individual outcomes of particular measurements are in specific pure states.

2.2 Herbert's FLASH

In his proposal of the FLASH Herbert [36] has clearly indicated that his "hookup" is based on the "novel" type of the quantum measurement performed on Bob's side. According to Herbert: "Superluminal message (alternation of individual events) can be sent but not decoded". Therefore the FLASH was supposed to operate in such a way that via a measurement of an *individual* quantum system (polarization states of a photon) one can determine the state of the system. The "novel" aspect of the FLASH was an idea to use *active* detectors such that before they register the incoming state they first multiply (copy, clone) this state into a large collection of particles all in the same (incoming) state. This would mean that the FLASH realizes the copying (cloning) cloning of an unknown pure state $|\psi\rangle$ of the incoming particle with the Hilbert space H, on N other particles of the same physical origin, i.e. the cloner is described by a map $\mathcal{C}(H) \to \mathcal{C}(H^{\otimes N})$ such that

$$|\psi\rangle|0\rangle^{\otimes(N-1)} \to |\psi\rangle^{\otimes N}, \tag{6}$$

where $|0\rangle$ is some *known* state of the systems onto which the information is going to be copied (see below).

Once this collection of clones is generated then with the help of an optimal measurement better (perfect) determination of the incoming state can be performed. So to quote Herbert "FLASH does not really deal with statistical aggregates of photons but with the aid of perfect xeroxing provided by the laser laser effect is able to examine the polarization of each individual photon." Herbert calls this as the *third type* of measurement (as we will see later this measurement us nothing else but the well known generalized POVM measurement [73, 74]). So the question is whether this type of the active measurement can be physically realized and whether via the improvement of the detection scheme faster-than-light communication can be established.

The main Herbert's idea is to substitute the standard projection measurement by a generalized measurement which includes also an amplifier amplifier (cloner). In quantum theory any amplification is inevitable accompanied by quantum noise which irreversibly spoils the signal [75, 76]. Herbert was aware that "a serious objection to FLASH concerns the noise ... of the copying process". Nevertheless he has not performed a detailed analysis of the problem. He just presented a vague argument that the noise induced by the stimulated emission via which copying of the incoming photon is performed, is after all not a serious obstacle.

3. No-cloning Theorem

So can it be that within a linear quantum mechanics the transformation (6) can be realized? In order to illuminate this question I will formulate the no-cloning cloning theorem. The issue of cloning, or (self) reproducing of quantum states has been first discussed by Wigner [77] (see below). The no-cloning theorem itself has been proved independently by Dieks [37], and Wootters and Zurek [38] and others [78-81]. I will also present Mandel's proposal [82] for cloning of polarization states of photons via down conversion. Mandel was the first to present an explicit calculation of the amount of noise which is inevitable for cloning of quantum states.

3.1 Pre-no-cloning history

Probably the first account on quantum cloning was done by E.P. Wigner [77] in his analysis of earlier work by W.M. Elsasser devoted to a discussion of the origin of life and the multiplication of organisms [83]. Wigner has presented a quantum-mechanical argument according to which "the probability is zero for existence of self-reproducing states". The argument is based on two assumptions: Firstly just systems with finite-dimensional Hilbert spaces are considered. Secondly, it is assumed that the Hamiltonian Hamiltonian which governs the behavior of a complicated system is a *random* symmetric matrix, with no particular properties except for its symmetric nature. Under this assumptions Wigner has shown that "it is infinitely unlikely that there be*any* state of the nutrient [5] which would permit the multiplication of any set of states which is much smaller than all possible states of the system." This conclusion is based on the fact that the input state of the system and the cloner and their ideal output states do not specify the S matrix describing the self-reproducing (cloning) process.

I will not go into further detail of Wigner's argument. The reason why I comment on this work is to show that the first idea of cloning cloning quantum states has not been related to the problem of signaling, but has been based on information aspects of quantum theory. [6]

3.2 Wootters-Zurek theorem

Wootters and Zurek [38] have presented a very simple proof that the perfect cloning transformation (6) for unknown quantum states is impossible.Their proof goes as follows: in order to clone an unknown state $|\psi\rangle$ a device (quantum cloner) is needed. This cloner is initially prepared in a state $|S\rangle$ which does not depend on $|\psi\rangle$. In addition, a set of $(N-1)$ particles onto which the information is going to be copied is available. These particles are prepared in a known state denoted as $|0\rangle$. Then the perfect copying transformation $\mathcal{U}$ can be written as

$$|\psi\rangle|0\rangle^{\otimes(N-1)}|S\rangle \xrightarrow{\mathcal{U}} |\psi\rangle^{\otimes N}|S'\rangle\,, \tag{7}$$

where $|S'\rangle$ is the state of the quantum copier after the cloning has been performed. Since the input is totally unknown, the transformation $\mathcal{U}$ has to work for an arbitrary input. So let us assume two input states $|\psi\rangle$ and $|\tilde{\psi}\rangle$ both transformed according to Eq. (7)

$$\begin{aligned} \mathcal{U}\left(|\psi\rangle|0\rangle^{\otimes(N-1)}|S\rangle\right) &= |\psi\rangle^{\otimes N}|S'\rangle\,; \\ \mathcal{U}\left(|\tilde{\psi}\rangle|0\rangle^{\otimes(N-1)}|S\rangle\right) &= |\tilde{\psi}\rangle^{\otimes N}|S'\rangle\,. \end{aligned} \tag{8}$$

Taking the inner product of the left-hand sides of above equations we find

$$\langle\psi|\tilde{\psi}\rangle = (\langle\psi|\tilde{\psi}\rangle)^2\,, \tag{9}$$

which can only be fulfilled if the two states $|\psi\rangle$ and $|\tilde{\psi}\rangle$ are either identical or orthogonal. So the ideal (perfect) cloning device for *arbitrary* states does not

[5]The "nutrient" is equivalent to two quantum systems the clone and the ancilla (i.e. the cloner itself) via which the information is distributed.

[6]Without any further exploration of the idea I just note that the impossibility of self-reproducing units has a direct consequence in a theory of reproducing quantum cellular automata which has not been properly explored yet.

exist [7]. Perfect cloning machines would require nonlinear dynamics [38]. It is well known (see Refs. [65, 66, 67]) that non-linear dynamics might lead to violation of the signaling constraint. So it is not surprising that the nonlinear dynamics allows for perfect cloning of unknown states, since this process again leads to the violation of the signaling constraint.

Very similar argument as the one presented by Wootters and Zurek [38] has been presented independently by Dieks [37]. Yet another version of the no-cloning theorem is due to Yuen [78] who pointed out a relation between cloning cloning and generalized measurements.

The no-cloning theorem has been generalized (the so-called no-broadcasting theorem) for statistical mixtures by Barnum *et al.* [79]. The extension of the no-cloning theorem for entangled systems has been presented by Koashi and Imoto [80]. Lindblad [81] has recently presented the most general version of the no-cloning theorem.

3.3 Mandel's analysis

The no-cloning theorem says us that unknown states cannot be *perfectly* cloned. But what is the limit on cloning? How good the copies can be made? If the imperfection is small enough then Herbert's FLASH can still be used for superluminal communication. The Wootters-Zurek theorem itself does not rule out a possibility of signaling due to *imperfect* cloning. Interestingly enough, this crucial question for the FLASH has not attracted due attention. To be more specific, in his short note Mandel [82] was the first to study an unavoidable noise due to amplification (cloning) of a polarization state of a photon via interaction with a two-level atom two-level atom. In particular, Mandel has considered an incoming photon with two possible polarizations $\vec{\varepsilon}_1$ and $\vec{\varepsilon}_2$. The state of a single incoming photon is denoted as $|1_{\vec{\varepsilon}_s}\rangle$ (with $s = 1, 2$). The atomic amplifier amplifier in the ideal case generates out of the incoming photon two photons with the same polarization, i.e. $|1_{\vec{\varepsilon}_s}\rangle \rightarrow |2_{\vec{\varepsilon}_s}\rangle$. So, following Mandel [82], let us consider a two-level atom with the transition dipole moment $\vec{\mu}$. The atom is initially prepared in the excited state $|e\rangle$. The interaction Hamiltonian Hamiltonian governing the interaction between the atom and an electromagnetic field is considered in the electric-dipole and the rotating-wave approximation, i.e.

$$H_I = g \sum_{s=1}^{2} \left[\vec{\mu} \cdot \vec{\varepsilon}_s^{\,*} \, \sigma^- a_s^\dagger + h.c. \right] , \tag{10}$$

[7] It has to be stressed that the linearity argument based on which the no-cloning theorem is proved does not forbid the amplification of *known* states. That is a cloning device designed specifically for a given input state can be constructed without any violation of the no-signaling constraint.

where $\sigma^{\pm}$ are Pauli spin-flip operators and $a_s^\dagger$ (a_s) is the creation (annihilation) operator of a photon with a polarization $\vec{\varepsilon}_s$. If we consider the initial state of the electromagnetic field to be $|1_{\vec{\varepsilon}_1}, 0_{\vec{\varepsilon}_2}\rangle$ then dynamics governed by the Hamiltonian (10) forces the field to evolve in a short-time limit into the state

$$|1_{\vec{\varepsilon}_1}\rangle|0_{\vec{\varepsilon}_2}\rangle \longrightarrow \frac{\sqrt{2}\vec{\mu}\cdot\vec{\varepsilon}_1^{\,*}|2_{\vec{\varepsilon}_1}, 0_{\vec{\varepsilon}_2}\rangle + \vec{\mu}\cdot\vec{\varepsilon}_2^{\,*}|1_{\vec{\varepsilon}_1}, 1_{\vec{\varepsilon}_2}\rangle}{\left(2|\vec{\mu}\cdot\vec{\varepsilon}_1^{\,*}|^2 + |\vec{\mu}\cdot\vec{\varepsilon}_2^{\,*}|^2\right)^{1/2}} . \qquad (11)$$

The state $|2_{\vec{\varepsilon}_1}, 0_{\vec{\varepsilon}_2}\rangle$ attributes to the stimulated emission while $|1_{\vec{\varepsilon}_1}, 1_{\vec{\varepsilon}_2}\rangle$ describes a spontaneous emission. So the perfect cloning cloning is accompanied with a noise due to spontaneous emission.

