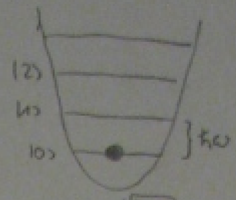
Ion trap:

Physical size of ground state

$$\left.\begin{matrix}\omega = 2\pi\, 1\text{MHz} \\ m = 40u\end{matrix}\right\} \langle x^2\rangle^{1/2} = \sqrt{\frac{\hbar}{2m\omega}} \approx 11\,\text{nm} \ll \text{optical wavelength}$$



$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$$

$$\hat{p} = -i\sqrt{\frac{\hbar\omega}{2m}}(a - a^\dagger)$$

Energy scale of interest

$$\hbar\omega = kT \rightarrow T \sim 50\,\mu K$$

Separation of ions $\sim 5\,\mu m$

Interaction with Light:

$$H = H^{int} + H^{ext} + H^{interaction}$$

H^{int}: $\frac{\hbar\omega_0}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$

H^{ext}: $\sum \hbar\omega_i a_i^\dagger a_i$

$H^{interaction}$: $\frac{1}{2}\hbar\Omega(|g\rangle\langle e| + |e\rangle\langle g|)\, e^{i(k\hat{x} - \omega t + \phi)}$

Ω: Rabi frequency; ω: Laser frequency; $\hat{x}$: position of ion, $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$

Interaction picture:

$$H_I^{int} = \frac{\hbar\Omega}{2} |e\rangle\langle g| e^{-i(\omega - \omega_0)t + i\phi}\, e^{i\eta(a e^{-i\nu t} + a^\dagger e^{i\nu t})} + h.c.$$

$\delta = \omega - \omega_0$ = detuning

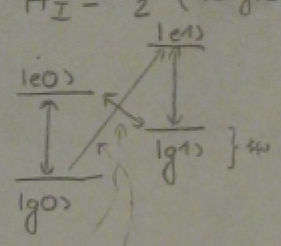
$$\eta = \sqrt{\frac{\hbar}{2m\omega}}\, k \quad \text{relative size of ground state to size of wavelength}$$

$$= \sqrt{\frac{E_{Recoil}}{E_{phonon}}}$$

E_{Recoil} – kinetic energy of atom from one photon $\frac{\hbar^2 k^2}{2m}$

E_{phonon} – energy of phonon

On resonance: $\delta = 0 \Rightarrow H_I \cong \frac{\hbar\Omega}{2}(|e\rangle\langle g| e^{i\phi} + |g\rangle\langle e| e^{-i\phi})$



~~Heuristic treat of photon number~~

$\delta = -\nu$: $H_{red} = \frac{\hbar\Omega}{2}(|e\rangle\langle g| a e^{i\phi} + |g\rangle\langle e| a^\dagger e^{-i\phi})$

$\delta = +\nu$: $H = \frac{\hbar\Omega}{2}(|e\rangle\langle g| a^\dagger e^{i\phi} + |g\rangle\langle e| a e^{-i\phi})$

~~Two qubit phase gate:~~

~~$|g0\rangle \rightarrow |g0\rangle$~~

~~$|e0\rangle \rightarrow -i|g1\rangle$~~

~~with $\delta = -\nu$~~

Exercise:

i) Show H_{red} conserves the operator $a^\dagger a + |e\rangle\langle e| = N$

ii) Consider subspace $\{|g1\rangle, |e0\rangle\}$ and write the action of $e^{-iH_{red}t/\hbar}$ in this subspace

iii) Show that for $t = \frac{\pi}{\Omega}$ we have

$$e^{-iH_{red}t/\hbar}|g1\rangle = -i|e0\rangle$$

$$e^{-iH_{red}t/\hbar}|e0\rangle = -i|g1\rangle$$

and for $t = \frac{2\pi}{\Omega}$

$$e^{-iH_{red}t/\hbar}|g1\rangle = -|g1\rangle$$

$$|e0\rangle = -|e0\rangle$$

iv) Write the operator for general t in this subspace

Taming the Quanta

Martin B. Plenio

September 16, 2008

Contents

1 Introduction

Almost all proposed quantum information processing implementations exploit at some point tools that are commonly regarded as being part of the quantum optical domain. These notes aim to provide a very brief overview of various approaches and introduce some basic concepts and notations. I make no claim or aim to provide a complete survey which will be heavily slanted towards the areas of expertise of the author and even in those areas there will certainly be glaring omissions.

Broadly speaking QI in QO can be separated into two main approaches:

• Qubits are stored in photonic (or generally bosonic) degrees of freedom and where matter provides the coupling between those qubits

• Qubits are stored in matter and a bosonic degree provides the interaction between those qubits.

Finally there may be a third class of systems where matter and bosonic degree of freedom play more even roles. But thats a matter of taste.

It might be oversimplifying things a little bit but a basic goal in many implementations is to try and clean and control the system sufficiently until the interaction between matter and the bosonic degree of freedom is accurately described by a Jaynes-Cummings Hamiltonian

$$H = g\left(\sigma^- a^\dagger + \sigma^+ a\right) \tag{1}$$

where σ^+ (σ^-) are the raising and lowering operators of the matter qubit while $a^\dagger$ (a) are the raising and lowering operators of the bosonic mode. Once a Hamiltonian like this is achieved we know that in principle we can use it to implement quantum gates between different matter qubits.

2 Basic criteria

All physical system are ultimately governed by quantum mechanics and one may hence hope that they are all possible candidates for a quantum information processor or quantum simulator. Evidently, a successful technology must combine a variety of properties in one physical realisation. But what are these properties? Around 10 years ago some of the most relevant requirements for were summarized succinctly in what is now known as DiVincecnzo's criteria. Necessary criteria for a viable quantum information processing technology are

1. a scalable physical system of well-characterized qubits;
2. the ability to initialize the state of the qubits to a simple fiducial state;
3. long (relative) decoherence times, much longer than the gate-operation times;
4. a universal set of quantum gates;
5. a qubit-specific measurement capability

and additional criteria for networkability of quantum information processors are

6. the ability to interconvert stationary and flying qubits;

7. the ability to faithfully transmit flying qubits between specified locations.

3 Brief selection of promising technologies

There is a wide variety of possible technologies that are currently under investigation and ideas for new realisations are still being developed. Before I talk in a little more detail about some of them I would like to present here some of the more popular technologies and give for each one or two useful articles (usually reviews) that allow you to delve into more detail.

- Trapped ions: Here ions are held in free space with the help of, possibly time-dependent, electric and magnetic fields. Lasers are used to drive optical transitions in the ions and controlled interactions between the ions are realised via coupling of the internal electronic degrees of freedom to the motional degrees of freedom of the ions. High efficiency readout of the ions is achieved by observing state selective resonance fluorescence. Scalability may be achieved in designs in which the ions are moved around the ion trap. This technology has been under development of QI since more than a decade now.
  Further reading: D.J. Wineland, C. Monroe, W.M. Itano, D. Leibfried, B.E. King and D.M. Meekhof, J. Res. Nat. Inst. Stand. Tech. **103**, 259 (1998) and quant-ph/9710025
  A.M. Steane, Appl. Phys. B **64**, 623 (1997) and quant-ph/9608011;
  M. Sasura and V. Buzek, J. Mod. Opt. **49**, 1593-1647 (2002) and quant-ph/0112041.

- Trapped atoms in optical lattices: Very cold neutral atoms may be held in free space by light forces exerted by an optical lattice. An optical lattice is formed of a number of counter-propagating optical fields that form standing waves. The atoms will be attracted to the points of highest (or lowest) intensity and to lowest order they will experience harmonic potentials at those sites. For sufficiently low temperatures the atoms will populate the lowest energy levels of those potentials. Controlled collisions may be achieved between neighboring atoms by employing state dependent light forces (e.g. shifting atoms in state $|1\rangle$ to the left and atoms in state $|2\rangle$ to the right until they start to come close and interact) Depending on the strength of the optical fields the atoms may also tunnel between neighboring sites which gives rise to interesting physical Hamiltonians such as the Bose-Hubbard model.
  Further reading: M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen (De) and U. Sen, Adv. Phys. **56**, 243 (2007) and cond-mat/0606771;
  I. Bloch, J. Dalibard, W. Zwerger, Rev. Mod. Phys. 80, 885 (2008) and arXiv:0704.3011

- Superconducting qubits: Small superconducting rings with Josephson junction elements in them can be used prepared in well-defined quantum states of either charge or flux. These state may then be manipulated and read out. Y. Makhlin, G. Schoen, A. Shnirman, Rev. Mod. Phys. 73, 357-400 (2001) and arXiv:cond-mat/0011269;

- Cavity QED: Trapped atoms and ions may be coupled to light by trapping them inside an optical cavity. This has the advantage that they are predominantly coupling to a specific light mode which may then be observed. This light mode may be used either to serve as a bus to let different atoms or ions interact with each other or it allows to transfer the state of the matter qubits to the photonic qubits which may then be transmitted to other distant sites. A great variety of cavity designs exist for many different wave-lengths. Broadely, one may distinguish technologies in the optical regime and those in the microwave regime. A further distinction is between bulk cavities (loosely speaking with large mirrors and sizes of mm or cm) and micro-cavities whose sizes may only be a few wavelengths. More recently, superconducting qubits have been coupled to microwave stripline resonators.
  Further reading: P.R. Berman, *Cavity QED*, Adv. At. Mol. and Opt. Phys., Academic Press;
  H. Mabuchi and A. Doherty, Science **298**, 1372 (2002); S. M. Spillane, T. J. Kippenberg, K. J. Vahala, K. W. Goh, E. Wilcut, H. J. Kimble, Phys. Rev. A **71**,

013817 (2001) and quant-ph/0410218;
S.M. Girvin, R.G. Huang, A. Blais, A. Wallraff, R.J. Schoelkopf, Proc. Les Houches Summer School, Session LXXIX 2003 and cond-mat/0310670
D. Schuster, PhD Thesis Yale 2007.

