

# QUANTUM INFORMATION

## with light and atoms

### *Lecture 2*

**Alex Lvovsky**

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# MAKING QUANTUM STATES OF LIGHT

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1. Photons
2. Biphotons
3. Squeezed states
4. **Beam splitter**
5. Conditional measurements

# Beam splitter transformation (Heisenberg picture)

- **Quadrature transformation**

$$\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} = \begin{pmatrix} t & r \\ -r & t \end{pmatrix} \begin{pmatrix} \hat{a}'_1 \\ \hat{a}'_2 \end{pmatrix}$$

where  $t^2$  is the beam splitter transmission,  $r^2$  reflection.  $|t|^2 + |r|^2 = 1$ .

- If  $t$  and  $r$  are real, we can assign  $t = \cos \theta$ ;  $r = \sin \theta$ ;

$$\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \hat{a}'_1 \\ \hat{a}'_2 \end{pmatrix}$$

Also valid for positions, momenta

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix}; \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} p'_1 \\ p'_2 \end{pmatrix}$$

→ **Beam splitter transformation = rotation in the phase space ⇒ entanglement**

Problem: Show that a beam splitter acting on a pair of coherent states will generate a pair of coherent states

# Example:

## Einstein-Podolsky-Rosen state

- **State preparation**

- Overlap  $X$ -squeezed ( $\langle \delta x_1^2 \rangle < 1/2$ ) and  $P$ -squeezed ( $\langle \delta p_2^2 \rangle < 1/2$ ) vacuum states on a symmetric beam splitter

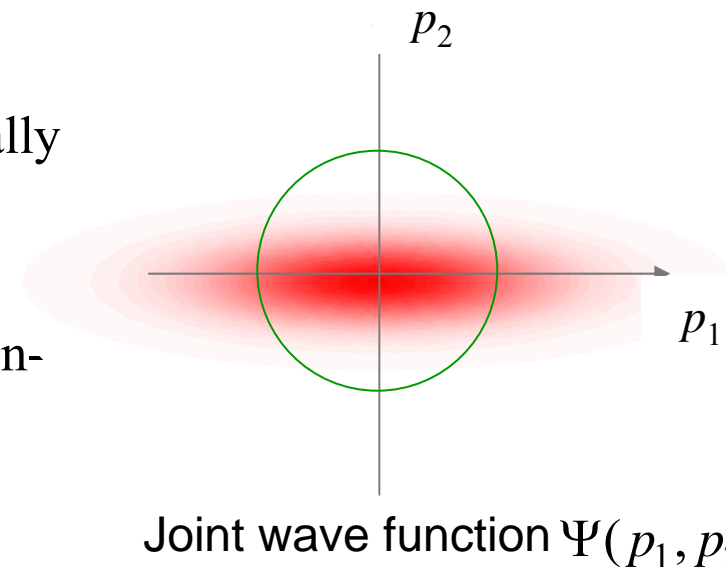
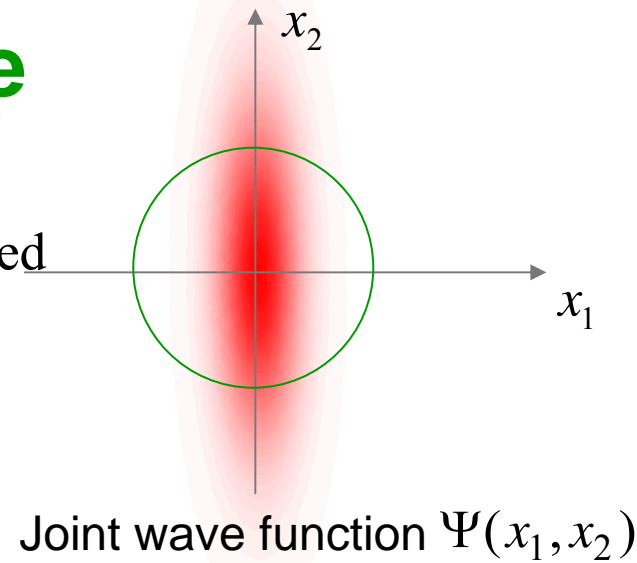
- Beam splitter transformation:

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} X_2 - X_1 \\ X_2 + X_1 \end{pmatrix}; \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} P_2 - P_1 \\ P_2 + P_1 \end{pmatrix}$$

- After beam splitter:

$$\langle \delta(x_1 - x_2)^2 \rangle < 1/2; \langle \delta(p_1 + p_2)^2 \rangle < 1/2$$

- Both positions and momenta are nonclassically correlated.
- Entangled state generated!
- This states approximates the original Einstein-Podolsky-Rosen state.



# By the way: the original EPR paradox

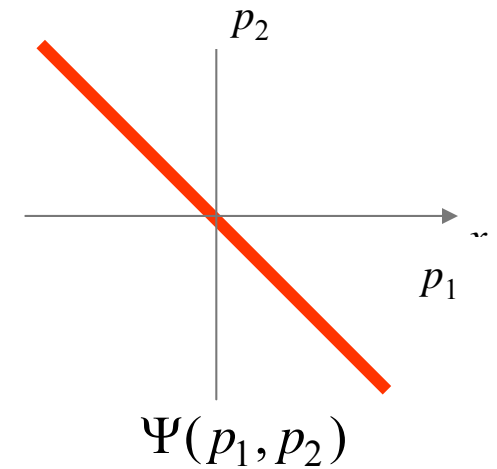
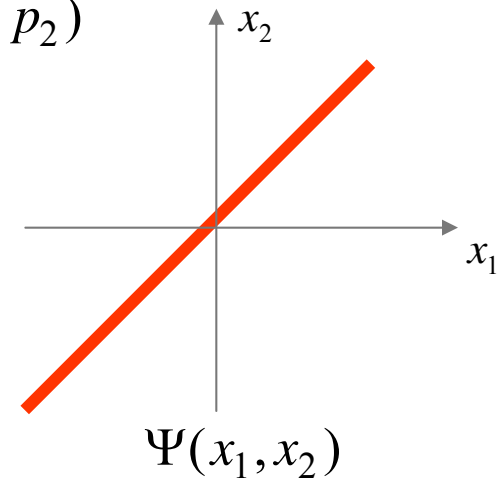
- **The ideal Einstein-Podolsky-Rosen state**

- **In position representation:**  $\Psi_{\text{EPR}}(x_1, x_2) = \delta(x_1 - x_2)$
- **In momentum representation:**  $\Psi_{\text{EPR}}(p_1, p_2) = \delta(p_1 + p_2)$

Problem: obtain the position representation wave function from the momentum representation