The amplifier amplifier (10) is designed for cloning of a specific input state. Mandel has also presented a generalization of the amplifier amplifier such that it clones an arbitrary polarization state with the same fidelity fidelity (i.e. this is a prototype of the universal cloning machine - see Section 4). This input-state independent amplifier amplifier consists of two resonant atoms with orthogonal transition dipole moments $\vec{\mu}_a = |\mu|\vec{\varepsilon}_a$; $\vec{\mu}_b = |\mu|\vec{\varepsilon}_b$, where $\vec{\varepsilon}_a$ and $\vec{\varepsilon}_b$ are two complex orthogonal unit polarization vectors. The interaction Hamiltonian Hamiltonian between the atoms and the electromagnetic field is taken in the form

$$H_I = g\sum_{s=1}^{2}\left[\left(\sigma_a^-\,\vec{\mu}_a + \sigma_b^-\,\vec{\mu}_b\right)\cdot\vec{\varepsilon}_s^{\,*}\,a_s^\dagger + h.c.\right] . \qquad (12)$$

If the two atoms are initially excited then in a short-time limit the dynamics (12) generates the electromagnetic field in a state described by a density operator (the atomic degrees of freedom are traced-off):

$$\rho_{\vec{\varepsilon}_1,\vec{\varepsilon}_1} = \frac{2}{3}|2_{\vec{\varepsilon}_1}, 0_{\vec{\varepsilon}_2}\rangle\langle 2_{\vec{\varepsilon}_1}, 0_{\vec{\varepsilon}_2}| + \frac{1}{3}|1_{\vec{\varepsilon}_1}, 1_{\vec{\varepsilon}_2}\rangle\langle 1_{\vec{\varepsilon}_1}, 1_{\vec{\varepsilon}_2}| \qquad (13)$$

As we will see later the fidelity fidelity of the cloning process (12) does not depend on the polarization of the incident photon and of two atomic transition dipole moments (providing these are orthogonal). So out of a state $|1_{\vec{\varepsilon}_1}, 0_{\vec{\varepsilon}_2}\rangle$ with an *arbitrary* polarization we obtain the two-photon state $|2_{\vec{\varepsilon}_1}, 0_{\vec{\varepsilon}_2}\rangle$ with a probability $2/3$ which does not depend on the polarization. In addition to this a noise, represented by the state $|1_{\vec{\varepsilon}_1}, 1_{\vec{\varepsilon}_2}\rangle$ is generated in this process. I will turn to this result of Mandel later [8]. Here, following Wootters and Zurek I note that to each "perfect clone" there is also one randomly polarized, spontaneously

[8]Mandel in his paper [82] refers to the Wootters-Zurek argument as to ingeniously simple. We might like to say that Mandel's ingenious insight into the physics of amplification lead him to a correct answer to the question raised many years later.

emitted photon. But still the question is not answered whether this spontaneous emission definitely rules out a possibility of superluminal signaling.

In order to have a clear answer to this question I present in Section 4 the optimal universal quantum cloner which introduces the smallest amount of noise during the cloning procedure and will show you that this cloner cannot be utilized for the FLASH.

Concluding this section I note that the recent proposal for experimental realization of the universal cloning machine presented by Simon *et al.* [54, 55], as well as an experimental demonstration of the quantum cloning machine due to Lamas-Linares *et al.* [56] and De Martini *et al.* [57, 58], share the basic features of the original Mandel's idea. I will describe in a more detail in Section 8 the experiment by De Martini *et al.* [59] in which universal NOT gate and quantum cloning has been realized.

4. Universal Quantum Cloners

Within classical physics we can imagine "machines" which take as an input a classical physical object in a state which corresponds to a classical information encoded in the system and perform an arbitrary operation prescribed by a specific map (transformation). After the transformation the output is obtained which corresponds to a result of a classical information processing. The fidelity fidelity of such a process in classical physics for an *arbitrary* unknown input can in principle always be equal to unity.

Quantum mechanics offers new perspectives in information processing [84], which in part is due to the fact, that quantum information can be represented by *qubits* which are two-level quantum systems with one level labeled $|0\rangle$ and the other $|1\rangle$. Qubits can not only be in one of the two levels, but in any superposition of them as well. This fact makes the properties of quantum information quite different from those of its classical counterpart. A typical example is the quantum cloner.

The impossibility of copying (cloning) quantum information puts fundamental limits on amount of information extractable from finite ensembles of identically prepared quantum systems [42]. Or *vice versa*, since finite ensembles do not allow for a complete determination of states [73,74,85-89], unknown quantum states cannot be copied perfectly.

So let us assume quantum machines which take as an input a qubit in an unknown state and generate at the output qubits in a state according to a specific prescription, e.g. one would like to have copies of the original. Important feature of the quantum cloning cloning machine is that it generates an output with the fidelity fidelity which does not depend on the input. Obviously, if the input state is known (i.e. complete classical information about the preparation of the given state is available) then an arbitrary transformation can be performed with the fidelity fidelity equal to unity. If the state is unknown then the transformation cannot be performed perfectly. In this case it is desirable that all states are transformed equally well (with the fidelity independent on a particular input state). This covariance covariance property of quantum machines with respect to unitary transformations of inputs makes the machines *universal.*

4.1 Quantum cloning $1 \longrightarrow 2$

Let $\mathcal{H} = \mathbf{C}^2$ denote a two-dimensional Hilbert space of a single qubit. The action of the cloning machine is equivalent to a a *completely positive trace preserving map* $\mathcal{T} : \mathcal{H} \rightarrow \mathcal{H}$ The machine is design so, that for any pure one-particle state ρ at the input, the output $\mathcal{T}(\rho)$ is as close as possible to an input state.

In order to derive the cloning transformation $\mathcal{T}$ we have to specify the constraints which have to be met:

(i) The state of the original system and its quantum copy at the output of the quantum copier, described by density operators $\rho_a^{(out)}$ and $\rho_b^{(out)}$, respectively, are identical, i.e.,

$$\rho_a^{(out)} = \rho_b^{(out)} . \tag{14}$$

(ii) If no *a priori* information about the *in*-state of the original system is available, then it is reasonable to require that *all* pure states should be copied equally well. One way to implement this assumption is to design a quantum copier such that the distances between density operators of each system at the output ($\rho_x^{(out)}$ where $x = a, b$) and the ideal density operator $\rho^{(id)}$ which describes the *in*-state of the original mode are input-state independent. Quantitatively this means that if we employ a fidelity fidelity

$$\mathcal{F} = \mathrm{Tr}\left[\rho^{(out)}\, \rho^{(id)}\right] \tag{15}$$

as a measure of distance between two operators, then the quantum copier should be such that $\mathcal{F} = const$ for all possible input states.

(iii) Finally, we would also like to require that the copies are as close as possible to the ideal output state, which is, of course, just the input state. This means that we want our quantum copying transformation to minimize the distance between the output state $\rho_x^{(out)}$ of the copied qubit and the ideal state $\rho_x^{(id)}$. The distance is minimized, that is the fidelity fidelity $\mathcal{F}$ is maximized, with respect to all possible unitary transformations $\mathcal{U}$ acting on the Hilbert space $\mathcal{H}$ of two qubits and the quantum copying machine (i.e., $\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b \otimes \mathcal{H}_c$)

$$\mathcal{F}(\rho_x^{(out)}; \rho_x^{(id)}) = \max\left\{\mathcal{F}^{(\mathcal{U})}(\rho_x^{(out)}; \rho_x^{(id)}); \forall \mathcal{U}\right\} , \tag{16}$$

where $x = a, b$.

It has been shown recently [39] that in the case of $1 \longrightarrow 2$ cloning the machine can be represented as a third qubit c via which the information from a is transfered to the qubit b. The cloning cloning transformation itself can be expressed in a covariant form as

$$|\psi\rangle_a|\Xi\rangle_{bc} \longrightarrow \sqrt{\tfrac{2}{3}}|\psi\rangle_a|\psi\rangle_b|\psi^\perp\rangle_c - \tfrac{1}{\sqrt{3}}|\{\psi^\perp,\psi\}\rangle_{ab}|\psi\rangle_c, \tag{17}$$

where $|\{\psi^\perp,\psi\}\rangle_{ab} = (|\psi\rangle_a|\psi^\perp\rangle_b + |\psi^\perp\rangle_a|\psi\rangle_b)/\sqrt{2}$ is a symmetrically entangled state of the two qubits which are in the states $|\psi\rangle$ and $|\psi^\perp\rangle$, respectively. The copy and the cloner qubits at the input are always prepared in the state $|\Xi\rangle_{bc}$ which I do not specify here. The two qubits a and b at the output of this cloner are both in the same state

$$\rho_x^{(out)} = \frac{2}{3}\rho + \frac{1}{6}\mathbf{1}; \qquad x = a, b. \tag{18}$$

The scaling factor $s = 2/3$ corresponds to the fidelity fidelity of the cloning equal to $\mathcal{F} = 5/6$ which is much higher than the fidelity fidelity of estimation of the state which is equal to $\mathcal{F} = 2/3$ (for more details see Section 8).

There exists a simple logical network [41] which realizes the transformation (17). This network is composed of four control-NOT gates via which the information from the original qubit is transfered to the qubit b. It is interesting to note that depending on a specific preparation of the qubits b and c we can control the flow of information in the network: we can even swap the roles of the output qubits or perform an asymmetric cloning, etc. The proof has been presented by Gisin and Massar [42], Bruß *et al.* [43], and Werner [46], that the transformation (17) is indeed optimal, in a sense that it generates best clones under the constrains (14)-(16).

4.2 Quantum cloning $N \longrightarrow N + M$

Let us now suppose that we have N qubits all prepared in the same state ψ at the input of the cloner, but we want to generate $N + M$ clones at the output under the same conditions as discussed above.

In order to perform $N \longrightarrow N + M$ cloning we need M "blank" qubits (we label them with the subscript b) and M additional qubits which play the role of the cloner (c) and are used for the transfer of information. These $2M$ qubits are alway prepared in the same state $|\Xi\rangle_{bc}$. The universal optimal cloning transformation for every vector $|\psi\rangle \in \mathcal{H}$ can be expressed as [42, 45]

$$U_{NM}|N\psi\rangle_a \otimes |\Xi\rangle_{bc} = \sum_{j=0}^{M} \gamma_j^{(N,M)} |\Xi_j(\psi)\rangle_{ab} \otimes |\{(M-j)\psi^\perp; j\psi\}\rangle_c \tag{19}$$

with

$$\gamma_j^{(N,M)} = (-1)^j \binom{N+M-j}{N}^{1/2} \binom{N+M+1}{M}^{-1/2}, \tag{20}$$

where $|N\psi\rangle_a = |\psi\rangle^{\otimes N}$ is the input state consisting of N qubits in the same state $|\psi\rangle$. On the right hand side of Eq.(19) $|\{(M-j)\psi^{\perp}; j\psi\}\rangle_c$ denotes symmetric and normalized states with $(M-j)$ qubits in the complemented (orthogonal) state $|\psi^{\perp}\rangle$ and j qubits in the original state $|\psi\rangle$. Similarly, the vectors $|\Xi_j(\psi)\rangle_{ab}$ consist of $N+M$ qubits, and are given explicitly by

$$|\Xi_j(\psi)\rangle_{ab} = |\{(N+M-j)\psi; j\psi^{\perp}\}\rangle_{ab}. \tag{21}$$

Let us note that with this choice of the coefficients $\gamma_j^{(M,N)}$, the scalar product of the right hand side with a similar vector, with ψ replaced by ϕ, becomes $\langle\psi,\phi\rangle^N$. This is consistent with the unitarity of the operator U_{NM}.