- Photons: If the photons are trapped between highly reflecting mirrors we have cavity QED again. Otherwise we have freely propagating radiation which possesses a wealth of useable degrees of freedom. The three most important are perhaps (i) polarization, (ii) photon number and (iii) which path. Photons travel well and some basic operations can be achieved by beam-splitters and phase shifters. However, photons do not interact strongly and interactions need to be assisted either by matter or using photo-detection to achieve non-linear interactions. Some basic demonstrations such as teleportation have been achieved but it is fair to say that photon memories need to be developed to make the technology scalable.
  P. Kok, W.J. Munro, K. Nemoto, T.C. Ralph, J.P. Dowling, G.J. Milburn, Rev. Mod. Phys. **79**, 135 (2007);
  S.L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005) and quant-ph/0410100;
  J. Eisert and M.B. Plenio, Int. J. Quant. Inf. **1**, 479 (2003);

- Quantum dots: A quantum dot is a semiconductor whose excitons are confined in all three spatial dimensions. As a result, they have properties that are between those of bulk semiconductors and those of discrete molecules. Neighboring quantum dots may interact via tunneling or the light that they are emitting may be used to create measurement based interactions.
  Further reading: D. Loss and D. DiVincenzo, Phys. Rev. A **57**, 120 (1998).

- Nuclear Magnetic Resonance: Here the nuclear spins in a molecule are addressed and manipulated by micro-wave radiation. Due to their proximity the nuclear spins are also interacting directly. Molecules with up to around 10 spins have been manipulated but there is some controversy as to whether these experiments actually constitute quantum information processing in the sense that they may achieve an exponential speedup compared to classical computation. The reason is the fact that the spins are actually not in a pure state but in a highly mixed state as they are in a room temperature environment. There may be ways around this problem but none of these have yet been demonstrated.
  Further reading: J.A. Jones, Prog. NMR Spectroscopy **38**, 325 (2001) and quant-ph/0009002; J. Baugh, J. Chamilliard, C. M. Chandrashekar, M. Ditty, A. Hubbard, R. Laflamme, M. Laforest, D. Maslov, O. Moussa, C. Negrevergne, M. Silva, S. Simmons, C. A. Ryan, D. G. Cory, J. S. Hodges, C. Ramanathan, to appear in Physics in Canada, Special Issue on Quantum Computing and Quantum Information, 2007 and arXiv:0710.1447

- Atomic ensembles: A collection of atoms with transverse dimensions large compared to the wavelength of light collectively couples very efficiently to any spatial light mode. In classical electrodynamics this is manifested by large absorption and dispersion that characterizes an atomic ensemble interacting with nearly resonant light. Luckily, the same scaling persists for some collective quantum properties of light and atoms. J. Sherson, B. Julsgaard, E.S. Polzik, Advances in Atomic, Molecular, and Optical Physics" Vol. 54. (2006) and arXiv:quant-ph/0601186.

4 'Pure' Optics

There are various approaches to quantum information processing where qubits are represented in photonic degrees of freedom, i.e. light or microwave radiation but potentially also oscillations of nano-mechanical devices. I will first concentrate on settings with freely propagating light and leave cavity QED schemes for later as they will usually involve matter in a different way and will be discussed under the class of hybrid systems. Most of todays work on implementations are using optical frequencies, i.e. near infra-red to near ultra-violet. There is some work employing micro-waves but this is mostly concerned with cavities. Above optical frequencies there is not much activity for the lack of coherent sources, the increasing losses as well as the lack of suitable detectors.

4.1 Degrees of freedom

It is important to note that a photon is a complex object with many potentially useful degrees of freedom. Thus there is no unique way to encode quantum information in light and a whole range of degrees of freedom may be used individually or jointly.

1. Polarization degree of freedom
2. Which path degree of freedom
3. Photon number degree of freedom
4. Spatial mode degree of freedom
5. Frequency mode degree of freedom
6. Angular momentum degree of freedom
7. Temporal degree of freedom (Time-bins)

Perhaps the currently most frequently used dofs are the first three (note that 1 and 2 can be inter-converted easily by polarizing beam-splitters) but there is much to be said for exploring the use of the others. Particularly interesting would be the simultaneous use of several degrees of freedom of a photon to carry information. This is sometimes termed hyperentanglement. As an example note that with passive linear optics ele-

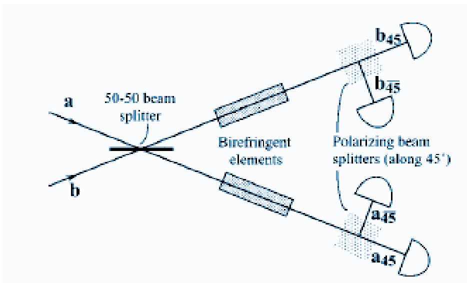


Figure 1: Setup to discriminate all four polarization Bell states, employing addition dofs. Interference at the 50-50 beam splitter renders ψ^- distinguishable from the others as one photon is emitted into each arm; the birefringent element with axes along the horizontal and vertical directions separates ψ^+ as it will lead to simultaneous clicks on the beam-splitters; interference at the polarizing beam splitters distinguishes ϕ^- (detection at different detectors on one side) from ϕ^+ (detection in the same detector at the same side).

ments, photon counters and feed-forward it is not possible to perfectly discriminate all the four Bell states when given in the polarization basis [Calsamiglia & Lütkenhaus, App. Phys. B **72**, 67 (2001)]. This theorem is correct but allowing for additional physical degrees of freedom to play a role permits you to overcome it and achieve 100% efficiency (at least in principle) This example shows that the use of several degrees of freedom may bring various potential benefits

- Higher information capacity per photon
- More freedom of encoding
- More operations are available (e.g. Bell state discrimination is impossible with linear optics and polarization dof alone)

but also has potential drawbacks

- Higher impact of absorption
- not all dofs equally well measurable
- may be hard to retrieve all information from photon

There are two media that can propagate photons: optical fibers and free space. Each of these two possible choices implies the use of the corresponding appropriate wavelength. For optical fibers, the classical telecom choices are 1300 and 1550 nm and any application in the real world of quantum communication in fibers has to stick to this choice. For free space the favored choice is either at shorter wavelengths, around 800 nm, where efficient detectors exist, or at much longer wavelengths, 410 microns, where the atmosphere is more transparent.

Inside fibres loss is higher than 0.05dB/Km but can be much lower in free space.

4.2 Sources, Optical Elements & Detectors

Quantum information processing with photonic degrees of freedom involves three main ingredients

4.2.1 Detectors

The final important building block in any optical QIP network are detectors. Despite many years of development detectors still have efficiencies that are quite a bit away from 100% and it seems unlikely that compact, fast high efficiency detectors will become available soon.

Yes/No detectors – These are photo-detectors that will either say that there was no photon (upto dark counts) or that there was at least one photon, i.e. the detector does not discriminate between photon numbers. Silicon avalanche photodetectors (Si APD's) can reach efficiencies of about 70%.

Photo-counters – The above detectors may be used to create photon counters by using a fibre optics based time-multiplexing approach. The current devices achieve about 40% detector efficiencies and can distinguish up to about 8 photons even though more may may be possible. [Achilles, Silberhorn and Walmsley, Phys. Rev. Lett. 97, 043602 (2006)].

Homodyne detection – Photon-counters are not sensitive to phase information. This may be obtained by mixing the signal beam on a beam-splitter with a strong coherent state (local oscillator) to provide a phase reference. This approach can yield very high effective detector efficiencies. In a balanced homodyne detector a $50/50$ beamsplitter takes a input the signal on one port and the coherent beam on the other. Two detectors measure the output and we subtract the signal. The consequence is a quadrature measurement i.e. a measurement of the expectation value $\langle ae^{i\phi} + a^{\dagger}e^{-i\phi}\rangle$ where ϕ is

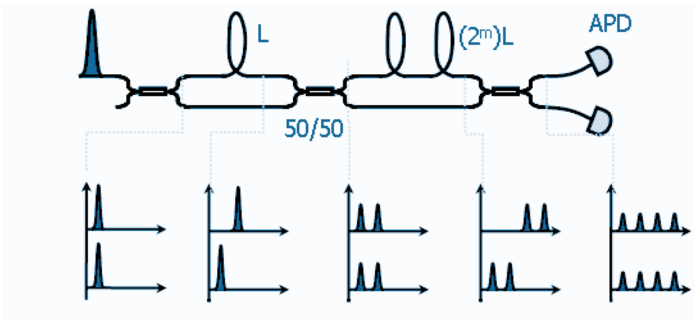


Figure 2: Fibre based beam-splitters with delay lines are used to split up wave packets which are then measured by standard yes/no avalanche photo-detectors.

the relative phase between local oscillator and signal mode. [Leonhardt, *Measuring the quantum state of light*, CUP]

Superconducting detectors – These are being developed and may eventually lead to high detector efficiencies. These work by photons breaking up copper pairs which is then detected in their effect on the current. The detectors may be a bit clunky though.