- **If shared between Alice and Bob:**

- **If Alice measures position  $x_1$**   
→ **Bob receives a position eigenstate  $|x_2\rangle = |x_1\rangle$**
  - **If Alice measures momentum  $p_1$**   
→ **Bob receives a momentum eigenstate  $|p_2\rangle = |-p_1\rangle$**
- **Alice can create two mutually incompatible physical realities at a remote location**



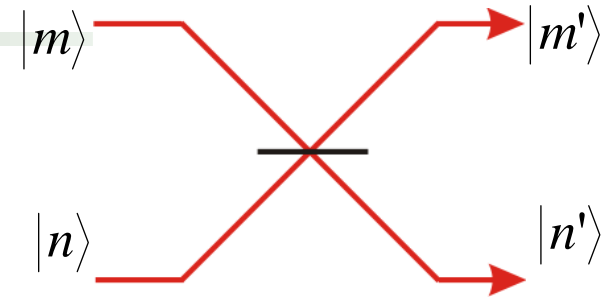
Theory: Einstein, Podolsky, Rosen, PRA **47**, 777 (1935)

Experiment: Z.Y.Ou et al., PRL 68, 3663 (1992)

# Beam splitter transformation (Schrödinger picture)

- Photon number transformation

$$|m, n\rangle \rightarrow \sum_{j,k=0}^{m,n} \sqrt{\frac{(j+k)!(m+n-j-k)!}{m!n!}} \binom{m}{j} \binom{n}{k} (-1)^k t^{n+j-k} r^{m-j+k} |m+n-j-k, j+k\rangle$$



- Simplest example: splitting a photon

$$|1\rangle \rightarrow t|1\rangle_A |0\rangle_B - r|0\rangle_A |1\rangle_B$$

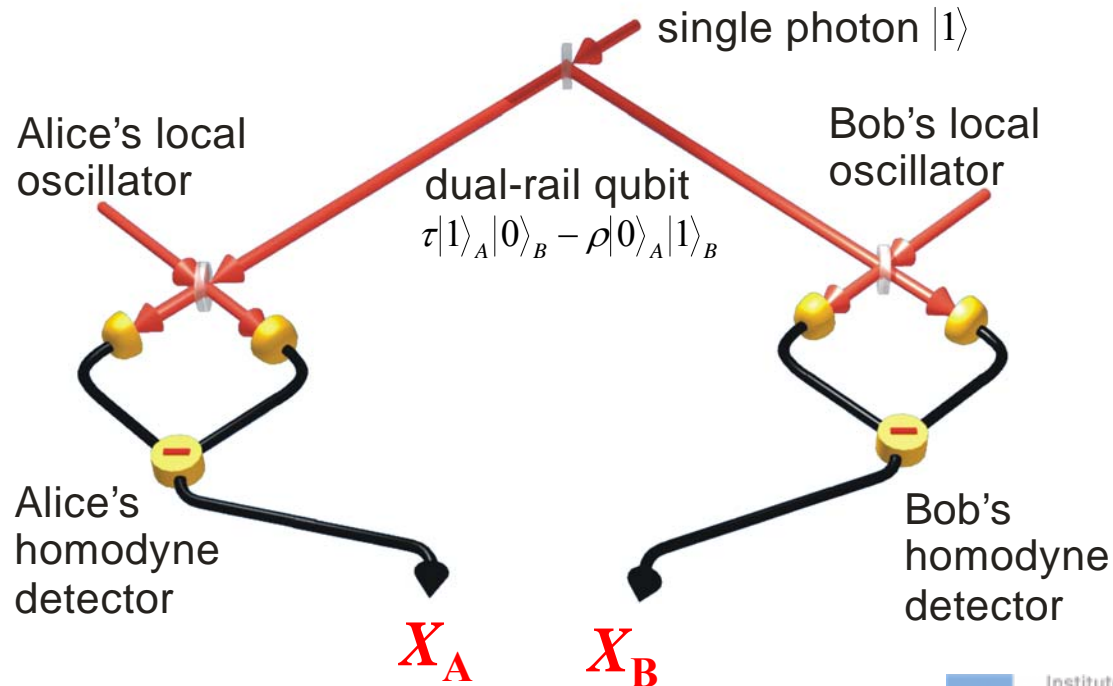
# Example:

## Tomography of a dual-rail qubit

- Photon hits a beam splitter → a **two-mode qubit** is generated

$$|1\rangle \rightarrow t|1\rangle_A|0\rangle_B - r|0\rangle_A|1\rangle_B$$

- Measure quadratures  $X_A$  and  $X_B$  via homodyne detectors
- Phase/dependent quadrature statistics → state reconstruction