The $N+M$ qubits in the systems a and b at the output of the gate are individually in the state described by the density operator

$$\rho_j^{(out)} = s\rho + \frac{1-s}{2}\mathbf{1}, \qquad j = 1, \ldots, N+M\,, \tag{22}$$

with the scaling factor

$$s = \frac{N}{N+2} + \frac{2N}{(N+M)(N+2)}, \tag{23}$$

i.e. these qubits are the *clones* of the original state with a fidelity fidelity of cloning larger than the fidelity of estimation (see Sec. 6). This fidelity depends on the number, M, of clones produced out of the N originals, and in the limit $M \to \infty$ the fidelity of cloning becomes equal to the fidelity of estimation. These qubits represent the output of the *optimal* $N \to N+M$ cloner introduced by Gisin and Massar [42].

So this is the optimal universal quantum cloning process [42, 46] which is possible within the linear quantum mechanics. Now we have to answer the question whether the minimal noise represented by the term $\frac{1-s}{2}\mathbf{1}$ in the expression for a single-qubit density operators at the out-put of the cloner, is enough to preserve the no-signaling condition.

5. No-signaling & Linearity of Quantum Mechanics

In this section I will show how the no-signaling condition determines bounds on possible dynamics of a physical system [65]. Before I proceeding I want to note that there is a long-standing discussion on the relation between the no-signaling constraint and the linearity of quantum mechanics [65-71].In what follows I will not study subtle details of this relation (I refer those who are interested in the details of the problem to Ref. [71]). The only argument from all these discussions I am going to use for the present purpose is that I will associate the no-signaling condition with the linearity of quantum mechanics. I will adopt the usual quantum kinematics. The possible states of the system are described by one-dimensional projectors $P_\psi = |\psi\rangle\langle\psi|$ in a Hilbert space. The projectors in this Hilbert space are the measurable physical quantities (taking only values 0 and 1).

The time evolution of the system is described by a map $\mathcal{G}$ on the set of pure states:

$$\mathcal{G} : P_\psi \rightarrow \mathcal{G}(P_\psi)\,. \tag{24}$$

Consequently, a mixture $\{P_{\psi_i}, x_i\}$ of states P_{ψ_i} with weights x_i evolves into a mixture of states $\mathcal{G}(P_{\psi_i})$ with the same weights x_i. Therefore the corresponding density operators evolve as follows:

$$\sum_i x_i P_{\psi_i} \rightarrow \sum_i x_i \mathcal{G}(P_{\psi_i}) \tag{25}$$

Now consider two mixtures $\{P_{\psi_i}, x_i\}$ and $\{P_{\phi_j}, y_j\}$ such that

$$\sum_i x_i P_{\psi_i} = \sum_j y_j P_{\phi_j} = \rho_0, \tag{26}$$

i.e., they correspond to the same density operator. Assume that

$$\sum_i x_i \mathcal{G}(P_{\psi_i}) \neq \sum_j y_j \mathcal{G}(P_{\phi_j}), \tag{27}$$

then the two mixtures can be distinguished after a finite time.

But, as shown in Ref. [65] and as I will show below, any two mixtures corresponding to the same density operator can be (instantaneously) prepared at a distance making use of an appropriate entangled state. entangled states As stated above, I assume the usual quantum kinematics, which in particular implies existence of entangled states. The assumption (27) thus contradicts the requirement that there should be no superluminal communication.

Under the assumption that relativity is correct, this implies that the time development of the system can only depend on the initial density matrix density matrix ρ_0 (and not on the specific mixture):

$$\mathcal{G} : \rho_0 \rightarrow \mathcal{G}(\rho_0) \tag{28}$$

where

$$\mathcal{G}(\rho_0) = \mathcal{G}(\sum_i x_i P_{\psi_i}) = \sum_i x_i \mathcal{G}(P_{\psi_i}) \tag{29}$$

for every mixture having a density operator ρ_0.

Let us notice that the assumption (27) would mean that the density operator formalism is not appropriate for determining the time evolution of mixtures, but it could remain useful for the computation of expectation values of observables. That any two mixtures corresponding to the same density operator can be prepared at a distance can be shown as follows: The equation (29) is based on two facts: (1) the time evolution depends only on the density operator; (2) the time evolution doesn't mix composite states of the mixture.

5.1 Bloch vectors and no-signaling

Let us consider a process in which a single particle input state is mapped into a two particle output state. The input state can be represented as

$$\rho^{(in)}(\vec{m}) = \frac{1}{2}(\mathbf{1} + \vec{m}\cdot\vec{\sigma}) \equiv |\vec{m}\rangle\langle\vec{m}|, \tag{30}$$

where $\vec{m}$ is a real vector whose length is less than or equal to unity. The most general two-particle output state, which is hermitian and has a trace equal to one, can be expressed in the basis of matrices $\{\mathbf{1}\otimes\mathbf{1},\ \mathbf{1}\otimes\sigma_i,\ \sigma_i\otimes\mathbf{1},\ \sigma_i\otimes\sigma_k\}$ as

$$\begin{aligned}\rho_{ab}^{(out)}(\vec{m}) &= \frac{1}{4}[\mathbf{1}\times\mathbf{1} + \vec{\eta}_1\cdot\vec{\sigma}\otimes\mathbf{1} + \mathbf{1}\otimes\vec{\eta}_2\cdot\vec{\sigma} \\ &+ \sum_{j,k=x,y,z} t_{jk}\sigma_j\otimes\sigma_k],\end{aligned} \tag{31}$$

where $\vec{\eta}_1$, $\vec{\eta}_2$, and t_{jk} are functions of $\vec{m}$. The requirement that the reduced density matrices of the two output particles be the same, which I shall impose, implies that $\vec{\eta}_1 = \vec{\eta}_2$.

Let us now impose the requirement of covariance. covariance This means that if $\rho_a^{(in)}(\vec{m})$ is mapped onto $\rho_{ab}^{(out)}(\vec{m})$, and if U is a matrix in $SU(2)$, then the input state $U\rho_a^{(in)}(\vec{m})U^{-1}$ will be mapped onto the output state $U\otimes U\rho_{ab}^{(out)}(\vec{m})U^{-1}\otimes U^{-1}$. Another way of stating this condition is obtained by noting that if we express U as

$$U = \exp(-i\theta\vec{e}\cdot\vec{\sigma}/2), \tag{32}$$

where $\vec{e}$ is a unit vector corresponding to the rotation axis and θ is the rotation angle, then

$$U\,(\vec{m}\cdot\vec{\sigma})\,U^{-1} = \vec{m}'\cdot\vec{\sigma}, \tag{33}$$

where $\vec{m}' = R(\vec{e},\theta)\vec{m}$. The rotation matrix, $R(\vec{e},\theta)$, is the 3×3 matrix which rotates a vector about the axis $\vec{e}$ by an angle θ, and it is given explicitly by

$$R(\vec{e},\theta) = \exp(\theta\vec{e}\cdot\vec{K}), \tag{34}$$

where

$$K_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$K_y = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \tag{35}$$
$$K_z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

We have that

$$U\rho_a^{(in)}(\vec{m})U^{-1} = \rho_a^{(in)}(R\vec{m}), \tag{36}$$

which will be mapped onto $\rho_{ab}^{(out)}(R\vec{m})$, so that the covariance covariance condition can now be expressed as

$$\rho_{ab}^{(out)}(R\vec{m}) = U \otimes U\rho_{ab}^{(out)}(\vec{m})\,U^{-1} \otimes U^{-1}. \tag{37}$$

Now let us examine the consequences of this relation: Let us first consider the terms linear in $\vec{\sigma}$ and let R be a rotation about $\vec{m}$ by a very small angle θ. We have that

$$\vec{\eta}_1(R\vec{m}) = R\,\vec{\eta}_1(\vec{m}), \tag{38}$$

which for our choice of rotation becomes

$$\vec{\eta}_1(\vec{m}) = (\mathbf{1} + \theta\,\hat{m}\cdot\vec{K})\vec{\eta}_1(\vec{m}), \tag{39}$$

or

$$\vec{e}_{\vec{m}} \cdot \vec{K}\vec{\eta}_1(\vec{m}) = 0, \tag{40}$$

where $\vec{e}_{\vec{m}}$ is a unit vector in the direction of $\vec{m}$. This implies that $\vec{e}_{\vec{m}} \times \vec{\eta}_1(\vec{m}) = 0$, so that $\vec{\eta}_1(\vec{m})$ is parallel to $\vec{m}$, and we can write $\vec{\eta}_1(\vec{m}) = \eta_1(\vec{m})\,\vec{m}$. If we now substitute this result back into Eq. (38) and consider a general rotation R, we have that

$$\eta_1(R\vec{m}) = \eta_1(\vec{m}). \tag{41}$$

This implies that $\eta_1(\vec{m})$ is a constant, which we shall denote by η_1. Analogous arguments lead to a conclusion that $\vec{\eta}_2 = \eta_2\,\vec{m}$.