4.2.2 Optical elements

Once the photons have left the detector they will propagate freely either in free space or inside a fibre. They may then encounter devices that may affect their properties. These devices may be grouped into ***passive linear***, ***active linear*** and ***non-linear*** optical devices. Linear optics devices are those that map effect a linear transformation on the photonic annihilation and creation operators. If these devices act unitarily then the corresponding Hamilton operator is quadratic in annihilation and creation operators. Of course such elements may be combined with partial tracing and the provision of vacuum modes to generate POVM's (see Lindblad 2000).

The ***passive optical elements*** are phase plates and beam-splitters and preserve the photon number. Polarizing beam-splitters are important for manipulating polarization degrees of freedom.

Active linear optical elements are squeezers that will not preserve the photon numbers. The best squeezing that has been achieved in optical devices is of the order of 9.7dB (ie a reduction of the noise level in one quadrature phase by about a factor of 10). Effecting squeezing in optics is not an easy task and tends to introduce noise into the system. Anything else, i.e. active optical elements, is even harder.

Note that some people will define active and passive differently, namely whether the device requires external control or not.

Possible problems – Apart from the difficulty of achieving strong noiseless squeezing, even it also not straightforward to string together a large number of linear optical elements as this introduces additional error sources even if the photons themselves are perfect for interference on the devices. This includes mode-matching problems in time, space and frequency as Photons have to overlap on beam-splitters perfectly to obtain interference. Feeding light from free space into an optical fibre is not straightforward as the spatial mode of the photon may not coincide with that supported by the fibre.

4.2.3 Fundamental Noise limits

One may ask the question whether such devices can, at least in principle, be arbitrarily close to perfection. This is a topic that is also of interest in the matter qubits systems. In the linear optics regime it is well-known that squeezers are noisy devices. But it may come as a surprise that even simple devices such as beam-splitters have some fundamental bounds on their error rates. While these bounds are unlikely to be relevant to the error thresholds they are nevertheless surprisingly large. Even for a particularly simple beam splitter geometry, a single planar slab surrounded by vacuum it can be shown that, using general causality requirements and statistical arguments, the lower bound depends on the frequency of the incident light and the transverse resonance frequency of a suitably chosen single resonance model only. For symmetric beam splitters and reasonable assumptions on the resonance frequency ω_T, the lower absorption bound is $p_{min} \approx 10^{-6}(\omega/\omega_T)^4$. [Scheel,PRA 73, 013809 (2006)] We will return to this topic in the purely matter section.

4.2.4 Mathematical description of infinite dimensional degrees of freedom –

While the description of finite dimensional degrees of freedom is fairly clear, the infinite dimensional setting has its subtleties.

Firstly, without any further constraints one looses reasonable properties such as the trace norm continuity of many functions such as entanglement measures. This manifests itself in problems such as the fact that in every neighborhood of a product state there is an infinitely entangled state. Such problems can be largely alleviated by introducing very reasonably a constraint on the mean energy of the quantum states that one wishes to consider [Eisert, Simon, Plenio, J. Phys. A. 35, 3911 (2002)].

An important class of quantum states in the continuous variable setting are the so-called Gaussian states as they are the class of states that is accessible via linear optics and homodyne detection, ie the set of operations that is reasonably readily accessible in the lab. Furthermore, many question concerning the interconversion of Gaussian states and their entanglement properties have been answered in full [Eisert, Plenio, Int. J. Quant. Inf. 1, 479 (2003)].

One key reason for these successes is the fact that Gaussian states are completely specified by their first and second moments so that questions concerning properties of Gaussian states can be translated into properties of (comparatively small) finite-dimensional matrices. The systems that will typically be discussed are quantum systems with n canonical degrees of freedom representing n field modes of light. The *canonical commutation relations* (CCR) between the $2n$ canonical self-adjoint operators corresponding to position and momentum of such a system with n degrees of freedom, may be written in a particularly convenient form employing the row vector,

$$O = (O_1, \ldots, O_{2n})^T = (X_1, P_1, \ldots, X_n, P_n)^T. \quad (2)$$

In terms of the familiar creation and annihilation operators of the modes choosing $\hbar = 1$, X_n and P_n can be expressed as $X_n = (a_n + a_n^\dagger)/\sqrt{2}$, $P_n = -i(a_n - a_n^\dagger)/\sqrt{2}$. so that

$$[O_j, O_k] = i\sigma_{j,k} \ , \quad (3)$$

where the skew-symmetric block diagonal real $2n \times 2n$-matrix σ given by

$$\sigma = \bigoplus_{j=1}^{n} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad (4)$$

is the so-called *symplectic* matrix. The phase space then becomes what is known as a symplectic vector space, equipped with the scalar product corresponding to this symplectic matrix. Instead of referring to states, i.e., density operators, one may equivalently refer to functions that are defined on phase space. There are many common choices of such functions in phase space, such as the Wigner function, the Q-function or the P-function, each of them is favorable in a particular physical context. For later purposes it is most convenient to introduce the *characteristic function*, which is the Fourier transform of the Wigner function. Using the Weyl operator

$$W_\xi = e^{i\xi^T \sigma O} \tag{5}$$

for $\xi \in R^{2n}$, we define the (Wigner-) characteristic function as

$$\chi_\rho(\xi) = \mathrm{tr}[\rho W_\xi]. \tag{6}$$

In quantum optics, the Weyl operator is typically referred to as phase space displacement operator or Glauber operator, but with a different convention concerning its arguments. For a single mode, let the complex number α be defined as $\alpha = -(\xi_1 + i\xi_2)/\sqrt{2}, \alpha^* = -(\xi_1 + i\xi_2)/\sqrt{2}$, then the phase space displacement operator D_α of quantum optics is most commonly taken as $D_\alpha = W_\xi$. Each characteristic function is uniquely associated with a state, and they are related with each other via a Fourier-Weyl relation. The state ρ can be obtained from its characteristic function according to

$$\rho = \frac{1}{(2\pi)^n} \int d^{2n}\xi \chi_\rho(-\xi) W_\xi. \tag{7}$$

In turn, the Wigner function as commonly used in quantum optics is related to the characteristic function via a Fourier transform, i.e.,

$$\mathcal{W}(\xi) = \frac{1}{(2\pi)^{2n}} \int d^{2n}\zeta e^{i\xi^T \sigma \zeta} \chi(\zeta). \tag{8}$$

Gaussian states are, as mentioned before, defined through their property that the characteristic function is a Gaussian function in phase space, i.e.,

$$\chi_\rho(\xi) = \chi_\rho(0) e^{-\frac{1}{4}\xi^T \Gamma \xi + D^T \xi}, \tag{9}$$

where Γ is a $2n \times 2n$-matrix and $D \in R^{2n}$ is a vector. As a consequence, a Gaussian characteristic function can be characterized via its first and second moments only, such that a Gaussian state of n modes requires only $2n^2 + n$ real parameters for its full description, which is polynomial rather than exponential in n. The first moments form a vector, the displacement vector $d \in R^{2n}$, where

$$d_j = \langle O_j \rangle_\rho = \mathrm{tr}[O_j \rho], \tag{10}$$

$j = 1, ..., 2n$. They are the expectation values of the canonical coordinates, and are linked to the above D by $D = \sigma d$. They can be made zero by means of a translation in phase space of individual oscillators. As a consequence the first moments do not carry any information about the entanglement properties of the state. The second moments are embodied in the real symmetric $2n \times 2n$ covariance matrix γ defined as

$$\gamma_{j,k} = 2\mathrm{Re}\,\mathrm{tr}\left[\rho \left(O_j - \langle O_j \rangle_\rho\right)\left(O_k - \langle O_k \rangle_\rho\right)\right]. \tag{11}$$

With this convention, the covariance matrix of the n-mode vacuum is simply $\mathbb{1}_{2n}$. Again, the link to the above matrix Γ is $\Gamma = \sigma^T \gamma \sigma$. Clearly, not any real symmetric $2n \times 2n$-matrix can be a legitimate covariance of a quantum state: states must respect the Heisenberg uncertainty relation. In terms of the second moments the latter can be phrased in compact form as the matrix inequality

$$\gamma + i\sigma \geq 0 \,. \tag{12}$$

In turn, for any real symmetric matrix γ satisfying the uncertainty principle (12) there exists a Gaussian state the second moments of which are nothing but γ. So Eq. (12) implies the only restriction on legitimate covariance matrices of Gaussian quantum states.

Note that a theory of generalized Gaussian operations may also be formulated. These can essentially be represented by a general linear map on the level of covariance matrices with sufficiently much added noise to make it physical [Lindblad, J. Phys. A **33**, 5059 (2000)].

4.2.5 The possible, the challenging and the impossible

Depending on the degrees of freedom that are being used various tasks become easier, more difficult or even impossible.

Gaussian quantum information processing – Gaussian states are fully characterized by their first and second moments. Thus if we employ only linear optics, vacuum modes, displacement operators and homodyne detection that are preserving the Gaussian character we can provide an efficient classical description of the dynamics of the system and thus cannot achieve universal quantum computation. Thus additional tools such as photon counters are required.

Quantum state teleportation – In the Gaussian setting quantum state teleportation has been achieved. It is based on our ability to perform measurements of quadrature phases as well as the generation of strong squeezing. In the lab experiment Kimble et al 1998], a verifier Victor checks the overlap with the teleported state and is still in the same lab. More challenging would be verify the result with an independent source as this would have be phase coherent. Phase locking of distant sources is possible however and has been achieved for example by the Vienna group.