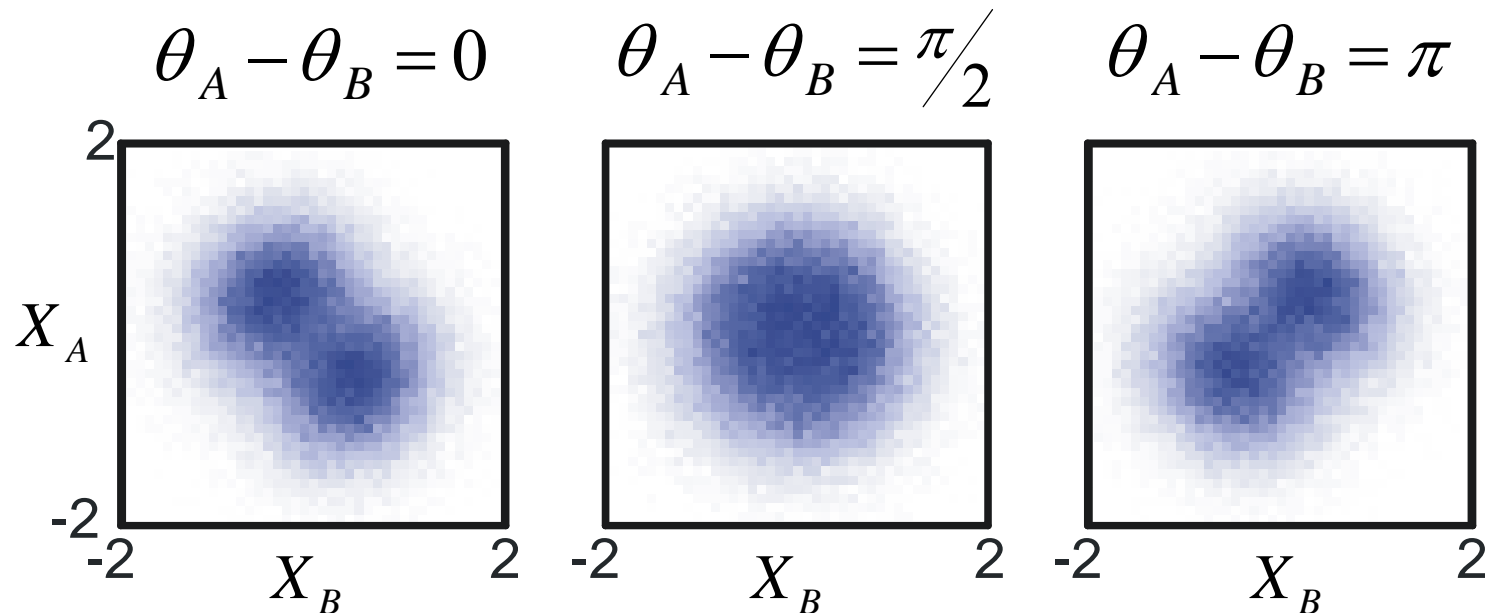


# Example:

## Tomography of a dual-rail qubit (...continued)

- **Probability distributions**  $\text{pr}(X_A, X_B)$

- Serve as marginal distributions for the 4-D Wigner function
- Entanglement  $\rightarrow$  Nonclassical, phase-dependent correlations

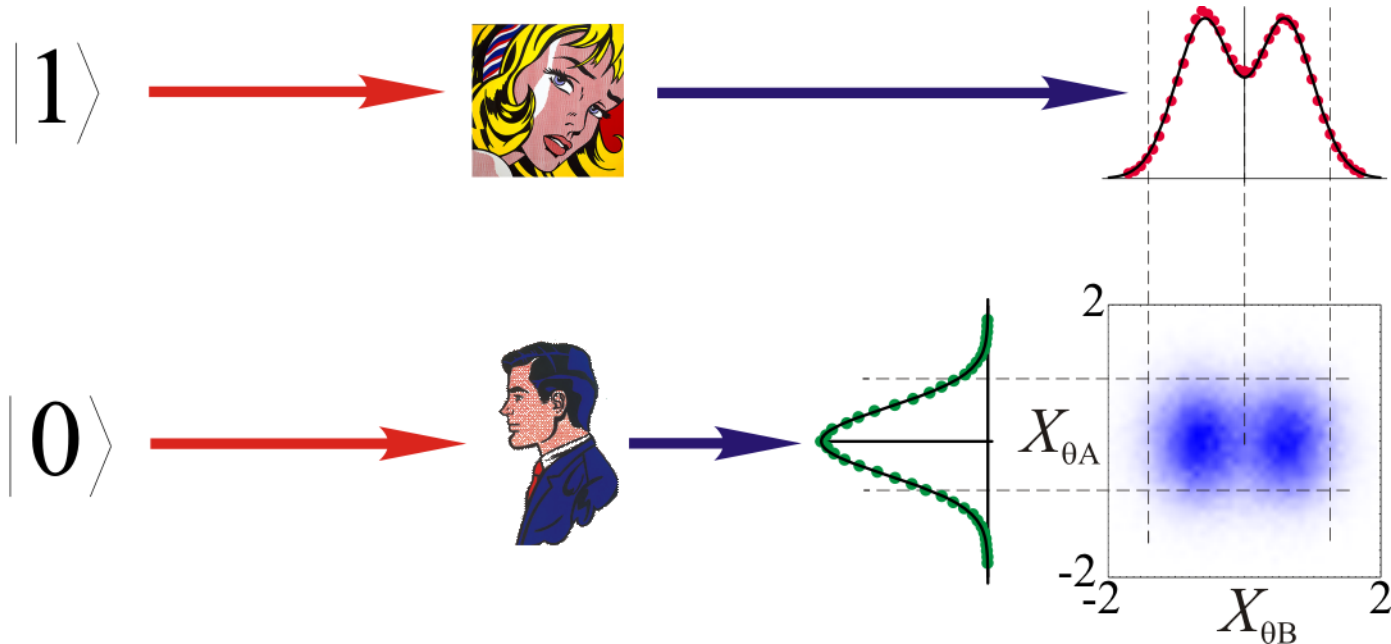


- **Probability distributions**  $\text{pr}(X_A, X_B)$  for all  $\theta_A, \theta_B$   
 $\rightarrow$  quantum state reconstruction



# Why do we see such distributions?

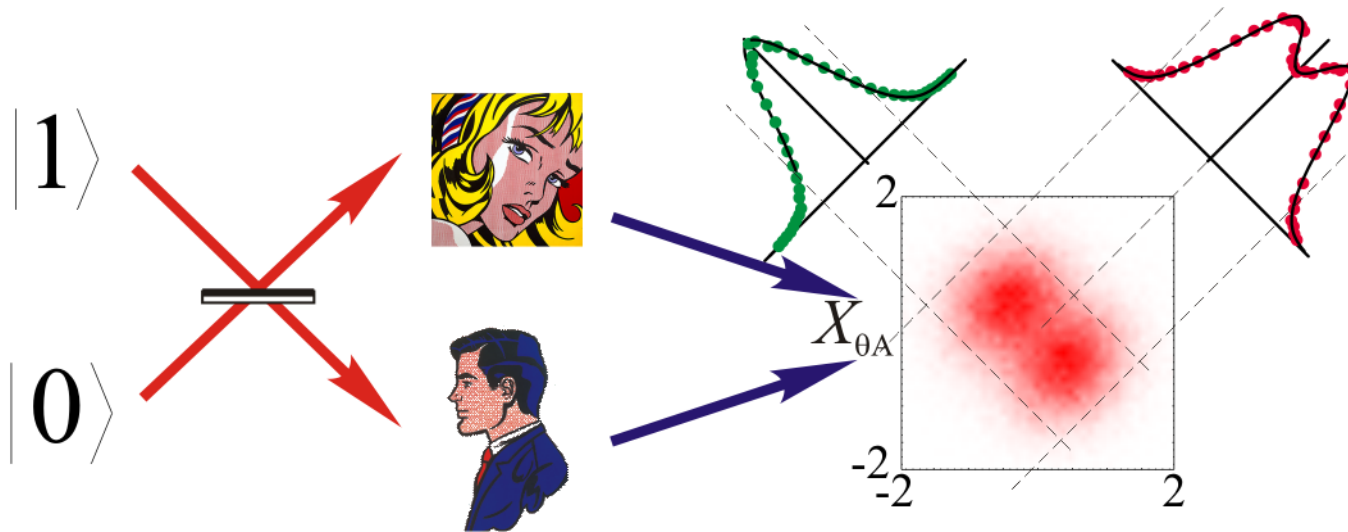
- If no beam splitter is present



- Alice measures the single-photon state, Bob measures the vacuum state
- Measurements are uncorrelated  $\rightarrow$  distributions are uncorrelated
- No entanglement
- No information about relative phase  $\theta_A - \theta_B$