Now let us see what covariance covariance implies about the terms quadratic in $\vec{\sigma}$. Application of the covariance condition, Eq. (37), to these terms gives

$$t_{jk}(R\vec{m}) = \sum_{j',k'} R_{jj'}R_{kk'}t_{j'k'}(\vec{m}). \tag{42}$$

If we again choose R to be a rotation about $\vec{m}$ by a small angle θ, we find the condition

$$0 = \sum_{j'} (\vec{e}_{\vec{m}} \cdot \vec{K})_{jj'} t_{j'k}(\vec{m}) + \sum_{k'} (\vec{e}_{\vec{m}} \cdot \vec{K})_{kk'} t_{jk'}(\vec{m}). \tag{43}$$

Since we consider universal machines we can assume without a loss of generality, that $\vec{m}$ be in the z direction, in particular, $\vec{m} = \vec{e}_z$. In this case we find that $t_{xx} = t_{yy}$, $t_{xy} = -t_{yx}$, and $t_{xz} = t_{zx} = t_{yz} = t_{zy} = 0$, where all of these are evaluated at $\vec{m} = \vec{e}_z$. We now want to impose the no signaling condition

$$\rho_{ab}^{(out)}(\vec{e}_z) + \rho_{ab}^{(out)}(-\vec{e}_z) = \rho_{ab}^{(out)}(\vec{e}_x) + \rho_{ab}^{(out)}(-\vec{e}_x), \tag{44}$$

and to do so we need to find all of the density matrixes in the above equation in terms of $t_{jk}(\vec{e}_z)$. This can be done by applying the covariance covariance condition, Eq. (37), to $\rho_{ab}^{(out)}(\vec{e}_z)$ and making the proper choice of R. When these results are substituted into Eq. (44) we find that $t_{xx}(\vec{e}_z) = t_{yy}(\vec{e}_z) = t_{zz}(\vec{e}_z)$, and I shall designate this common value by t. We then can rewrite the general expression for the two-qubit density operator satisfying the covariance covariance condition (37) as (here we again use the general orientation of the vector $\vec{m}$)

$$\rho_{ab}(\vec{m}) = \frac{1}{4}\left(\mathbf{1}\otimes\mathbf{1} + \eta_1 \vec{m}\vec{\sigma}\otimes\mathbf{1} + \eta_2 \mathbf{1}\otimes\vec{m}\vec{\sigma} + t\,\vec{\sigma}\otimes\vec{\sigma} + t_{xy}\vec{m}[\vec{\sigma}\times\vec{\sigma}]\right) \tag{45}$$

where $\eta_1, \eta_2, t, t_{xy}$ are real parameters. In order for $\rho(\vec{m})$ to be a physical density matrix, density matrix its eigenvalues have to be non-negative. A simple calculation shows that this implies constraints

$$\begin{gathered} 1 + t \pm (\eta_1 + \eta_2) \\ 0 \\ 1 - t \pm \sqrt{4t^2 + 4t_{xy}^2 + (\eta_1 - \eta_2)^2} \\ 0 \end{gathered} \tag{46}$$

In what follows I will apply the no-signaling constraint (46) to the universal quantum cloners of Section (4).

5.2 Bounds on cloning due to signaling

In the case of the $1 \longrightarrow 2$ cloning, cloning the task is to optimize the fidelity fidelity $\mathcal{F} = \mathrm{Tr}[\rho_{ab}^{(out)}(\vec{m})\rho_a^{(in)} \otimes \mathbf{1}] = \mathrm{Tr}[\rho_{ab}^{(out)}(\vec{m})\ P_{\vec{m}} \otimes \mathbf{1}]$, where $P_{\vec{m}} = |+\vec{m}\rangle\langle+\vec{m}|$, assuming $\eta_1 = \eta_2 \equiv \eta$. A simple solution of Eqs. (46) then leads to the optimal values $t_{xy} = 0, t = 1/3, \eta = 2/3$, for which $\mathcal{F} = \frac{5}{6}$, which corresponds to a single-qubit density operator at the output given by Eq. (18). Note that this also optimizes the fidelity describing generation of two clones, i.e. $\mathrm{Tr}[\rho_{ab}^{(out)}(\vec{m})\ P_{\vec{m}} \otimes P_{\vec{m}}] = \frac{2}{3}$ These are exactly the bounds that are valid in quantum mechanics as discussed in Section 4 and analyzed by Werner [46]. Therefore I can conclude that the *optimal* universal quantum cloning one cannot violate the no-signaling constraint. It is exactly the noise [represented by the term $1/6$ in Eq. (18)] induced by cloning procedure which prevents the FLASH to transmit signals superluminally.

One may wonder whether there still might be a possibility, that generating more than two clones out of an incoming qubit one would be able to operate the FLASH. In order to rule out this possibility one can check that the bound on $1 \longrightarrow 1 + M$ cloning imposed by the no-signaling (linearity) constraint is satisfied by the optimal universal cloners.

6. Cloning, Signaling & POVM

To understand the connection between the optimal universal cloning and the no-signaling constraint let us turn our attention to a problem of optimal estimation of states of quantum systems. Let us consider a finite ensemble of N qubits all prepared in the same pure state $|\psi\rangle$. If the state is totally unknown, i.e. we have no *a priori* information about its preparation, then we have to assume that all pure state are equally probable. This corresponds to a uniform probability distribution on a state space of a given system, i.e. in the case of qubits - the Bloch sphere (see Fig. 1).

It is well known [73, 74, 85, 86, 87, 88] (for a review see Ref. [89]) that there exists an optimal measurement of the finite set of N qubits via which the *best* possible estimation of the state $|\psi\rangle$ can be performed. Holevo [73] has shown that it is possible to realize the best estimation via the so-called *covariant* measurement, which is a continuous POVM measurement performed on the whole finite ensemble. Obviously, in this case the problem is, that physically it is difficult to perform experimentally a measurement with a continuous number of observables. Later it has been shown by Massar and Popescu [85] and Derka *et al.* [86], that the optimal measurement on a finite ensemble of qubits can be realized via a finite-dimensional POVM. Such POVM can be realized when we imagine projective measurements performed on the whole set of N qubits (that is the qubits are not measured sequentially, but simultaneously, in one "shot"). Once this optimal measurement is performed then the best possible estimation

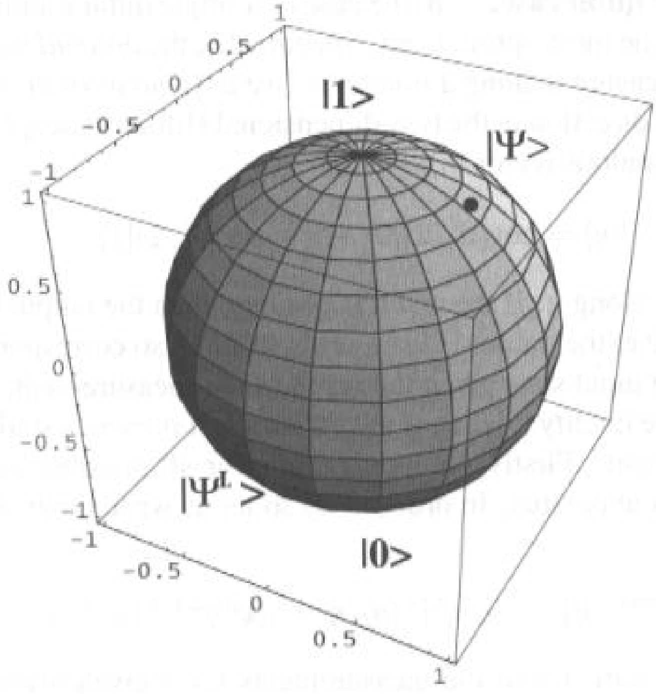

Figure 1. The state space of a qubit is a Bloch sphere. Pure states $|\psi\rangle$ are represented by points on the sphere, while statistical mixtures are points inside the sphere. The state $|\psi^{\perp}\rangle$ that is orthogonal to $\psi\rangle$ is its antipode.

of the measured state can be expressed in a form of the density operator

$$\rho^{(est)} = s_N \rho + \frac{1 - s_N}{2} \mathbb{1}, \tag{47}$$

where the "scaling" factor s_N is given by the expression

$$s_N = \frac{N}{N+2}, \tag{48}$$

and is directly related to the mean fidelity

$$\overline{\mathcal{F}} = \int d\Omega_\rho \langle\psi|\rho^{(est)}|\psi\rangle, \tag{49}$$

where the integration is performed over all input states ρ and $d\Omega_\rho = \sin\vartheta d\vartheta\, d\varphi/4\pi$ is the integration measure associated with the state space , i.e. the Bloch sphere. When we insert $\rho^{(est)}$ given by Eq.(47) into Eq.(49) we find

$$\overline{\mathcal{F}} = s_N + \frac{1 - s_N}{2} = \frac{N+1}{N+2}. \tag{50}$$

There does not exist a measurement which would give us more information than the one POVM just considered (for more details see the review article [89]).

6.0.1 Single qubit case. In the case of a single qubit a simple projective measurement is the most optimal one. Specifically, the *optimal* way to estimate the state, is to measure it along a *randomly* (we have no prior knowledge about the state) chosen direction in the two-dimensional Hilbert space[73, 85, 86]. So let us choose a random vector $|\eta\rangle$, where

$$|\eta\rangle = \cos(\vartheta'/2)|0\rangle + e^{i\varphi'}\sin(\vartheta'/2)|1\rangle, \tag{51}$$

and measure $|\psi\rangle$ along it. If the result is positive, then the output is taken to be $|\eta\rangle$, and if negative, the output is $|\eta^{\perp}\rangle$. This would also correspond to our best estimation of the input state given the result of the measurement.

To evaluate the fidelity fidelity of the estimation I present a statistical picture of the measurement. Firstly, let us average over all possible orientations of the measurement apparatus. In order to do so let us write down a single-qubit density operator

$$\rho^{(meas)}(\eta) = |\langle\psi|\eta\rangle|^2\,|\eta\rangle\langle\eta| + |\langle\psi|\eta^{\perp}\rangle|^2\,|\eta^{\perp}\rangle\langle\eta^{\perp}|\,. \tag{52}$$

which describes statistics of the measurements for a given orientation of the measurement apparatus. To get the final output density matrix density matrix we average (52) over all possible choices of the measurement (i.e. over all vectors $|\eta\rangle$)

$$\rho^{(est)} = \int d\,\Omega_\eta \rho^{(meas)}(\eta). \tag{53}$$

where $d\,\Omega_\eta = \frac{1}{4\pi}\sin\vartheta' d\vartheta'\, d\varphi'$ is the integration measure on the state space of the "measurement" apparatus. After the integration is performed we find

$$\rho^{(est)} = s\rho + \frac{1-s}{2}\mathbf{1}, \tag{54}$$

where for a single input qubit we have $s = 1/3$ and $\rho = |\psi\rangle\langle\psi|$.

In order to find the mean fidelity fidelity of the estimation itself we have to average the fidelity, i.e. $\langle\psi|\rho^{(est)}|\psi\rangle$ over all possible preparations, i.e.

$$\overline{\mathcal{F}} = \int d\,\Omega_\rho \langle\psi|\rho^{(out)}|\psi\rangle = \frac{2}{3}. \tag{55}$$

Obviously, instead of the projective measurement one can consider some other optimal generalized measurement to be performed on the input qubit. We can even consider a continuous POVM. Nevertheless, since in the given case the projective measurement, described above, is the optimal one, no other measurement can give us more information about the input state $|\psi\rangle$.