Gaussian entanglement purification – A less obvious consequence of the restriction

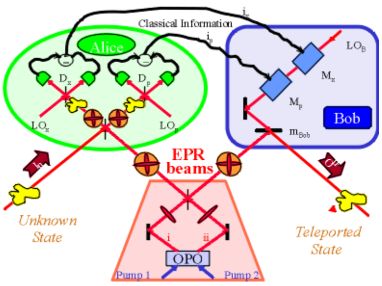


Figure 3:

to the set of Gaussian states and operations concerns entanglement purification. Indeed, if we distribute Gaussian entanglement states along a channel that preserves the Gaussian character, then it is impossible to subsequently concentrate or distill the entanglement of several copies into a more strongly entangled but fewer pairs. This result can be obtained mathematically but a a very intuitive argument yielding this result is not available [Eisert, Scheel, Plenio, Phys. Rev. Lett. **89**, 137903 (2002)]. This no-go result may be overcome by adding simple non-Gaussian operations such as photo-detectors. [Browne, Eisert, Scheel, Plenio, Phys. Rev. A **67**, 062320 (2003)] At the time of writing no entanglement purification step has been implemented experimentally in this setting even though work is in progress in Oxford(Walmsley)/Imperial and in Paris (Grangier).

Quantum cryprography – It is remarkable that despite the above results there are useful QI tasks that can be achieved with Gaussian states. This includes continuous variable cryptography. In a possible protocol, the sending party, Alice, chooses at random to send either a state with a well defined position q or momentum p. Then Alice chooses a value of q or p by sampling a probability distribution, prepares a narrow wave packet centered at that value, and sends the wave packet to the receiving party, Bob. Bob decides at random to measure either q or p. Through public discussion, Alice and Bob discard their data for the cases in which Bob measured in a different basis than Alice used for her preparation, and retain the rest. To correct for possible errors, which could be due to eavesdropping, to noise in the channel, or to intrinsic imperfections in Alices preparation and Bobs measurement, Alice and Bob apply a classical error correction and privacy amplification scheme, extracting from the raw data for n oscillators a number $k < n$ of key bits. [Ralph, Phys. Rev. A **61**, 044301 (2000);Phys. Rev. A **62**, 062306 (2000); Gottesman and Preskill, quant-ph/0008046] More work on security aspects of these schemes has been carried out e.g. in Brussels and in Barcelona.

It is important to note that Gaussian continuous variable cryptography has been demonstrated experimentally e.g. by the Grangier group.

Beyond the Gaussian setting – If one wishes to implement fully fledged quantum computation one needs to go beyond the Gaussian regime. This may be done either by using non-Gaussian resources or by using non-Gaussian operations.

The KLM scheme for example employs various non-Gaussian resources. In this scheme single photon sources, passive linear optics and photo-detectors are sufficient for implementing reliable quantum algorithms. Feedback from the detectors to the optical elements is required for this implementation. (Knill, Laflamme and Milburn, arXiv:quant-ph/0006088v1). Considerable improvements to the efficiency of this scheme have been made using the cluster state approach [Nielsen, quant-ph/0402005, Rudolph, Browne,] but fully fledged quantum computation will still require very daunting resources. The quality of the photons have to be very high and essentially Fourier limited (jitter free) [Rohde, Ralph, Nielsen, quant-ph/0505139;Kiraz, Atatuere, Imamoglu, PRA 69, 032305 (2004).]

Cat states for quantum information processing – Here one considers superpositions of the form $|\alpha\rangle \pm |\alpha\rangle$ to encode the qubit. Various gates are actually quite simple, including a phase gate which is just a rotation in phase space and a NOT gate which is simply a change of the relative phase. Conditional phase gates can also be implemented with moderate effort. Some threshold results have been proven for this setting an it does not appear worse in terms of resource needs than KLM. [Ralph, Gilchrist, Milburn, Munro, Glancy, Phys. Rev. A 68, 042319 (2003); Gilchrist, Nemoto, Munro, Ralph, Glancy, Braunstein, Milburn, J. Opt. B: Quantum Semiclass. Opt. 6, S828 (2004)]

Cluster states with Polarization qubits – Entangled polarization states may be created non-deterministically and have been successfully used in various experiments for example by the Vienna group to create cluster states [Kiesel et al, Phys. Rev. Lett. 95, 210502 (2005); Walther et al, Nature 434, 169 (2005)] and the feedforward necessary for the execution of gates on that state has also been demonstrated [Prevedel et al, Nature 445, 65 (2006)]. Entanglement purification using polarizin beam-splitters has also been achieved.

It should be noted however that in all of these experiments *all* the photons are being measured so that one can only infer from the measurement data that these processes have been implemented on the qubits. Given the stochastic character of the generation of polarization qubits this does *not* imply that if one was not to measure the output qubits one would have the correct states. As such these experiments are not scalable as there is no output and the probability of success decreases exponentially with the number of qubits.

Polarization qubit quantum key distribution and quantum teleportation Quantum key distribution with polarisation qubits has also been achieved over remarkable distances in free space. Steps are being taken to aim vertically towards a satellite. The reflection of a single photon off a satelite has been achieved [R. Ursin et al, submitted 2006]

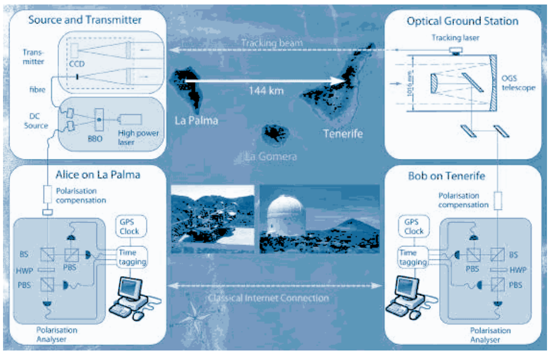


Figure 4:

Likewise teleportation has been achieved using polarization qubits sent along a fibre [O. Landry et al., quant-ph/0605010]

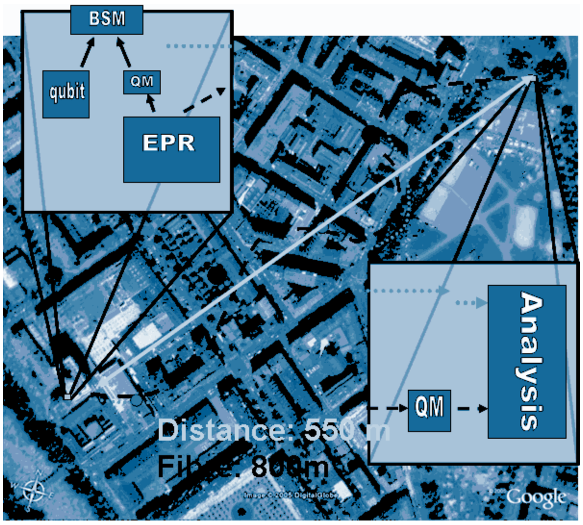


Figure 5:

4.2.6 Photon Sources

Important for various applications are sources both of single photons, multi-photon states as well as entangled states.

Entangled photon sources There are in principle many ways to create entangled photons. Perhaps the currently most frequently used ones come from the use of non-linear crystals. But there are many other approaches. Also there is again the distinction between free space and in fibre generation.

Spontaneous parametric down-conversion – A nonlinear crystal splits incoming photons into pairs of photons of lower energy whose combined energy and momentum are equal to the energy and momentum of the original photon.

Optical parametric oscillator – Resonator+non-linear crystal – Driven by classical field with frequency 2ω which interacts with two modes of lower frequency ω_1, ω_2. Output frequencies $700nm - 5000nm$ can be adjusted by tuning pump or phase matching conditions. Pump at $1.064\mu m$ or half that.

Micro-structured fibres – The development of microstructured and photonic crystal fibres with very small solid cores can have zero dispersion wavelength (ZDW) in the visible and near infra-red region of the spectrum while the very small guided mode area leads to extremely high optical intensity giving rise to ultrahigh optical nonlinearities. Recently pumping slightly blue shifted into the normal dispersion region has been shown to generate widely sep- arated, phase matched, parametric amplification peaks [23]. In such systems a bright source of heralded single photons and a bright source of entangled photon pairs can be created. A pair of pump photons produces a correlated pair of photons at widely spaced wavelengths (583 nm and 900 nm), via a four-wave mixing process. Non-classical interference between heralded photons from independent sources with a visibility of 95%, and an entangled photon pair source, with a fidelity of 89% with a Bell state.

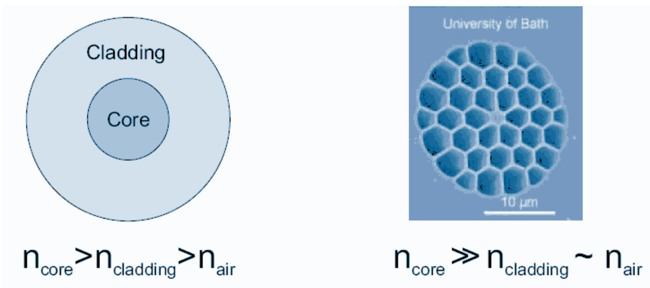


Figure 6: From Fulconis, Alibart, O'Brien, Wadsworth, Rarity, quant-ph/0611232, see also Li, Chen, Voss, Sharping and Kumar, Opt. Express 12 3737 (2004) for fibre based sources.