# Why do we see such distributions?

- If beam splitter is present



- Alice and Bob measure the same quadrature ( $\theta_A - \theta_B = 0$  or  $\pi$ )  
→ uncorrelated distribution rotates by  $45^\circ$
- Alice and Bob measure different quadratures ( $\theta_A - \theta_B = \pi/2$ )  
→ distribution remains uncorrelated

# Tomography of an optical qubit

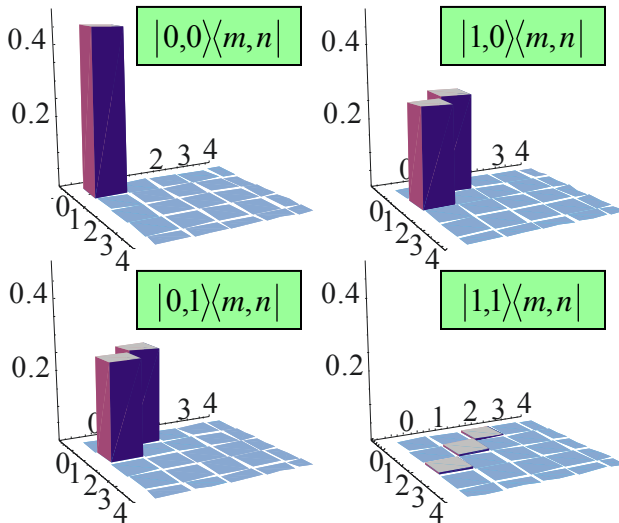
## Results

S. Babichev, J. Appel, A. I. Lvovsky, PRL **92**, 193601 (2004)

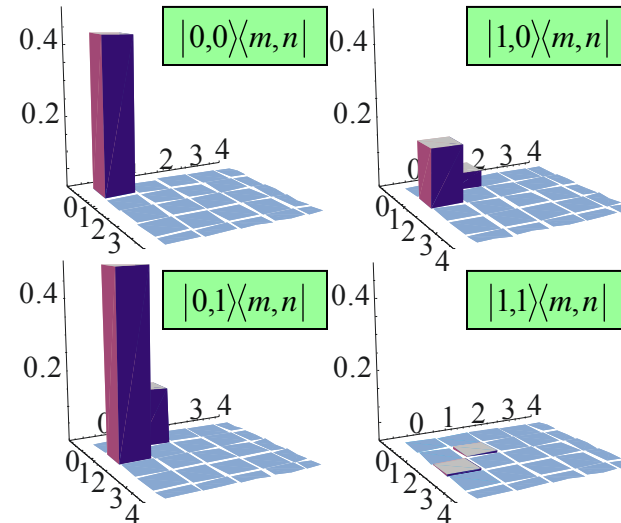
- **Density matrix**

- Serve as marginal distributions for the 4-D Wigner function
- Entanglement → Nonclassical, phase-dependent correlations

- Beam splitter 50 - 50 %



- Beam splitter 8 - 92 %



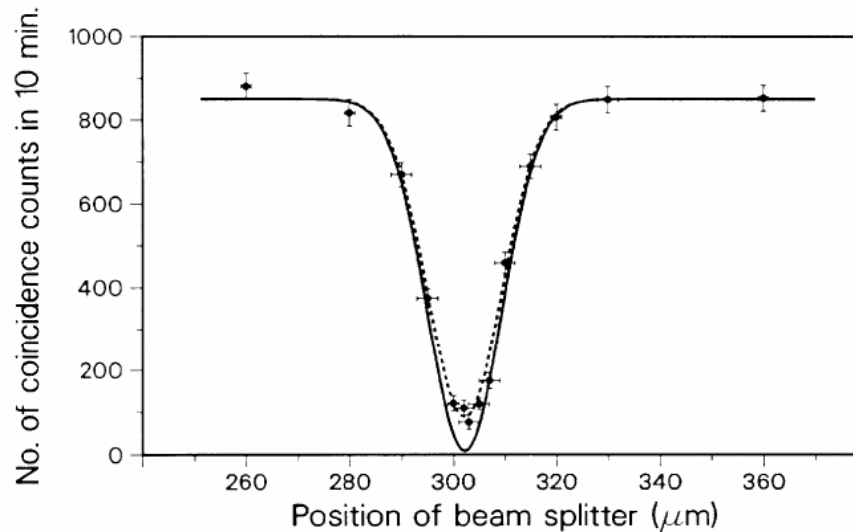
**First complete (not postselected) reconstruction of an optical qubit**

# Another example: Hong-Ou-Mandel dip

- Two photons "colliding" on a beam splitter will stick together

$$|1,1\rangle \rightarrow (|2,0\rangle - |0,2\rangle)/\sqrt{2}$$

- **Hong-Ou-Mandel effect: correlation count in the beam splitter output vanishes when the two photons arrive simultaneously.**

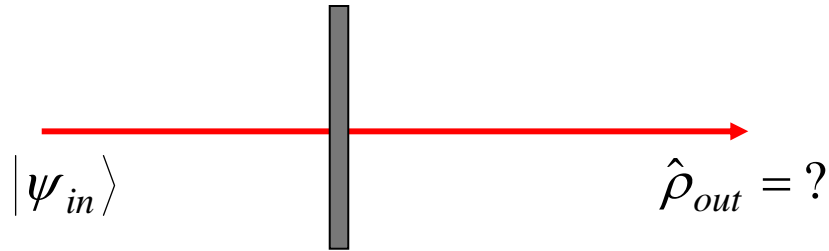


Hong, Ou, Mandel, PRL **59**, 2044 (1987)

# Beam splitter model of absorption

- **The problem**

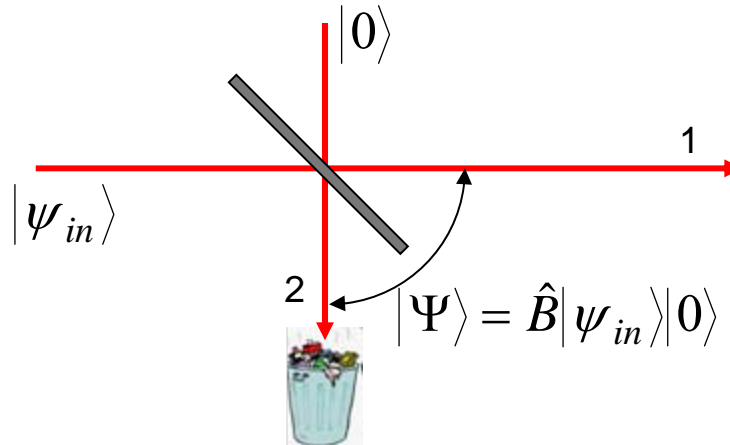
- A quantum state of light propagates through an attenuator.  
What is the transmitted state?