Now it is clear that the quantum cloning cloning can be represented as a specific generalized POVM measurement. It is a particular physical realization of the Naimark theorem [73] - the information contained in the original qubit (i.e. the state $|\psi\rangle$) is spread in between many clones. But when the optimal measurement on these clones is performed [44] the mean fidelity fidelity of the estimation is again equal to 2/3. In other words we cannot generate information via cloning and therefore we cannot violate the no-signaling constraint. This is the reason why the FLASH does not work. The argument can be generalized when the optimal $N \to N+M$ cloning cloning is considered [42, 46] Information about the input qubit(s) cannot be "generated". It only can be redistributed [94]. This is in perfect accordance with the no-signaling constraint.

7. Flipping Qubits: Universal NOT Gate

As follows from our previous discussion, quantum cloning can be considered as a form of a redistribution of quantum information from a set of incoming qubits to a large set of output qubits. In the cloning the main task has been to generate a copy (copies) of the original input qubit which are as close as possible to the input. But we can also assume other tasks, such as spin-flipping of unknown qubits.

One of the most striking difference between the classical and quantum information is as follows: it is not a problem to flip a classical bit, i.e. to change the value of a bit, a 0 to a 1 and vice versa. This is accomplished by a NOT gate. Flipping a qubit, however, is another matter: there exists the fundamental bound which prohibits to flip a qubit prepared in an *arbitrary* state $|\psi\rangle = \alpha|0\rangle + \beta|0\rangle$ and to obtain the state $|\psi^\perp\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$ which is *orthogonal* to it, i.e. $\langle\psi^\perp|\psi\rangle = 0$.

Let us assume the Bloch sphere which represents a state space of a qubit. The points corresponding to $|\psi\rangle$ and $|\psi^\perp\rangle$ are antipodes of each other. The desired spin-flip operation is therefore the *inversion of the Bloch sphere* (see Fig. 1).

It is well known that this inversion preserves angles (which is related to the scalar product $|\langle\phi,\psi\rangle|$ of rays). Therefore, by the arguments of the Wigner theorem the ideal spin-flip operation must be implemented either by a unitary or by an anti-unitary operation. Unitary operations correspond to proper rotations of the Bloch sphere, whereas anti-unitary operations correspond to orthogonal transformations with determinant -1. The spin-flip is an anti-unitary operation, i.e. it is not completely positive.

Due to the fact that the tensor product of an anti-linear and a linear operator is not correctly defined the spin-flip operation cannot be applied to a qubit while the rest of the world is governed by unitary evolution. On the other hand if we consider a spin-flip operation we should have in mind a universal NOT gate flipping an input qubit to its orthogonal state. The gate itself is an operation applied to the qubit, that is just a subsystem of a "whole universe". Therefore it must be represented a completely positive operation. It is well known that any completely positive operation on a qubit can be realized by a unitary operation performed on the qubit and the ancillary system. Following this arguments we see that the ideal *universal* NOT gate which would flip a qubit in an *arbitrary* state does not exist.

Obviously, if the state of the qubit is known, then we can always perform a flip operation. In this situation the classical and quantum operations share many similar features, since the knowledge of the state is a classical information, which can be manipulated according to the rules of classical information processing (e.g. known states can be copied, flipped etc). But, the universality of the operation is lost. That is, the gate which would flip the state $|0\rangle \to |1\rangle$, is not able to perform a flip $|(0\rangle + |1\rangle)/\sqrt{2} \to (|0\rangle - |1\rangle)/\sqrt{2}$.

Since it is not possible realize a perfect Universal-NOT gate which would flip an arbitrary qubit state, it is of interest to study, what is the best approximation to the perfect Universal-NOT gate. Here one can consider two possible scenarios. The first one is based on the measurement of input qubit(s) – using the results of an optimal measurement one can manufacture an orthogonal qubit, or any desired number of them. Obviously, the fidelity fidelity of the NOT operation in this case is equal to the fidelity of estimation of the state of the input qubit(s). The second scenario would be to approximate an anti-unitary transformation on a Hilbert space of the input qubit(s) by a unitary transformation on a larger Hilbert space which describes the input qubit(s) and ancillas.

It has been shown recently, that the best achievable fidelity fidelity of both flipping scenarios is the same [90, 91, 92, 93]. That is, the fidelity of the optimal Universal NOT gate is equal to the fidelity of the best state-estimation performed on input qubits [73, 85, 86] (one might say, that in order to flip a qubit we have to transform it into a bit). In what follows I briefly describe the unitary transformation realizing the quantum scenario for the spin-flip operation, that is, I present the optimal Universal NOT gate. Then I describe the recent experiment by De Martini *et al.* (see Ref. [59]) in which qubits encoded in polarization states of photons have been flipped.

7.1 Theoretical description of spin flipping

Let $\mathcal{H} = \mathbf{C}^2$ denote the two-dimensional Hilbert space of a single qubit Then the input state of N systems prepared in the pure state $|\psi\rangle$ is the N-fold tensor product $|\psi\rangle^{\otimes N} \in \mathcal{H}^{\otimes N}$. The corresponding density matrix density matrix is $\sigma \equiv \rho^{\otimes N}$, where $\rho = |\psi\rangle\langle\psi|$ is the one-particle density matrix. An important observation is that the vectors $|\psi\rangle^{\otimes N}$ are invariant under permutations of all N sites, i.e., they belong to the symmetric, or "Bose"-subspace $\mathcal{H}_+^{\otimes N} \subset \mathcal{H}^{\otimes N}$. Thus as long as we consider only pure input states we can assume all the input states of the device under consideration to be density operators on $\mathcal{H}_+^{\otimes N}$. I will denote by $\mathcal{S}(\mathcal{H})$ the density operators over a Hilbert space $\mathcal{H}$. Then the U-NOT gate must be a completely positive trace preserving map $T : \mathcal{S}\left(\mathcal{H}_+^{\otimes N}\right) \to \mathcal{S}(\mathcal{H})$. My aim is to design T in such a way that for any pure one-particle state $\rho \in \mathcal{S}(\mathcal{H})$ the output $T(\rho^{\otimes N})$ is as close as possible to the orthogonal qubit state $\rho^{\perp} = \mathbf{1} - \rho$. In other words, I am trying to make the fidelity fidelity $\mathcal{F} := \mathrm{Tr}[\rho^{\perp} T(\rho^{\otimes N})] = 1 - \Delta$ of the optimal complement with the result of the transformation T as close as possible to unity for an arbitrary input state. This corresponds to the problem of finding the minimal value of the error measure $\Delta(T)$ defined as

$$\Delta(T) = \max_{\rho,\mathrm{pure}} \mathrm{Tr}\left[\rho\, T(\rho^{\otimes N})\right]. \tag{56}$$

Note that this functional Δ is completely unbiased with respect to the choice of input state. More formally, it is invariant with respect to unitary rotations (basis changes) in $\mathcal{H}$: When T is any admissible map, and U is a unitary on $\mathcal{H}$, the map $T_U(\sigma) = U^* T(U^{\otimes N} \sigma U^{*\otimes N}) U$ is also admissible, and satisfies $\Delta(T_U) = \Delta(T)$. I will show you later on that one may look for optimal gates T, minimizing $\Delta(T)$, among the *universal* ones, i.e., the gates satisfying $T_U = T$ for all U. For such U-NOT gates, the maximization can be omitted from the definition (56), because the fidelity $\mathrm{Tr}\left[\rho\, T(\rho^{\otimes N})\right]$ is independent of ρ.

7.2 Measurement-based scenario

An estimation device (see also previous section) by definition takes an input state $\sigma \in \mathcal{S}(\mathcal{H}_+^{\otimes N})$ and produces, on every single experiment, an "estimated pure state" $\rho \in \mathcal{S}(\mathcal{H})$. As in any quantum measurement this will not always be the same ρ, even with the same input state ρ, but a random quantity. The estimation device is therefore described completely by the probability distribution of pure states it produces for every given input. Still simpler, I will characterize it by the corresponding probability density with respect to the unique normalized measure on the pure states (denoted "$d\phi$" in integrals), which is also invariant under unitary rotations. For an input state $\sigma \in \mathcal{S}(\mathcal{H}_+^{\otimes N})$, the value of this probability density at the pure state $|\phi\rangle$ is

$$p(\phi, \sigma) = (N+1)\langle \phi^{\otimes N}, \sigma\, \phi^{\otimes N}\rangle. \tag{57}$$

To check the normalization, note that $\int d\phi\, p(\phi,\sigma) = \mathrm{Tr}[X\sigma]$ for a suitable operator X, because the integral depends linearly on σ. By unitary invariance of the measure "$d\phi$" this operator commutes with all unitaries of the form $U^{\otimes N}$, and since these operators, restricted to $\mathcal{H}_+^{\otimes N}$ form an irreducible representation of the unitary group of $\mathcal{H}$ [for $d = 2$, it is just the spin spin $N/2$ irreducible representation of SU(2)], the operator X is a multiple of the identity. To determine the factor, one inserts $\sigma = 1$, and uses the normalization of "$d\phi$" to verify that $X = 1$.

Note that the density (57) is proportional to $|\langle\phi,\psi\rangle|^{2N}$, when $\sigma = |\psi^{\otimes N}\rangle\langle\psi^{\otimes N}|$ is the typical input to such a device: N systems prepared in the same pure state $|\psi\rangle$. In that case the probability density is clearly peaked sharply at states $|\phi\rangle$ which are equal to $|\psi\rangle$ up to a phase.