A terminology that will often appear in this context is the following:
type-I downconversion – Downconverted photons have parallel polarization while pump beam is polarized orthogonal to these. beam emerge with orthogonal polarization, energy conservation and phase matching conditions need to be satisfied

type-II downconversion – Beam emerge with orthogonal polarization, energy conservation and phase matching conditions need to be satisfied

Single photon sources There are various ways in which to create single photon sources and I present some of the more important or promising methods.

Single photons from entangled photon sources – If an entangled photon source emits a polarization entangled photon pair, then the measurement on one part of the output will lead to a single photon output in the other arm.

In the continuous variable regime this approach also allows for the generation of two-photon states based on the conditional detection with photon counters (time-multiplexed detectors). See [Achilles, Silberhorn, Walmsley, Phys. Rev. Lett. 97, 043602 (2006)].

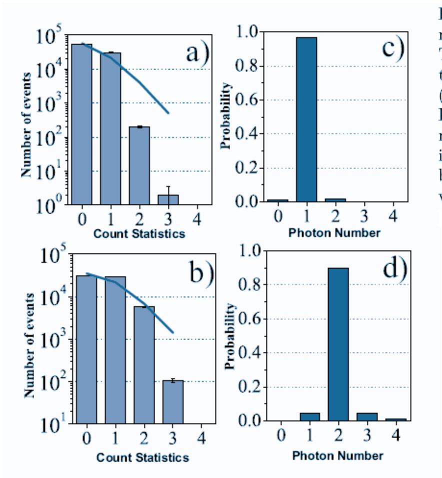


FIG. 2: (Color online) Count statistics detected (logarithmic scale) using (a) single and (b) double APD as a trigger. The solid line (used to guide the eye) shows the Poisson distribution with the same mean photon number as the data (0.376 and 0.623, respectively). This line illuminates the sub-Poissonian nature of our measured statistics. The photon number distribution obtained by using a maximum likelihood inversion with constraints $\rho(n) \geq 0$ for (c) single and (d) double APD trigger by taking into account the efficiencies, which were 37.3% and 31.5%, respectively.

Figure 7: From Achilles, Silberhorn, Walmsley, Phys. Rev. Lett. 97, 043602 (2006)

The advantage of this approach is that it works even if the detector has low efficiency as this only affects the rate. The disadvantage is that this is approach is non-deterministic as the spontaneous parametric down-conversion is random.

NV-colour centers in diamond – [Kurtsiefer, Mayer, Zarda, Weinfurter, Phys. Rev.

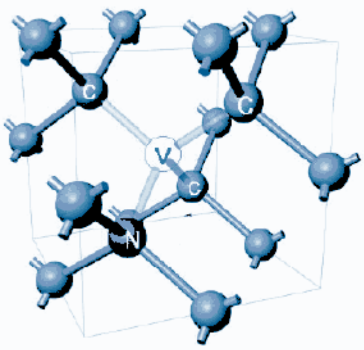


Figure 16. Atomic structure of an NV-centre in a diamond crystal. One of the carbon atoms is replaced by a nitrogen (N), and one of its neighbours is missing (vacancy V). The electronic structure of the centre has not been completely elucidated yet, but presents a triplet ground state (copied from http://www.physik.uni-stuttgart.de/institute/pi/3/ags/jelezko/forschung/).

Figure 8:

Lett. 85 290 (2000); Beveratos, Kuhn, Brouri, Gacoin, Poizat, Grangier, Eur. Phys. J.

D 18 191 (2002)]

Single Quantum Dots - Even though micro-cavities provide high quality single photons these devices are quite large. For integration into fibre optics networks devices based on solid state sources would be desirable. In that context quantum dots are quite relevant and have seen recent progress. Here bi-excitons are created electrically and then decay into two polarization entangled photons currently with about 70% fidelity [Young, Stevenson, Atkinson, Cooper, Ritchie, Shields, NJP **8**, 29 (2006), Stevenson,Young, Atkinson, Cooper, Ritchie, Shields, Nature **439** 179 (2006)]

Possible problems – Note that it is not good enough that your source reliably produces photons (or photon pairs in the case of entangled photon sources). It is also important that the other degrees of freedom of the photon are also well controlled. For example, if one uses different single photon sources, then one needs to ensure that those photons may interfere. This requires that the photon wave-packets are very similar in their spatial and frequency mode as well. Furthermore, the photon wave-packet has to be coherent 'from beginning to end' i.e. it has to be Fourier-transform limited. Note also that photon sources will sometimes fail to produce a photon so that the output of such a source is a mixture of one photon and the vacuum. It is not at all straightforward to improve the single weight of the single photon portion for such a source with post-processing [Berry, Scheel, Myers, Sanders, Knight, Laflamme, NJP 6, 93 (2004)].

4.3 Cavity QED

One further possible way for creating single photons are trapped particles in cavities. Important parameters in cavity QED are [Spillane et al, quant-ph/0410218]

- g is the atom to cavity photon coupling rate
- κ is the cavity decay rate
- γ is the spontaneous decay rate of the ion

These combine to the important quantities

- $(g/\gamma)^2$ = inverse of the critical photon number, which is the number of photons required to saturate an intra-cavity atom.
- κ/γ: If > 1 we have 'bad cavity regime'
- $\frac{g^2}{\kappa\gamma}$ = cooperativity factor (Purcell factor -1). The inverse is the critical atom number which gives the number of atoms required to have an appreciable effect on the cavity transmission.
- $Q = \pi c/(\lambda\kappa)$: Cavity quality factor, field frequency over decay rate.

4.3.1 Single atoms and ions trapped in cavities

Somewhat less demanding that high-quality single photon sources are experiments that demonstrate atom-photon entanglement.

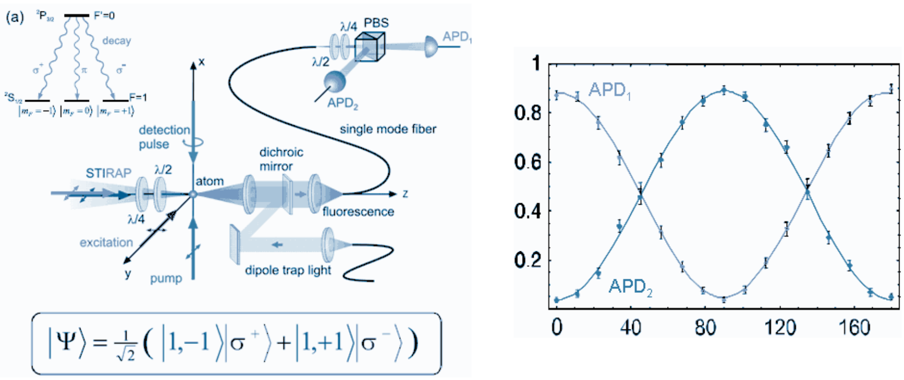


Figure 9: Atom photon entanglement, works even for bad cavities as the reconstruction may be conditioned on the detection of a photon. Can observe 87% fidelity [Volz, Weber et al.. PRL 96, 030404 (2006)].

Single atoms trapped in cavities may be used as a single photon source. Perhaps the currently most impressive experiments in this context is [Hijlkema, Weber, Specht, Webster, Kuhn, Rempe, quant-ph/0702034] where they trap a single ^{85}Rb atom which radiates at $780nm$ inside a cavity with $(g, \kappa, \gamma) = 2\pi \times (5, 5, 3)MHz$.

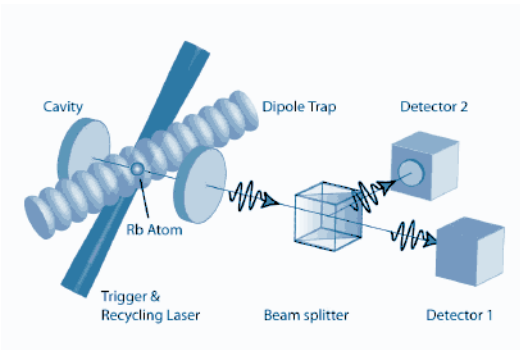


Figure 10: A single ^{85}Rb atom is trapped in a high-finesse optical microcavity by means of a two-dimensional optical lattice. The atom-cavity system is excited by a sequence of laser pulses and single photons emitted from the system are detected by two avalanche photodiodes in Hanbury Brown & Twiss configuration. [Hijlkema, Weber, Specht, Webster, Kuhn, Rempe, quant-ph/0702034]

Single ions trapped in cavities Naturally, combining ion traps with optical cavities is also a viable approach to single photon sources.

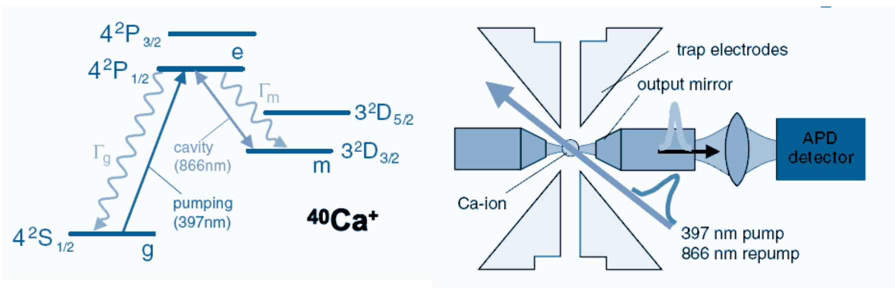


Figure 11: Level scheme and set-up for single photon source from a single ion tarpped inside an optical cavity. [Keller, Lange, Hayasaka, Lange, Walther, NJP 6, 95 (2004); see also see focus issue for single photons on demand in New Journal of Physics 2004 (no subscription needed)].