# Beam splitter model of absorption

- **The solution**

- **Replace the absorber with a beam splitter.**
- **The second input is vacuum**



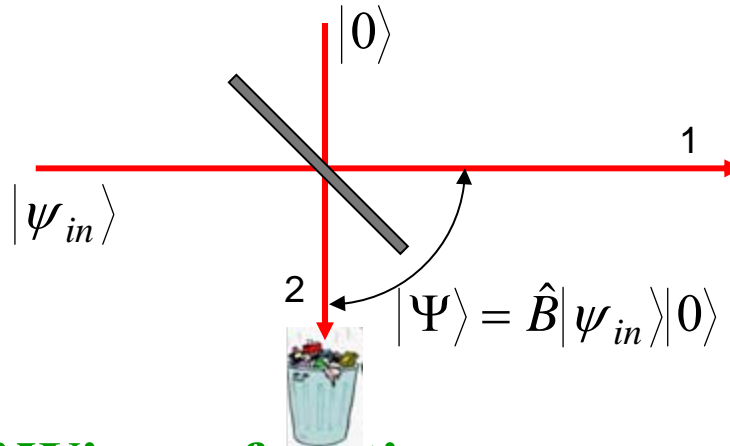
- **Beam splitter output  $|\Psi\rangle$  may be entangled**
- **Mode 2 in the beam splitter output is lost**
- **To find  $\hat{\rho}_{out}$ , trace over the lost mode in the beam splitter output**

$$\hat{\rho}_{out} = \text{Tr}_2 |\Psi\rangle\langle\Psi|$$

# Beam splitter model of absorption

- **The solution**

- Replace the absorber with a beam splitter.
- The second input is vacuum



Suppose you have a squeezed state with  $\langle x^2 \rangle = 1/4$ ,  $\langle p^2 \rangle = 1$ . How will these parameters change after propagating through a 50% absorber?

- **In terms of Wigner functions**

- Beam splitter input Wigner function:  $W_{|\psi\rangle|0\rangle} = W_{|\psi\rangle}(x_1, p_1)W_{|0\rangle}(x_2, p_2)$ .
- To find the beam splitter output Wigner function  $W_{|\Psi\rangle}(x_1, p_1, x_2, p_2)$  apply phase-space rotation.
- To find the Wigner function of mode 1, integrate over mode 2:

$$W_{out}(x_1, p_1) = \int_{-\infty}^{+\infty} W_{|\Psi\rangle}(x_1, p_1, x_2, p_2) dx_2 dp_2.$$

# MAKING QUANTUM STATES OF LIGHT

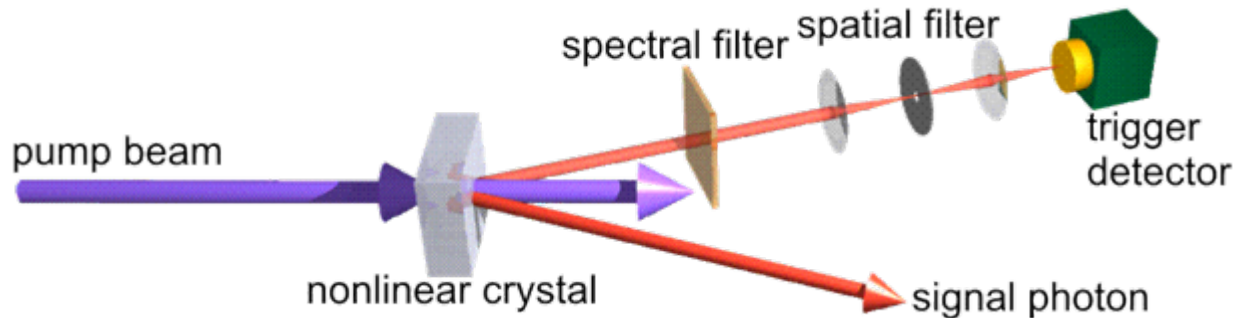
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1. Photons
2. Biphotons
3. Squeezed states
4. Beam splitter
5. **Conditional measurements**



# Conditional preparation of a photon

- **Parametric down-conversion**
  - “Red” photons are always born in pairs
  - Photon detection in one emission channel  
→ there must be a photon in the other channel as well



☹️ Not a single photon “on demand”

😊 To date, this is the only method which provides a single photon with a high efficiency in a certain spatiotemporal mode

# Schrödinger cat

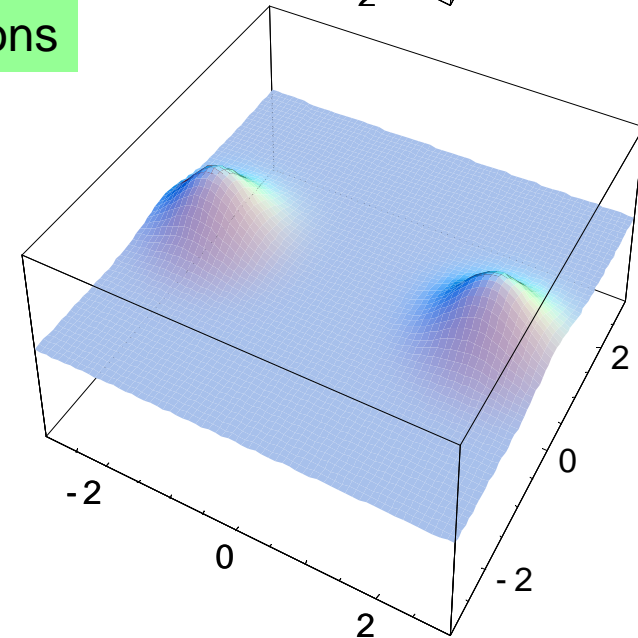
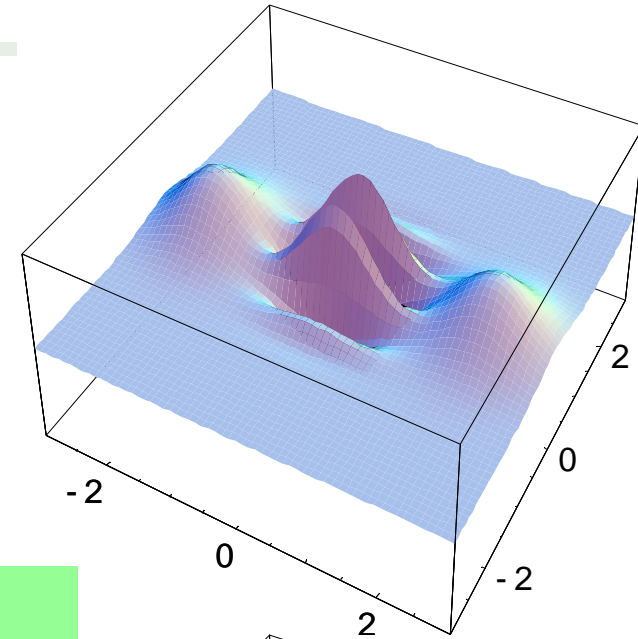
What does it mean in optics?