Suppose now that we combine the state estimation with the preparation of a new state, which is some function of the estimated state. The overall result will then be the integral of the state valued function with respect to the probability distribution just determined. In the case at hand the desired function is $f(\phi) = (\mathbf{1}-|\phi\rangle\langle\phi|)$. So the result of the whole measurement-based ("classical") scheme is

$$\rho^{(est)} = T(\sigma) = \int d\phi\, p(\phi,\sigma)\ (1-|\phi\rangle\langle\phi|)\,. \tag{58}$$

The fidelity fidelity required for the computation of Δ from Eq.(56) is then equal to (see also [85, 86])

$$\Delta = (N+1)\int d\phi\, |\langle\phi,\psi\rangle|^{2N}(1-|\langle\phi,\psi\rangle|^2) = \frac{1}{N+2}, \tag{59}$$

where I have used that the two integrals have exactly the same form (differing only in the choice of N), and that the first integral is just the normalization integral. Since this expression does not depend on ρ, we can drop the maximization in the definition (56) of Δ, and find $\Delta(T) = 1/(N+2)$, from which we find that the fidelity fidelity of creation of a complement to the original state ρ is

$$\mathcal{F} = \frac{N+1}{N+2}. \tag{60}$$

Finally I note, that the result of the operation (58) can be expressed in the form

$$\rho^{(out)} = s_N\rho^{\perp} + \frac{1-s_N}{2}\mathbf{1}, \tag{61}$$

with the "scaling" parameter $s_N = \frac{N}{N+2}$. From here it is seen that in the limit $N \to \infty$, perfect estimation of the input state can be performed, and, consequently, the perfect complement can be generated. For finite N the mean fidelity fidelity is always smaller than unity. The advantage of the measurement-based scenario is that once the input qubit(s) is measured and its state is estimated an arbitrary number M of identical (approximately) complemented qubits can be produced with the same fidelity, simply by replacing the output function $f(\phi) = (\mathbf{1}-|\phi\rangle\langle\phi|)$ by $f_M(\phi) = (\mathbf{1}-|\phi\rangle\langle\phi|)^{\otimes M}$.

7.3 Quantum scenario

In follows I will present a transformation which produces complements whose fidelity is the same as those produced by the measurement-based method. Assume we have N input qubits in an unknown state $|\psi\rangle$ and we are looking for a transformation which generates M qubits at the output in a state as close as possible to the orthogonal state $|\psi^{\perp}\rangle$. The universality of the proposed transformation has to guarantee that all input states are complemented with the same fidelity. If we want to generate M approximately complemented qubits at the output, the U-NOT gate has to be represented by $2M$ qubits (irrespective of the number, N, of input qubits), M of which serve as ancilla, and M of which become the output complements. Let us denote these subsystems by subscripts "a"=input, "b"=ancilla, and "c"=(prospective) output. The U-NOT gate transformation, U_{NM}, acts on the tensor product of all three systems. The gate is always prepared in some state $|\Xi\rangle_{bc}$, independently of the input state $|\psi\rangle^{\otimes N}$. Interestingly enough the optimal U-NOT gate is realized by the same transformation as the quantum cloning. cloning That is, the U-NOT is described by Eq. (19).

Each of the M qubits under consideration at the output of the U-NOT gate is described by the density operator (61) with $s_N = \frac{N}{N+2}$, *irrespective* of the number of complements produced. The fidelity fidelity of the U-NOT gate depends only on the number of inputs. This means that this U-NOT gate can be thought of as producing an approximate complement and then cloning it, with the quality of the cloning independent of the number of clones produced. The universality of the transformation is directly seen from the "scaled" form of the output operator (61).

Let us stress that the fidelity of the U-NOT gate (19) is exactly the same as in the measurement-based scenario. Moreover, it also behaves as a classical (measurement-based) gate in a sense that it can generate an arbitrary number of complements with the same fidelity. In fact, the transformation (19) represents the *optimal* U-NOT gate via quantum scenario. That is, the measurement-based and the quantum scenarios realize the U-NOT gate with the same fidelity. In Appendix A I will present a proof of the following theorem:

Theorem I. *Let $\mathcal{H}$ be a Hilbert space of dimension $d = 2$. Then among all completely positive trace preserving maps $T : \mathcal{S}\left(\mathcal{H}_{+}^{\otimes N}\right) \rightarrow \mathcal{S}(\mathcal{H})$, the measurement-based U-NOT scenario (58) attains the smallest possible value of the error measure defined by Eq.(56), namely $\Delta(T) = 1/(N+2)$.*

I conclude this section by saying that in the quantum world governed by unitary operations anti-unitary operations can be performed with the fidelity which is bounded by the amount of classical information potentially available about states of quantum systems.

8. Experimental Realization of U-NOT Gate

In what follows I will describe the experiment by DeMartini *et al.* [59] in which a flipping of a single qubit has been realized. In this case of just single input qubit, the flipping transformation reads exactly the same as the cloning cloning transformation (17) To be specific this transformation describes a process when the original qubit is encoded in the system a while the flipped qubit is in the system c. The density operator describing the state of the system c at the output is

$$\rho_c^{(out)} = \frac{1}{3}|\psi^{\perp}\rangle\langle\psi^{\perp}| + \frac{1}{3}\mathbf{1}, \tag{62}$$

and the fidelity of the spin flipping is $\mathcal{F} = 2/3$.

A natural way to encode a qubit into a physical system is to utilize polarization polarization states of a single photon. In this case the Universal NOT gate can be realized via the stimulated emission. The key idea of DeMartini's experiment is based on the proposal that universal quantum machines [40] such as quantum cloner can be realized with the help of stimulated emission in parametric down conversion [82, 54, 56]. Specifically let us consider a qubit to be encoded in a polarization state of a photon. This photon is injected as the input state into an optical parametric amplifier amplifier (OPA) physically consisting of a nonlinear (NL) BBO (β-barium-borate) crystal cut for Type II phase matching and excited by a pulsed mode-locked ultraviolet laser laser UV having pulse duration $\tau \approx 140f$ sec and wavelength (wl) $\lambda_p = 397.5nm$ associated to pulse duration. The relevant modes of the NL 3-wave interaction were the spatial modes with wave-vector (wv) $k_1$and k_2 each supporting the two horizontal (H) and vertical (V) linear-polarizations ((Π) of the interacting photons, e.g. Π_{1H} is the horizontal polarization unit vector associated with k_1. The OPA was frequency degenerate, i.e. the interacting photons had the same wl's $\lambda = 795nm$. The action of OPA under suitable conditions can be described by a simplified Hamiltonian

$$\hat{H}_{int} = \kappa\left(\hat{a}^{\dagger}_{\psi}\hat{b}^{\dagger}_{\psi^{\perp}} - \hat{a}^{\dagger}_{\psi^{\perp}}\hat{b}^{\dagger}_{\psi}\right) + h.c. \tag{63}$$

A property of the device, of key importance in the context of the present work, is its amplifying behavior with respect to the polarization Π of the interacting photons. It has been shown theoretically in Ref. [54] and in a recent experiment on universal quantum cloning [56, 59] that the amplification efficiency of this type of OPA under injection by *any* externally injected quantum field, e.g. consisting of a single photon or of a classical "coherent" field, can be made *independent* of the polarization state of the field. In other word the OPA "gain" is independent of any (*unknown)* polarization state of the injected field: This precisely represents the necessary *universality* *(U)* property of the U-NOT gate. For this reason in Eq. (63) we have denoted the creation $\hat{a}^\dagger_\psi$ ($\hat{b}^\dagger_\psi$) and annihilation $\hat{a}_\psi$ ($\hat{b}_\psi$) operators of a photon in mode k_1 (k_2) with subscripts ψ (or $\psi^\perp$) indication the invariance of the process with respect to polarization states of the input photon.

Let us consider the input photon in the mode k_1 to have a polarization ψ. I will describe this polarization state as $\hat{a}^\dagger_\psi|0,0\rangle_{k_1} = |1,0\rangle_{k_1}$, where I have used notation introduced by Simon et al. [54], i.e. the state $|m,n\rangle_{k_1}$ represents a state with m photons of the mode k_1 having the polarization ψ, while n photons have the polarization $\psi^\perp$. Initially there are no excitations in the mode k_2. The initial polarization state of these two modes reads $|1,0\rangle_{k_1} \otimes |0,0\rangle_{k_2}$ and it evolves according the Hamiltonian Hamiltonian (63):

$$\begin{aligned} &\exp(-i\hat{H}_{int}t)|1,0\rangle_{k_1} \otimes |0,0\rangle_{k_2} \simeq |1,0\rangle_{k_1} \otimes |0,0\rangle_{k_2} \\ - \quad & i\kappa t\left(\sqrt{2}|2,0\rangle_{k_1} \otimes |0,1\rangle_{k_2} - |1,1\rangle_{k_1} \otimes |1,0\rangle_{k_2}\right) \end{aligned} \tag{64}$$

This approximation for the state vector describing the two modes at times $t > 0$ is sufficient since the values κt are usually very small (see below). The zero order term corresponds to the process when the input photon in the mode k_1 did not interact in the nonlinear medium, while the second term describes the first order process in the OPA. This second term is formally equal (up to a normalization factor) to the right-hand side of Eq. (17). Here the state $|2,0\rangle_{k_1}$ describing two photons of the mode k_1 in the polarization state ψ corresponds to the state $|\psi\psi\rangle$. This state-vector describes the cloning cloning of the original photon [54, 56]. The vector $|0,1\rangle_{k-2}$ describes the state of the mode k_2 with a single photon in with the polarization $\psi^\perp$. That is, this state vector represents the flipped version of the input.

To see that the stimulated emission is indeed responsible for creation of the flipped qubit, let us compare the state (64) with the output of the OPA when the vacuum is injected into the nonlinear crystal. In this case, to the same order of approximation as above we obtain

$$\begin{aligned} &\exp(-i\hat{H}_{int}t)|0,0\rangle_{k_1} \otimes |0,0\rangle_{k_2} \simeq |0,0\rangle_{k_1} \otimes |0,0\rangle_{k_2} \\ & - \\ & i\kappa t\left(|1,0\rangle_{k_1} \otimes |0,1\rangle_{k_2} - |0,1\rangle_{k_1} \otimes |1,0\rangle_{k_2}\right) \end{aligned} \tag{65}$$

We see that the flipped qubit described by the state vector $|0,1\rangle_{k_2}$ in the right-hand sides of Eqs.(64) and (65) does appear with different amplitudes corresponding to the ratio of probabilities to be equal to $1:2$. This ratio has been measured in the experiment [59].