Cavity QED approaches – In addition it is possible in principle to use trapped ions inside optical cavities to create entangled photons. One example here is the generation of a train of photons that are in a matrix product state.

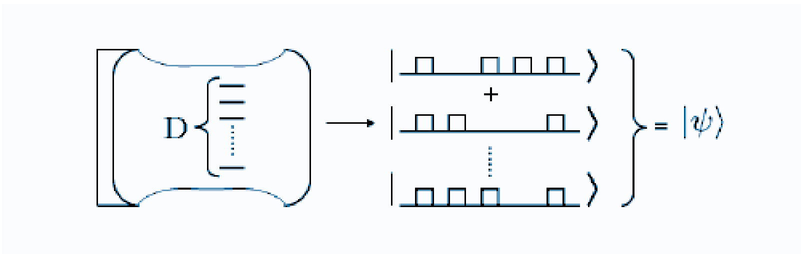


Figure 12: A trapped D-level atom is coupled to a cavity qubit, determined by the energy eigenstates $|0\rangle$ and $|1\rangle$. After bipartite source-qubit operations, photonic time-bins are sequentially and coherently emitted at the cavity output, creating a desired entangled multi-qubit stream. [Schön, Hammerer, Wolf, Cirac, Solano, quant-ph/0612101]

For atom-cavity single photon sources the quality of photons will be high enough for QC only if the co-operativity factor is well above unity [Kiraz, Atature, and Imamoglu, Phys. Rev. A 69, 032305 (2004).]

4.3.2 Quantum information processing in cavity QED

Useful message, if you want to use a noisy degree of freedom, then couple to it in a way that avoids much population in it. Quantum mechanically this is possible as the above approach shows. Generally the underlying principle is that of using a far-detuned Λ-system If $\Omega_i \ll \delta$ and $\delta >> \gamma$ then we obtain and effective dynamics

$$H = \frac{\Omega_1 \Omega_2}{\delta} (|e\rangle\langle g| + |g\rangle\langle e|) \tag{13}$$

The population in the upper level is of the order of $\frac{(\Omega_1+\Omega_2)^2}{\delta^2}$. Thus the spontaneous emission over a single Rabi cycle is $\gamma/\delta \ll 1$. You have moved from e to g without being in the intermediate level.

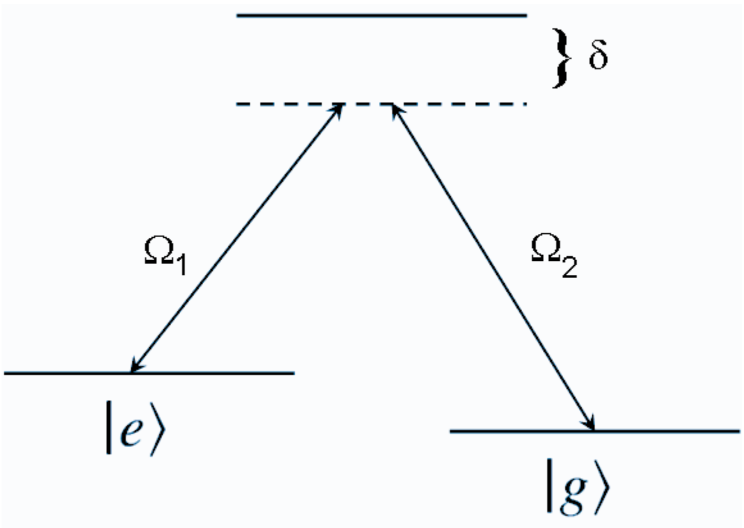


Figure 13:

Atom-cavity Hamiltonian for strrong detuning is of the type

$$H = g(a|e\rangle\langle g|e^{i\delta} + a^{\dagger}|g\rangle\langle e|e^{-i\delta}) \tag{14}$$

Second order perturbation theory now allows for coupling to another atom with only virtual excitation of the phonon mode. Effective transition rate $g_{eff} = \frac{g^2}{\delta}$, effective cavity decay rate $\kappa_{eff} = \kappa\frac{g^2}{\delta^2}$. Thus gate time $\tau = \delta/g^2$, cavity decays $\kappa_{eff}\tau = \kappa/\delta \ll 1$ and thus spontaneous decay $\gamma\tau = \gamma\delta/g^2 \gg \gamma\kappa/g^2$. Thus we need $g^2/(\kappa\gamma) \gg 1$.

4.3.3 Use cavity decay to implement Bell projections

The leakage of photons from the cavity may be used to obtain information about the atoms inside and thus effect state changes on these atoms. This is a special case of generalized measurements and in fact using linear optics networks one may implement complex generalized measurements of the atoms in the cavity.

Two atoms in a one cavity

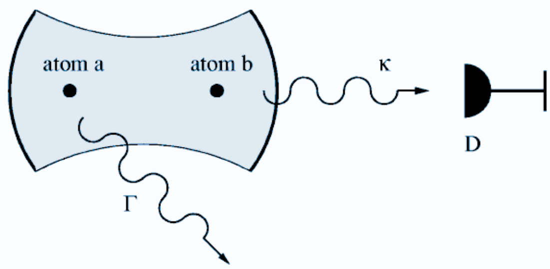


FIG. 1. Experimental setup. The system consists of two two-level atoms placed inside a leaky cavity. The decay rate Γ describes the spontaneous emission of the atoms, while the rate κ refers to photons leaking through the cavity mirrors. The latter can be monitored by the detector D.

Figure 14: Atoms placed symmetrically so they see the same cavity field. Excite one atom (or assymetric excitation) then wait if a photon emerges. If not then the atoms are in a singlet state. If we see a photon, then atoms are in ground state. With detector inefficiency, we end up in a mixture of singlet and ground state. Purify mixture by applying symmetric pulse. This will excite cavity photons if atoms are in ground state. Leakage will detect those, if there is no leakage then weight of the singlet has increased.[Plenio, Huelga, Beige, Knight, PRA 59, 2468 (1999)]

Atoms in distant cavities Single count at the detectors leads to Bell projection of

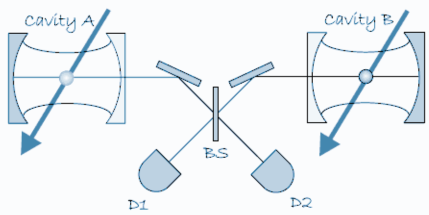


FIG. 1 (color online). We consider a setup in which individual ions are trapped inside two spatially separated optical cavities A and B. Photons can leak out of the cavities and are then mixed on a beam splitter BS and subsequently detected by photodetectors D_1 and D_2.

Figure 15:

photon mode. $c = (a+b)/\sqrt{2}$ and $d = (a-b)/\sqrt{2}$. [Bose, Knight, Plenio, Vedral, PRL 83, 5158 (1999); Duan, Kimble, Phys. Rev. Lett. 90, 253601 (2003); Browne, Plenio, Huelga, PRL 91, 067901 (2003)]

Generally, a complicated beam-splitter network outside of the cavity allows for the implementation of complex generalized measurements of the atoms inside of the cavity.

4.3.4 BEC in cavity

A BECs may be coupled to a strong-coupling cavities. This is made possible by combining a new type of fibre-based cavity with atom chip technology. This allows single-

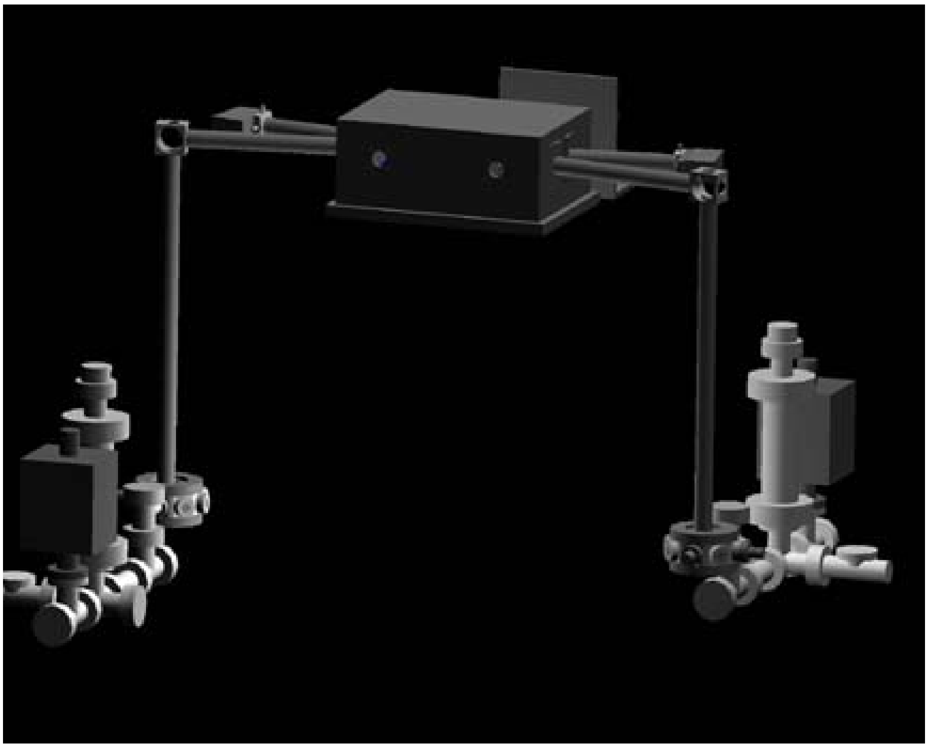
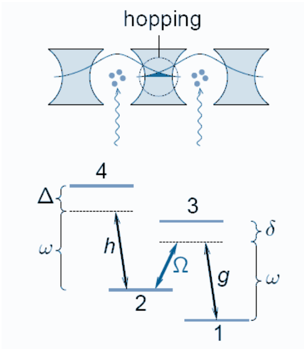