- **Coherent superposition of two coherent states**

$$|\text{cat}_{\pm}\rangle = |\alpha\rangle \pm |-\alpha\rangle$$

- Useful for quantum teleportation quantum computation, and error correction
- Fundamentally important

Problem. Calculate these Wigner functions



- **Compare: incoherent superposition of two coherent states**

$$\hat{\rho} = |\alpha\rangle\langle\alpha| \pm |-\alpha\rangle\langle-\alpha|$$

- Boring, classical state

# Schrödinger cat

## How to make one?

- Easy for small  $\alpha$ 's

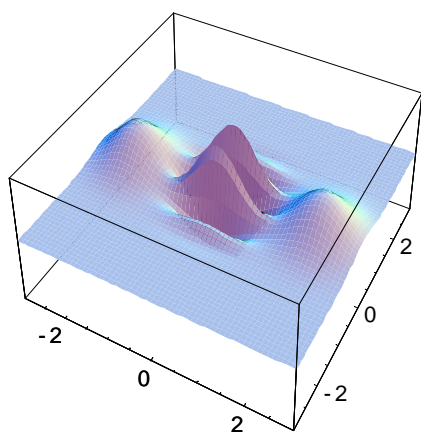
$$|\alpha\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \alpha_3|3\rangle + \dots$$

$$|-\alpha\rangle = \alpha_0|0\rangle - \alpha_1|1\rangle + \alpha_2|2\rangle - \alpha_3|3\rangle + \dots$$

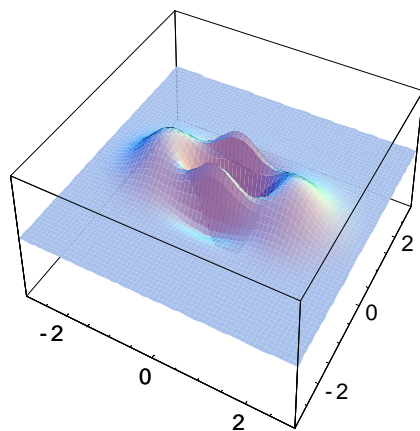
neglect  
high-number terms

$$|\text{cat}_-\rangle = \alpha_1|1\rangle - \alpha_3|3\rangle + \dots$$

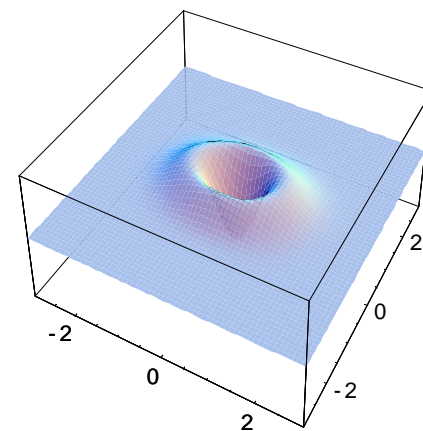
...just a squeezed single-photon state!



$\alpha = 2.1$



$\alpha = 1.4$



$\alpha = 0.7$

# Schrödinger cat

## How to make one?

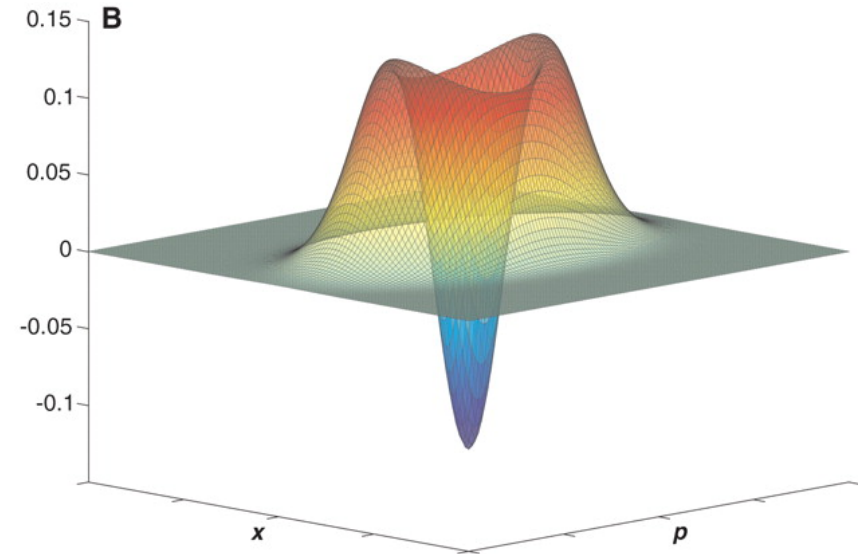
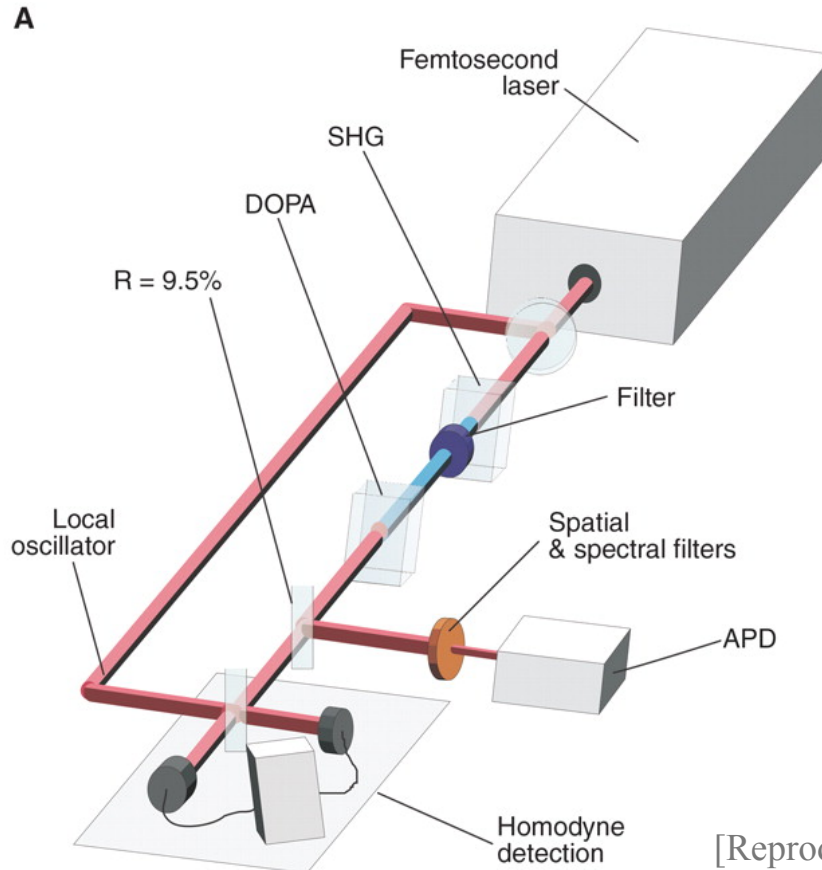
[A. Ourjoumtsev *et al.*, *Science* **312**, 83 (2006)]