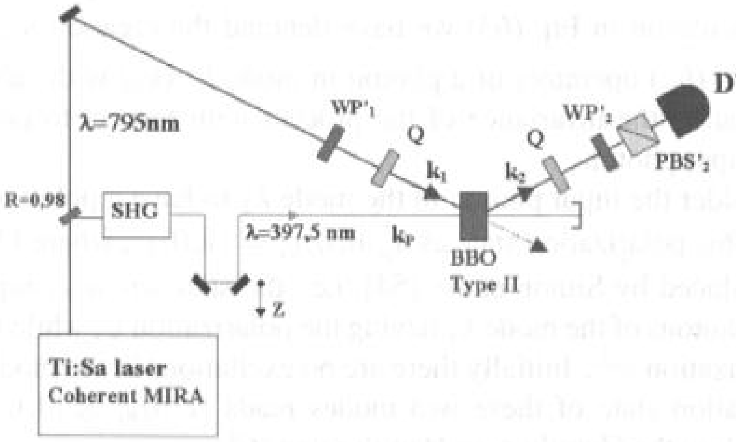

Figure 2. Schematic description of the experimental verification of the universality of the flip operation. A coherent state coherent state of attenuated laser laser field with wl $\lambda = 795nm$ has been used in the experiment. The source is Ti:Sa Coherent MIRA pulsed laser providing by Second Harmonic Generation Second harmonic generation(SHG) the OPA "pump" field associated with the spatial mode with wv k_p and wl λ_p. A small portion of the laser radiation at wl λ was directed along the OPA injection mode k_1. The parametric amplification, with calculated "gain" $g = 0.31$, was detected at the OPA output mode k_1 by D_1', a Si linear photodiode SGD100. The time superposition in the NL crystal of the "pump" and of the "injection" pulses was assured by micrometric displacements (Z) of a 2-mirror optical "trombone". Various $\Pi-$states of the injected pulse were prepared by the set $(WP\prime_1 + Q)$ consisting of a Wave-plate (either $\lambda/2$ or $\lambda/4$) and of a 4.3mm X-cut Quartz plate. These states were then analyzed after amplification and before detection on mode k_2 by an analogous optical set $(WP\prime_2 + Q + \Pi-analyzer)$, the last device being provided by the *Polarizing Beam Splitter* PBS_2'. In the experiment all the 4.5mm thick X-cut quartz plates (Q) provided the compensation of the unwanted beam walk-off effects due to the birefringence of the NL crystal.

8.1 Universality

On the "microscopic" quantum level the justification of this U-property of the OPA amplifier amplifier resides in the SU(2) rotational invariance of the NL interaction Hamiltonian Hamiltonian when the spatial orientation of the OPA NL Type II crystal makes it available for the generation of 2-photon entangled "singlet" states by Spontaneous Parametric Down Conversion (SPDC), i.e. by injection of the "vacuum field" [54]. However I should note that in the present context the *universality* property, i.e. the $\Pi-$*insensitivity* of the parametric amplification "gain" g, is a "macroscopic" classical feature of the OPA device. As a consequence, it can be tested equally well either by injection of "classical", e.g. coherent (Glauber) fields or of a "quantum" states of radiation, e.g. a single-photon Fock-state. De Martini *et al.* [59] carried out successfully both tests, leading to identical results. Below I describe the experiment

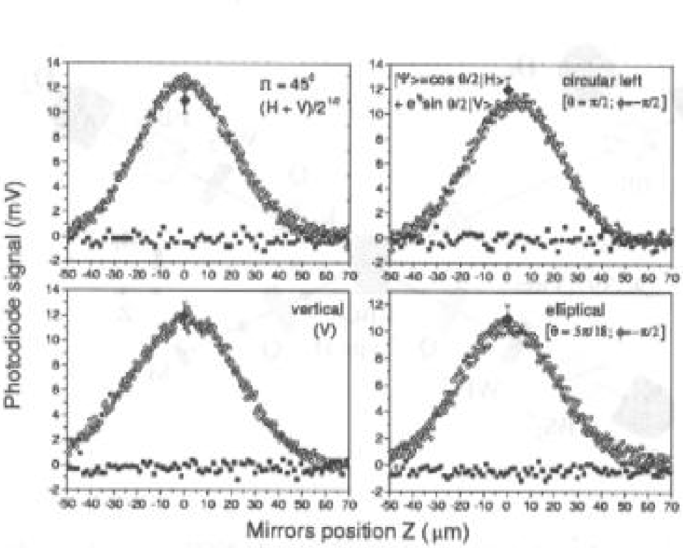

Figure 3. Experimental verification of the universality of OPA. The plots show the amplification pulses detected by D_2' on the OPA output mode, k_2. Each plot corresponds to a definite Π−state, $|\psi\rangle = [\cos(\vartheta/2)\,|H\rangle + \exp(i\phi)\, sin(\vartheta/2)\,|V\rangle]$, either $linear - \Pi$, i.e. $\vartheta = 0, \pi/2, \pi$; $\phi = 0$, or $circular{-}\Pi$, i.e. $\vartheta = \pi/2$; $\phi = \pm\pi/2$, or $elliptical - \Pi$, in the very general case: $\vartheta = 5\pi/18$; $\phi = -\pi/2$.

corresponding to the injection by attenuated "coherent" laser laser field – see Fig. 2

The *universality* condition is demonstrated by the plots in Fig. 3 showing the amplification pulses detected by D_2' on the OPA output mode, k_2. Each plot corresponds to a definite Π−state, $|\psi\rangle = [\cos(\vartheta/2)\ |H\rangle + \exp(i\phi)\ sin(\vartheta/2)$ $|V\rangle]$, either $linear - \Pi$, i.e. $\vartheta = 0, \pi/2, \pi$; $\phi = 0$, or $circular{-}\Pi$, i.e. $\vartheta = \pi/2$; $\phi = \pm\pi/2$, or $elliptical - \Pi$, in the very general case: $\vartheta = 5\pi/18$; $\phi = -\pi/2$. We may check that the corresponding amplification curves, each corresponding to a standard injection pulse$_1$ with an average photon number $N \approx 5 \times 10^3$, are almost identical. For more generality, the *universality* condition as well as the insensitivity of this condition to the value of N is also demonstrated by the *single* experimental data reported, with different scales, at the top of each amplification plot and corresponding to injection pulses with: $N \approx 5 \times 10^2$. Single-photon tests of the same conditions were also carried out with a different experimental setup, as said.

8.2 Optimality

Let us move to the main subject of the experiment under consideration, i.e. the quantum U-NOT gate. In virtue of the tested *universality* of the OPA amplification, it is of course sufficient to consider here the OPA injection by a *single-photon* in *just one* Π−state, for instance in the $vertical - \Pi$ state.

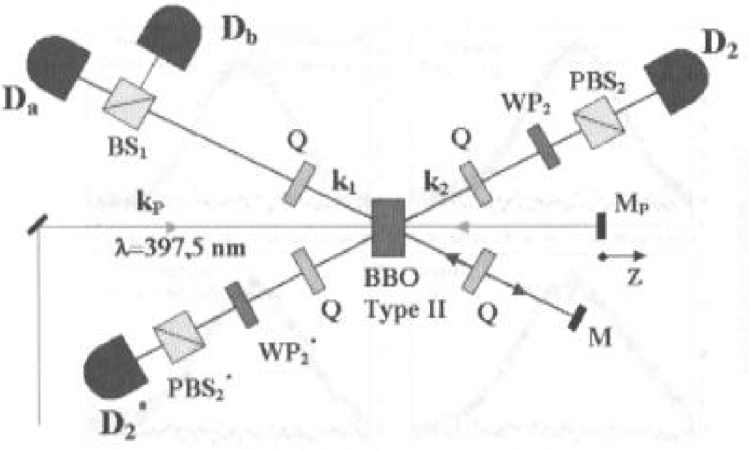

Figure 4. Experimental realization of the quantum U-NOT gate. Consider the k_p pump mode, i.e., the "towards R" excitation. A SPDC process created single photon-pairs with wl $\lambda = 795nm$ in entangled *singlet* $\Pi - states$, i.e. rotationally invariant, as said. One photon of each pair, emitted over k_1 was reflected by a spherical mirror M onto the NL crystal where it provided the $N = 1$ *quantum injection* into the OPA amplifier amplifier excited by the UV "pump" beam associated to the back reflected mode $-k_p$. In the experiment the flipping of a single photon in a state $|\psi\rangle = |V\rangle$ has been considered. In the experiment, owing to a spherical mirror M_p with 100% reflectivity and micrometrically adjustable position Z, the UV pump beam excited the same NL OPA crystal amplifier amplifier in both directions k_p and $-k_p$, i.e. correspondingly oriented towards the Right (R) and Left (L) sides of the figure. Because of the low intensity of the UV beam, the 2 photon injection probability, $N = 2$ has been evaluated to be $\approx 3.5 \times 10^{-4}$ smaller that for the $N = 1$ condition. The twin photon emitted over k_2 was $\Pi - selected$ by the devices $(WP_2 + PBS_2)$ and then detected by D_2, thus providing the "trigger" of the overall *conditional experiment*. All detectors in the experiment were equal active SPCM-AQR14 with quantum efficiency: $QE \approx 55\%$. Because of the EPR non-locality non-locality implied by the singlet state, the $\Pi - selection$ on channel k_2 provided the realization on $k_1$of the state $|\psi\rangle = |V\rangle$ of the injected photon. All the X-cut quartz plates Q provided the compensation of the unwanted beam walk-off effects due to the birefringence of the NL crystal. Consider the "towards L" amplification, i.e. the amplification process excited by the mode $-k_p$, and do account in particular for the OPA output mode k_2. The $\Pi-$state of the field on that mode was analyzed by the device combination $(WP_2^* + PBS_2^*)$ and measured by the detector D_2^*. The detectors D_a, D_b were coupled to the field associated with the mode k_1. The experiment was carried out by detecting the rate of the 4-coincidences involving all detectors $[D_2^* D_2 D_a D_b]$.

Accordingly, Fig. 4 shows a layout of the single-photon, $N = 1$, quantum-injection experiment with input state $|\psi\rangle = |V\rangle$.

From the analysis presented by Simon *et al.* [54, 56] it follows that the state of the field emitted by the OPA indeed realizes the U-NOT gate operation, i.e. the "optimal" realization of the "anti-cloning" of the original qubit originally encoded in the mode k_1. The flipped qubit at the output is in the mode k_2. As it has been shown earlier the state created by the U-NOT gate is not pure. The is a minimal amount of noise induced by the process of flipping which is inevitable

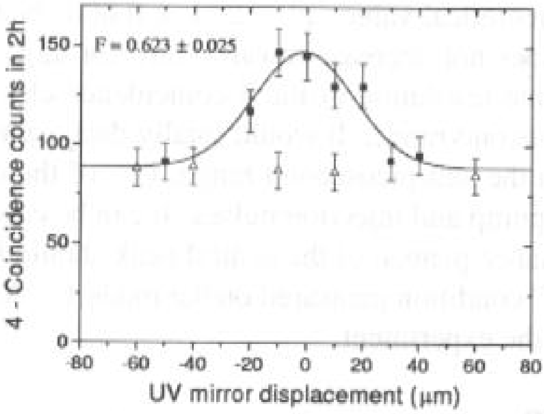

Figure 5. Experimental verification of the optimality of the U-NOT gate. The plots show experimental 4-coincidence data as function of the time superposition of the UV pump and of the injected single-photon pulses. That superposition was expressed as function of the micrometric displacement Z of the back-reflecting mirror M_p. The height of the central peak expresses the rate measured with the $\Pi - analyzer$ of mode k_2 set to measure the "correct" horizontal (H) polarization, i.e. the one orthogonal to the (V) polarization of the $\Pi - state$, $|\psi\rangle = |V\rangle$ of the injected, input single photon, $N = 1$. By turning by 90^0 the $\Pi - analyzer$, the amount of the "noise" contribution is represented by a "flat" curve. In the experiment the "noise" was provided by the OPA amplification of the unavoidable "vacuum" state associated with the mode k_1.

in order to preserve complete-positiveness of the Universal NOT gate. This mixed state is described by the density operator (62). The polarization state of the output photon in the mode k_2 in the experiment performed by De Martini *et al.* [59] is indeed described by this density operator.