Figure 16: Experiment by [Maunz, Moehring, Olmschenk, Younge, Matsukevich, Monroe, Nature Physics June 2007]

atom cQED experiments with a simplified setup, but moreover realizes the new situation of N atoms in a cavity each of which is identically and strongly coupled to the cavity mode. The BEC can be positioned deterministically anywhere within the cavity and localized entirely within a single antinode of the standing-wave cavity field. This gives rise to a controlled, tunable coupling rate, as we confirm experimentally. [Colombe, Steinmetz, Dubois, Linke, Hunger, Reichel, arxiv:0706.1390]

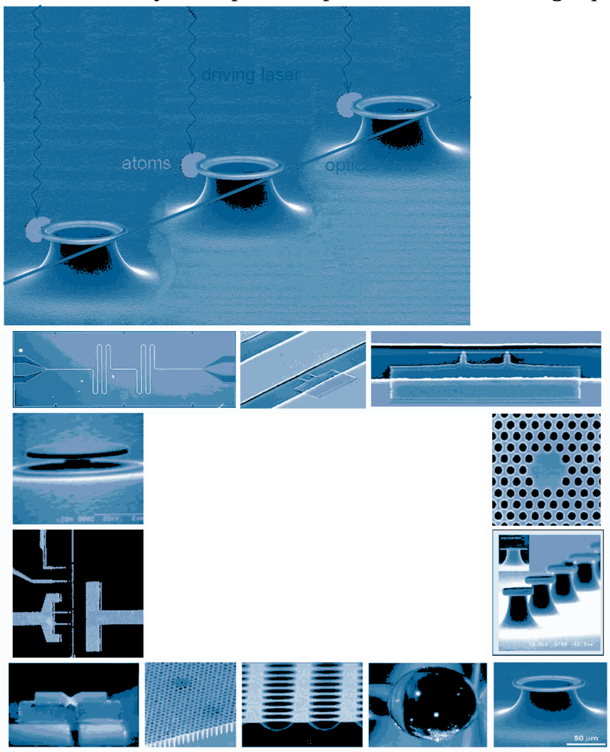
4.3.5 Arrays of coupled micro-cavities

Cavities may be arranged in arrays, e.g. in photonic crystals, or coupled via fibres. Then they will form effective many body systems. With effective non-linearities inside the cavities generated by atoms these systems are capable of generating Bose-Hubbard models, spin chains etc for, in principle, arbitrary geometries. [Hartmann, Brandao, Plenio, Nature Physics 2, 849 (2006)]



4.3.6 Examples of cavities

There are many examples of optical cavities with high quality factors.



5 ’Pure’ Matter

Strictly speaking there is no pure matter quantum information processing really (except perhaps in the solid state domain) as we need light to prepare, manipulate and measure the qubits which are usually of atomic nature.

I will only mention trapped particles which fall into two categories, charged and not charged.

5.0.7 Trapped Ions

Trapped ions are one of the most advances technologies when it comes to the controlled manipulation of few-particle systems at the quantum level. Currently the device of choice is the linear ion trap based on the Paul trap but the Penning trap is also still of interest and may see a revival due to some advantages.

See $http : //monroelab2.physics.lsa.umich.edu/research_info/TIQCworkshop/proceedings.html$ for a nice collection of lectures.

Linear ion traps – In a linear ion trap all particles are held by time dependent electric potential that give rise to a effective potential that is restoring in all three spatial dimensions (think of a saddle that is rotating along the vertical axis). If the confining potential is stronger in the xy-plane than the z axis, the particle will line up along the axis of the trap. The distance between neighboring ions depends on the strength of the confining potentials and their position in the chain. Typical distances are several microns which makes the ions individually addressable in principle.

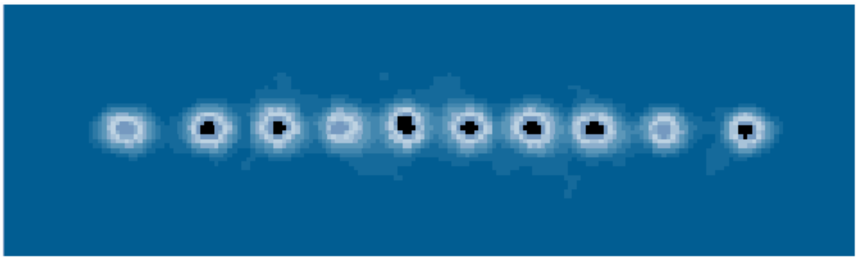


Figure 17: From Blatt, Innsbruck

The ions have internal and external degrees of freedom.

There are two principal modes of oscillation for the ions. Either they oscillate along the cavity axis which gives rise to center of mass modes, stretch modes etc or they may oscillate orthogonal to the cavity axis. In that case we have a set of weakly coupled harmonic oscillators with a long range coupling that falls of like $1/r^3$. This may be problematic for quantum information processing but may not be so bad for quantum simulations [Deng, Porras, Cirac, quant-ph/0703178]. This allows you to create Bose-Hubbard Hamiltonians and you may observe phases like the Mott-phase, the superfluid phase and a Tonks gas regime.

Important parameter is Lamb-Dicke parameter $\eta = kx_0 = k\sqrt{\frac{\hbar}{2m_0\omega}}$. When it is small than unity then the spontaneous emission of a photon will be unlikely to excite a phonon $p \cong 1 - e^{\eta^2}$ like in the Moessbauer effect the trap as a whole takes up the recoil rather than the individual ion.

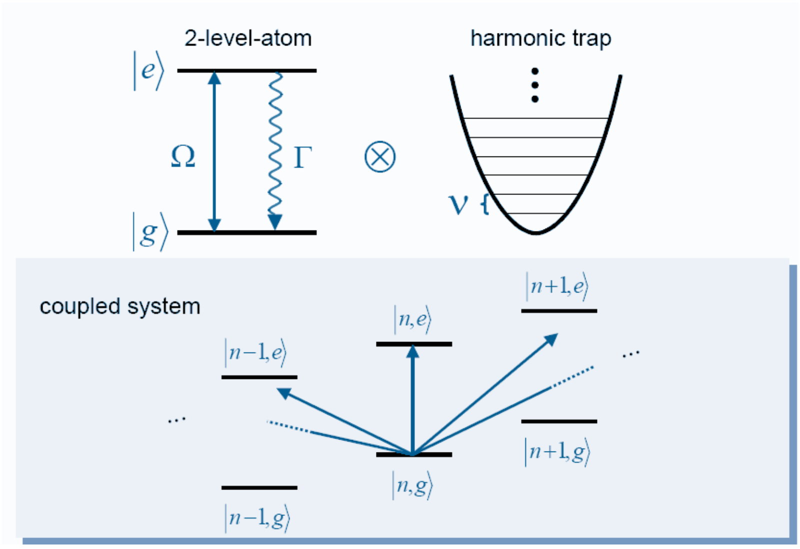


Figure 18: From Blatt LesHouches Lectures. To see the sidebands the decay rate has to be smaller than the motional frequency.

5.0.8 Detection in ion traps

Detection is important and can be achieved very reliably via quantum jump techniques

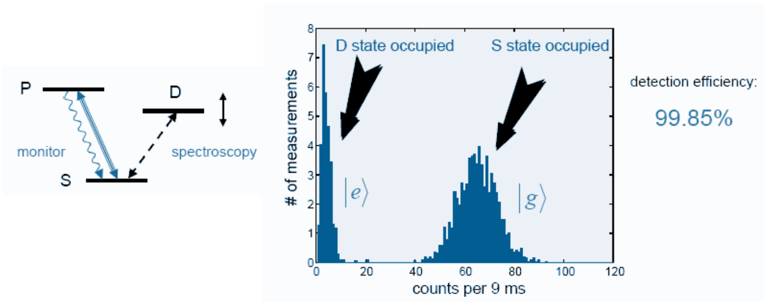


Figure 19: From Blatt LesHouches Lectures. Electronic state of ions can be detected very reliably.

5.0.9 Ion trap quantum gates

The ions feel each via their electric charge. In particular their center of mass mode may be used to create quantum gates using laser pulses on ions to create ion-motion interactions. Through individual addressing it is possible to affect individual ions while the phonon that is generated is felt by all ions in the same way. More precisely, one first swaps the state of one ion into the phonon mode, then affects a CNOT gate between the mode and another ion and then swaps the state of the mode back inot the first ion. That is the basic principle of the Cirac Zoller gate [Cirac, Zoller, Phys. Rev. Lett. 74, 4091 (1995)].

As such the gate is sensitive to various inaccuracies including finite temperatures of the phonon mode. Various improvements have been proposed.