J. S. Neergaard-Nielsen *et al.*, *PRL* **97**, 083604 (2006)

K. Wakui *et al.*, *quant-ph/0609153*]

- **Making a squeezed single-photon state**

- **Create a squeezed state**  $|\psi_s\rangle = \beta_0|0\rangle + \beta_2|2\rangle + \beta_4|4\rangle + \dots$
- **Subtract a photon**  $\hat{a}|\psi_s\rangle = \sqrt{2}\beta_2|1\rangle + 2\beta_4|3\rangle + \dots$



[Reproduced from A. Ourjoumtsev *et al.*, *Science* 312, 83 (2006)]

# Summary to part 2:

## Classification of quantum state preparation methods

- **“On demand”**:  
State is readily available  
when required by the user  
Example: photon from a quantum dot
- **“Heralded”**:  
State produced randomly; system provides user  
with a classical signal when the state is produced  
Example: heralded single photon
- **“Postselected”**:  
State is not known to have been produced  
until it is detected  
Example: photon pair from a down-converter

simplicity

usefulness



**Postselected + conditional measurement = Heralded (maybe)**

**Heralded + memory = On demand**

# QUANTUM REPEATER

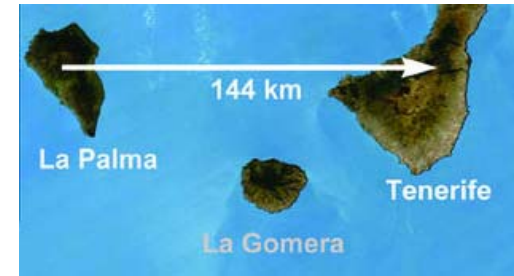
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and memory for light

# Quantum cryptography: here and now

## Secure communication up to 100-150 km

- Free space
- Optical fibers



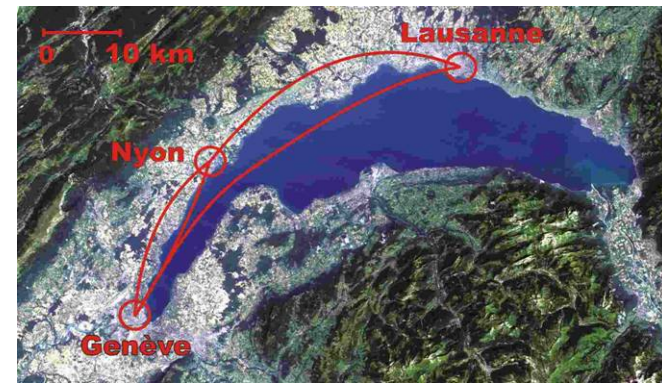
## Commercialization begins

- Id Quantique (Switzerland)
- MagiQ (Boston)
- BBN Technologies (Boston)



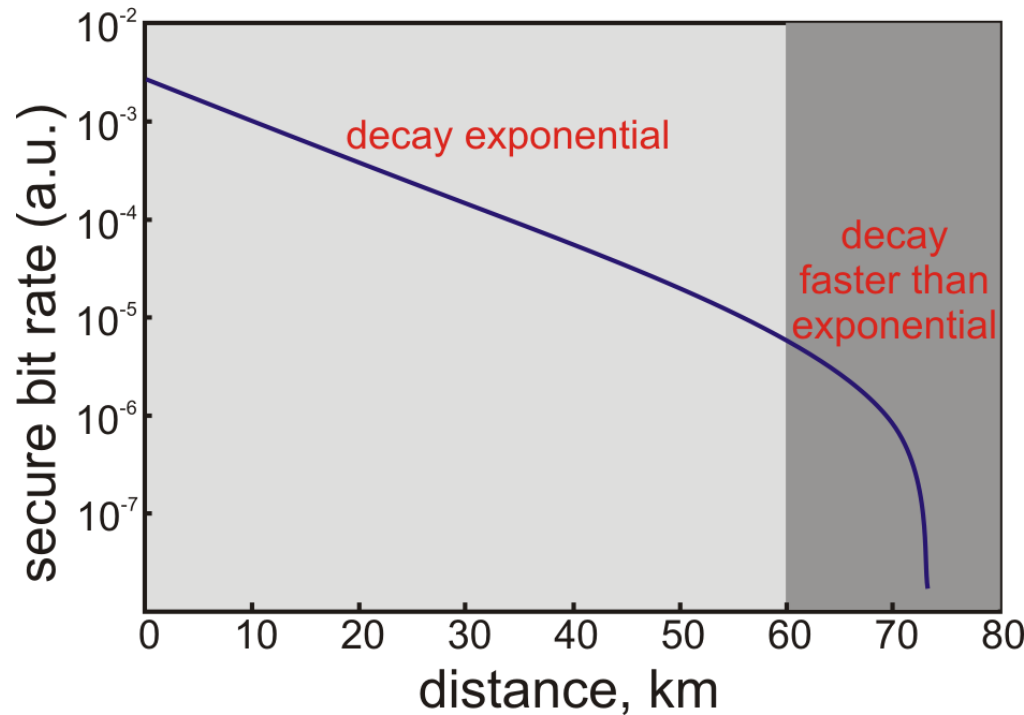
## Metropolitan quantum communication networks