The plots of Fig. 5 report the experimental 4-coincidence data as function of the time superposition of the UV pump and of the injected single-photon pulses.

The main result of the experiment [59] consists of the determination of the ratio R^* between the height of the central peak and the one of the flat "noise" contribution. To understand this ratio let us firstly note that the most efficient stimulation process in the OPA is achieved when a perfect match (overlap) between the input photon and the photon produced by the source is achieved. This situation corresponds the value of the mirror position Z equal to zero [see Eq.(64)]. As soon as the mirror is displaced from the position the two photons do not overlap properly and the stimulation is less efficient. Correspondingly, the spin flip operation is more noisy. In the limit of large displacements Z the spin flipping is totally random due to the fact that the process corresponds to injecting the vacuum into the crystal [see Eq.(65)]. The theoretical ration between the corresponding probabilities is 2.In the experiment [59] the ratio has been found to be $R^* = (1.66 \pm 0.20)$. This corresponds to a measured value of the fidelity fidelity of the U-NOT apparatus: $\mathcal{F}^* = (0.623 \pm 0.025)$ to be compared with the theoretical value: $F = 2/3 = 0.666$. Note that the height of the central peak does not decrease towards zero for large Z's. This effect is due to the finite time-resolution of the 4-coincidence electronic apparatus, which is in the nanosecond range. It would totally disappear if the resolution could be pushed into the sub-picosecond range, i.e. of the order of the time duration of the OPA pump and injection pulses. It can be easily found that the spurious out of resonance plateau of the central peak should indeed reproduce the size of the "noise" condition measured on the mode k_2. As we can see, this is indeed verified by the experiment.

9. Conclusion

In these notes I have overviewed two types of universal quantum machines - universal quantum cloners and the universal NOT gate. In order to motivate my discussion I have briefly discussed the role of no-signaling condition in quantum mechanics and I have shown that quantum cloning cannot be used for super-luminal signaling as originally suggested by Nick Herbert. The main message of my notes can be summarized as follows: Even though quantum mechanics does open new perspectives in information processing it also imposses new bounds on how well we can manipulate with information encoded in quantum systems. In quantum processes we can redistribute information according to specific rules, but we cannot generate it (e.g. in a sense that we would be able to perform better state estimation).

Acknowledgments

I would like to thank many of my friends for long standing collaboration and discussions on issues presented in these notes. First of all I thank Mark Hillery for his contribution to my understanding of universal quantum machines. I want to thank Reinhard Werner, Radoslav Derka, Christoph Simon, Artur Ekert, Peter Knight, Francesco De Martini, Nicolas Gisin, and Sam Braunstein. This work was supported by the IST project EQUIP under the contract IST-1999-11053. I also acknowledges a support from the Science Foundation Ireland offered to me via the E.T.S. Walton award.

Appendix: Proof of Theorem I

I have already shown in Section 7 [see Eq.(59)] that for the measurement-based strategy the error Δ attains the value $1/(N+2)$. The more difficult part, however, is to show that no other scheme [i.e., quantum scenario] can do better. In what follows I will largely follow the arguments in Ref. [50].

Note first that the functional Δ is invariant with respect to unitary rotations (basis changes) in $\mathcal{H}$: When T is any admissible map, and U is a unitary on $\mathcal{H}$, the map $T_U(\sigma) = U^* T(U^{\otimes N} \sigma U^{*\otimes N})U$ is also admissible, and satisfies $\Delta(T_U) = \Delta(T)$. Moreover, the functional Δ is defined as the maximum of a collection of linear functions in T, and is therefore convex. Putting these observations together we get

$$\Delta(T) \leq \int dU \, \Delta(T_U) = \Delta(T), \tag{A.1}$$

where $T = \int dU \, T_U$ is the average of the rotated operators T_U with respect to the Haar measure on the unitary group. Thus T is at least as good as T, and has the additional "covariance property" $T_U = T$. Without loss we can therefore assume from now on that $T_U = T$ for all U.

An advantage of this assumption is that a very explicit general form for such covariant operations is known by a variant of the Stinespring Dilation Theorem (see [50] for a version adapted to our needs).

The form of T is further simplified in our case by the fact that both representations involved are irreducible: the defining representation of SU(2) on $\mathcal{H}$, and the representation by the operators $U^{\otimes N}$ restricted to the symmetric subspace $\mathcal{H}_+^{\otimes N}$. Then T can be represented as a discrete convex combination $T = \sum_j \lambda_j T_j$, with $\lambda_j \geq 0, \sum_j \lambda_j = 1$, and T_j admissible and covariant maps in their own right, but of an even simpler form. Covariance covariance of T already implies that the maximum can be omitted from the definition (56) of Δ, because the fidelity no longer depends on the pure state chosen. In a convex combination of covariant operators we therefore get

$$\Delta(T) = \sum_j \lambda_j \Delta(T_j). \tag{A.2}$$

Minimizing this expression is obviously equivalent to minimizing with respect to the discrete parameter j.

Let us write the general form of the extremal instruments T_j in terms of expectation values of the output state for an observable X on $\mathcal{H}$:

$$\mathrm{Tr}(T(\sigma)X) = \mathrm{Tr}\big[\sigma \, V^*(X \otimes \mathbf{1})V\big], \tag{A.3}$$

where $V : \mathcal{H}_+^{\otimes N} \to \mathcal{H} \otimes \mathbf{C}^{2j+1}$ is an isometry intertwining the respective representations of SU(2), namely the restriction of the operators $U^{\otimes N}$ to $\mathcal{H}_+^{\otimes N}$ (which has spin $N/2$) on the one hand, and the tensor product of the defining representation (spin-$1/2$) with the irreducible spin-j representation. By the triangle inequality for Clebsch-Gordan reduction, this implies $j = (N/2) \pm (1/2)$, so only two terms appear in the decomposition of T. It remains to compute $\Delta(T_j)$ for these two values.

The basic idea is to use the intertwining property of the isometry V for the generators S_α, J_α, and L_α, $\alpha = 1, 2, 3$ of the SU(2)-representations on $\mathcal{H}$, $\mathbf{C}^{2j+1}$ and $\mathcal{H}_+^{\otimes N}$, respectively. We will show that

$$V^*(S_\alpha \otimes 1_j)V = \mu_j \, L_\alpha, \tag{A.4}$$

where μ_j is some constant depending on the choice of j. That such a constant exists is clear from the fact that the left hand side of this equation is a vector operator (with components labeled by $\alpha = 1, 2, 3$), and the only vector operators in an an irreducible representation of SU(2) are multiples of angular momentum Angular momentum (in this case L_α). The constant μ_j can be expressed in terms of a $6j$-symbol, but can also be calculated in an elementary way by using the intertwining property, $VL_\alpha = (S_\alpha \otimes 1 + 1 \otimes J_\alpha)V$ and the fact that the angular momentum

Angular momentum squares $\mathbf{J}^2 = \sum_\alpha J_\alpha^2 = j(j+1)$, $\mathbf{S}^2 = 3/4$, and $\mathbf{L}^2 = N/2(N/2+1)$ are multiples of the identity in the irreducible representations involved, and can be treated as scalars:

$$\begin{aligned} \mu_j \mathbf{L}^2 &= \sum_\alpha V^*(S_\alpha \otimes \mathbf{1}_j) V L_\alpha \\ &= \mathbf{S}^2 + \sum_\alpha V^*(S_\alpha \otimes J_\alpha) V. \end{aligned} \tag{A.5}$$

The sum on the right hand side can be obtained as the mixed term of a square, namely as

$$\frac{1}{2}\Big(\sum_\alpha V^*(S_\alpha \otimes \mathbf{1} + \mathbf{1} \otimes J_\alpha)^2 V - \mathbf{S}^2 - \mathbf{J}^2\Big) \tag{A.6}$$

$$= (\mathbf{L}^2 - \mathbf{S}^2 - \mathbf{J}^2). \tag{A.7}$$

Combining these equations we find

$$\mu_j = \frac{1}{2} + \frac{\mathbf{S}^2 - \mathbf{J}^2}{2\mathbf{L}^2} = \begin{cases} \frac{1}{N} & for j = \frac{N}{2} + \frac{1}{2} \\ \frac{-1}{N+2} & for j = \frac{N}{2} - \frac{1}{2}. \\ (A.8) \end{cases}$$

Let us combine equations (A.3) and (A.4) to get the error quantity Δ from equation (56), with the pure one-particle density matrix density matrix $\rho = \frac{1}{2}1 + S_3$:

$$\Delta(T) = \mathrm{Tr}(V^*(\rho \otimes \mathbf{1}) V \rho^{\otimes N}) = \frac{1}{2}(1 + N\mu_j). \tag{A.9}$$

With equation (A.8) we find

$$\Delta(T) = \begin{cases} 1 & for j = \frac{N}{2} + \frac{1}{2} \\ \frac{1}{N+2} & for j = \frac{N}{2} - \frac{1}{2}. \\ (A.10) \end{cases}$$

The first value is the largest possible fidelity fidelity for getting the state ρ from a set of N copies of ρ. The fidelity 1 is expected for this trivial task, because taking any one of the copies will do perfectly. On the other hand, the second value is the minimal fidelity, which we were looking for. This clearly coincides with the value (59), so the Theorem is proved.

The Theorem as it stands concerns the task of producing just one particle in the U-NOT state of the input. From previous results we see that it is valid also in the case of many outputs. We see that the maximum fidelity is achieved by the classical process via estimation: in equation (58) we just have to replace the output state $(\mathbf{1} - |\phi\rangle\langle\phi|)$ by the desired tensor power. Hence once again the optimum is achieved by the scheme based on classical estimation. Incidentally, this shows that the multiple outputs from such a device are completely unentangled, although they may be correlated.

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