1. Temperature: The Sorensen-Molmer gate is an approach that allows for quantum gates that are far less sensitive to finite temperatures. They employ interference to cancel terms that depend on the phonon number. [Sorensen, Molmer, Phys. Rev. Lett. 82 1971 (1999); PRA 62, 022311 (2000)]

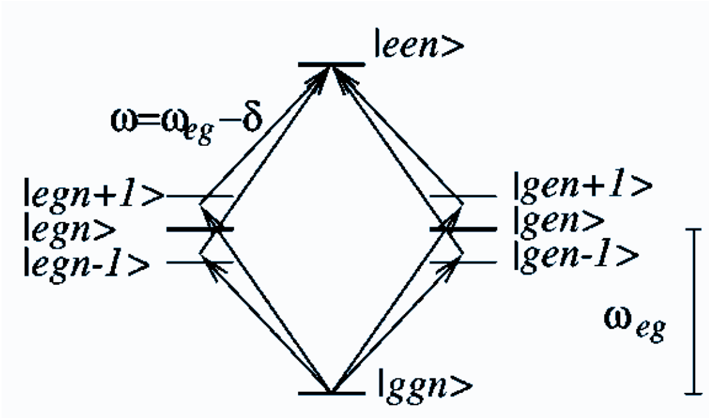


Figure 20:

$$H \cong \eta\Omega a|e\rangle\langle g|e^{i\delta t} + \eta\Omega a^\dagger|g\rangle\langle e|e^{-i\delta t} + \eta\Omega a^\dagger|e\rangle\langle g|e^{-i\delta t} + \eta\Omega a|g\rangle\langle e|e^{i\delta t} \quad (15)$$

Second order perturbation theory yields

$$\tilde{\Omega} = 2\sum_m \frac{\langle een|H_{int}|m\rangle\langle m|H_{int}|ggn\rangle}{E_m - (E_{ggn} + \omega_m)} = -\frac{(\Omega\,\eta)^2}{\nu - \delta}$$

The gate is not very sensitve to heating either. Have $\eta\Omega \ll \delta \ll \nu$ and heating rate $\ll \delta$.

2. Geometric phase gates: In fact the above gate may be interpreted as a geometric phase gate. This was also recognized by [Milburn, Schneider, James, Fortschr. Phys. 48, 801 (2000)] realized by [Leibfried et al, Nature **422**, 412 (2003)] Other geometric phase gates have been proposed by ...

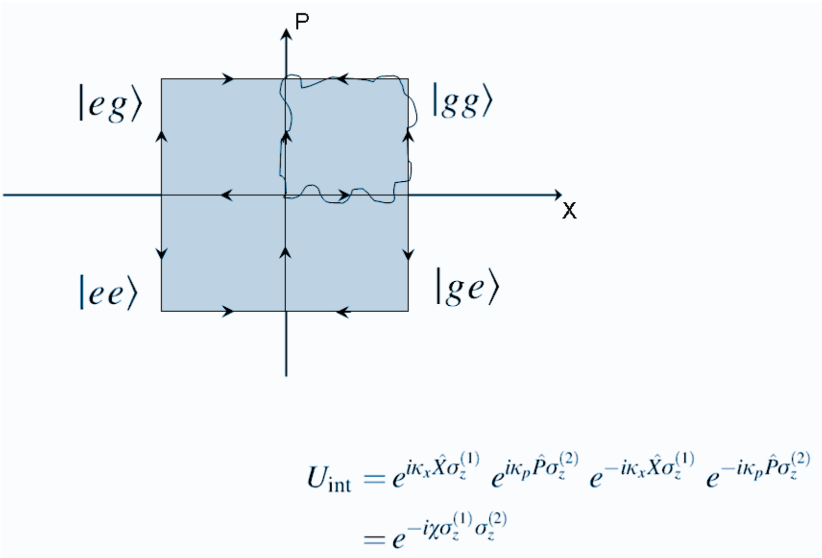


Figure 21: Displacements in phase space lead to the accumulation of a phase. Detailed path is not important just the area.

3. Composite pulse techniques: From NMR one may learn that composite pulses may be more fault tolerant to small errors than are single pulses [Gulde, Riebe, Lancaster, Becher, Eschner, Haffner, Schmidt-Kaler, Chaung, and Blatt, Nature **421**, 48 (2003)]

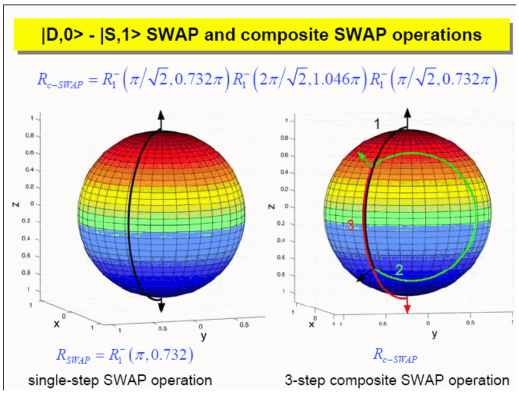


Figure 22:

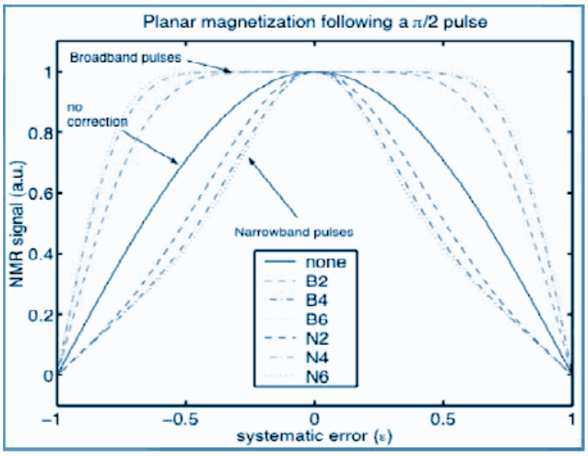


Figure 23: Brown, Chuang and Harrow

5.0.10 Limits

The motional degree of freedom has long lifetimes up to a few 100ms. The electronic degree of freedom is affected by stray fields etc but this is aided considerably by using encoding techniques such as $|01\rangle$ and $|10\rangle$ to represent a single qubit.

Ion trap quantum computing will suffer from some 'fundamental' error rates for example due to off-resonant couplings. But these error rates will be small, of the order of 10^{-6} per gate. [Plenio, Knight, PRA 53, 2986 (1996); Proc Roy Soc 453, 2017 (1997); Hughes, James, Knill, Laflamme, Petschek, PRL 77, 1996]. Going to lower frequencies helps.

5.0.11 Microwave traps

Remarkably it is possible to generate quantum gates in ion traps without the use of laser beams. In fact microwaves are sufficient if one uses an additional magnetic field. Changing the internal state of the ions now leads to a displacement, ie a coupling of internal and external degree of freedom. The other ions feel this change via their electromagnetic repulsion. This then allows for the implementation of quantum gates but also the creation of e.g. Ising model Hamiltonians.

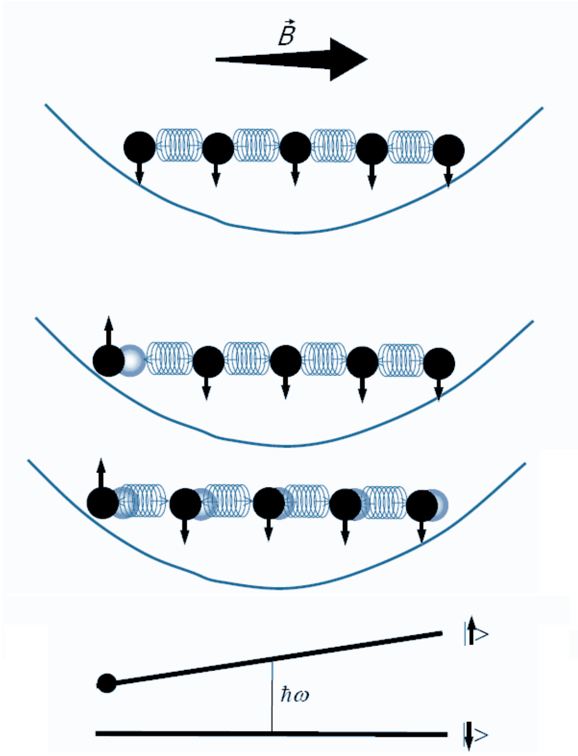


Figure 24:

5.0.12 Large crystals in optical cavities

There is another interesting development in ion trap physics for example in Aarhus. Here large numbers of ions are trapped (possibly inside a resonator) and one may observe various types of crystal order. This may not be useful for quantum information

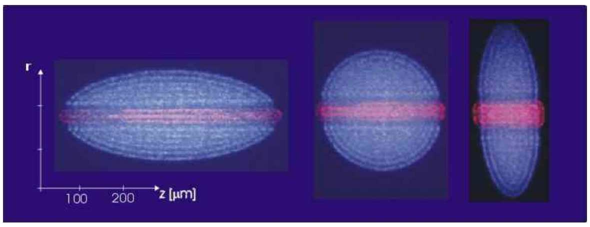


Figure 25:

processing but interesting for many body physics.

5.0.13 Penning traps

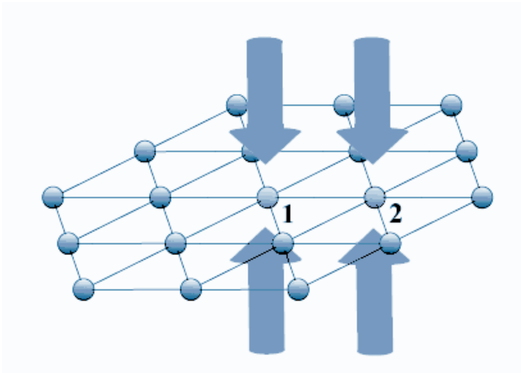


Figure 26: An 2-D ion crystal may be formed in a Penning trap and the oscillations of ion orthogonal to this plane may be used. Interactions may be induced for example via state-dependent dipole forces due to standing wave lasers. [Porras, Cirac, quant-ph/. 0601148]

References

[1] ...