- Geneve
- Boston
- Vienna
- Calgary



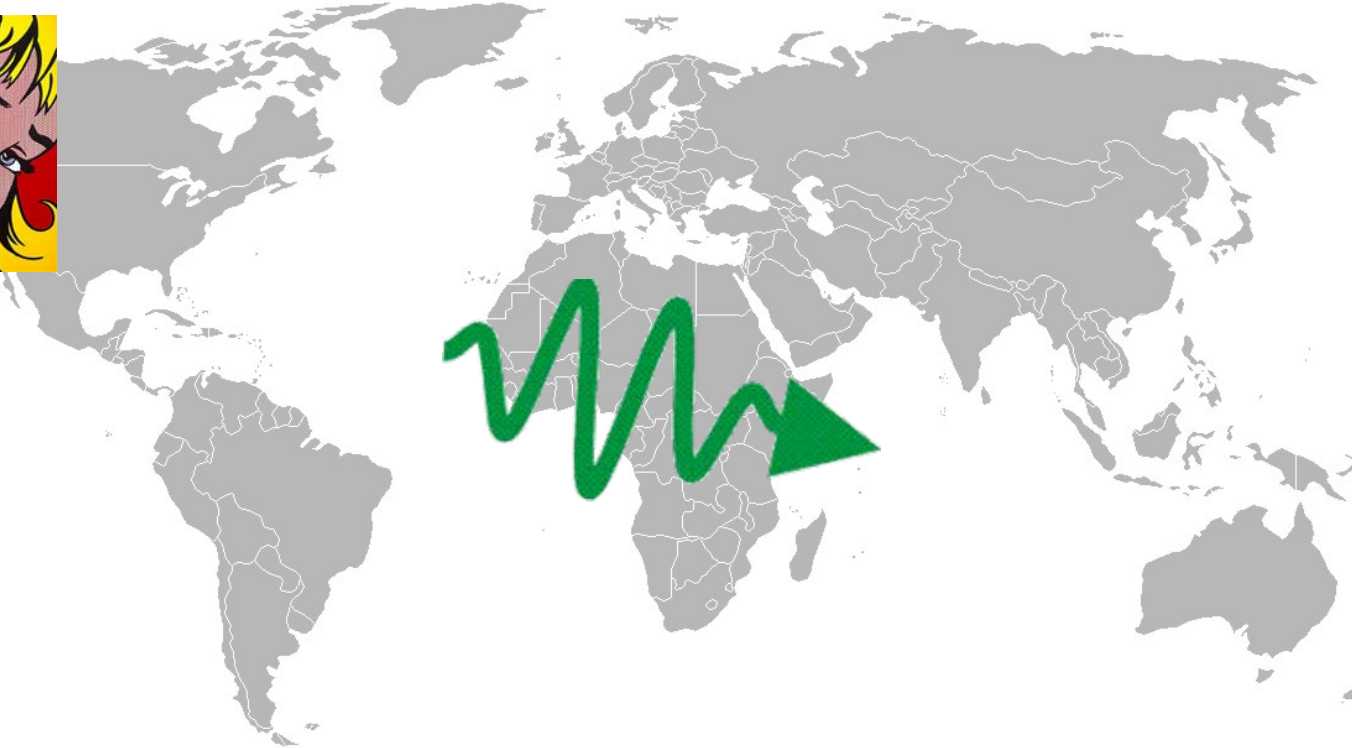
# Problems with quantum cryptography

- **Preparation of single photons**
  - Must ensure absence of two-photon pulses
- **Losses in optical fibers**
  - **0.2-0.3 dB/km**: half of photons are lost over 10-15 kilometers.
  - **Example: Dubai to Kish, 300 km, only 1 in 30,000,000 photons will reach destination**
  - Can't use amplifiers
- **"Dark counts" of detectors**
  - Sometimes a photon detector will "click" without a photon
  - Dark clicks cause errors
  - Too many errors  
→ can't detect eavesdropping



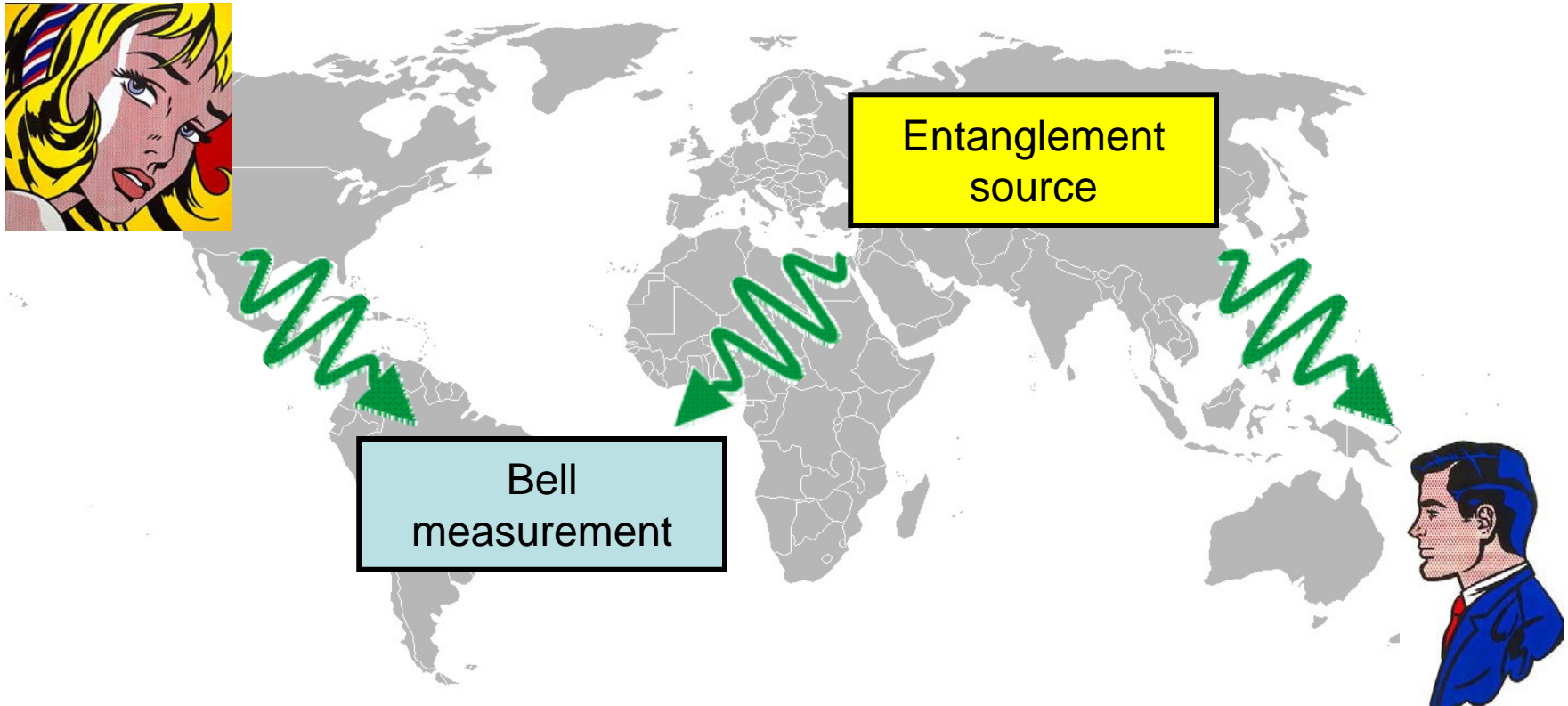


# Suppose Alice wants to send a photon to Bob...



☹️ The photon is likely to get lost on its way

# Quantum relay

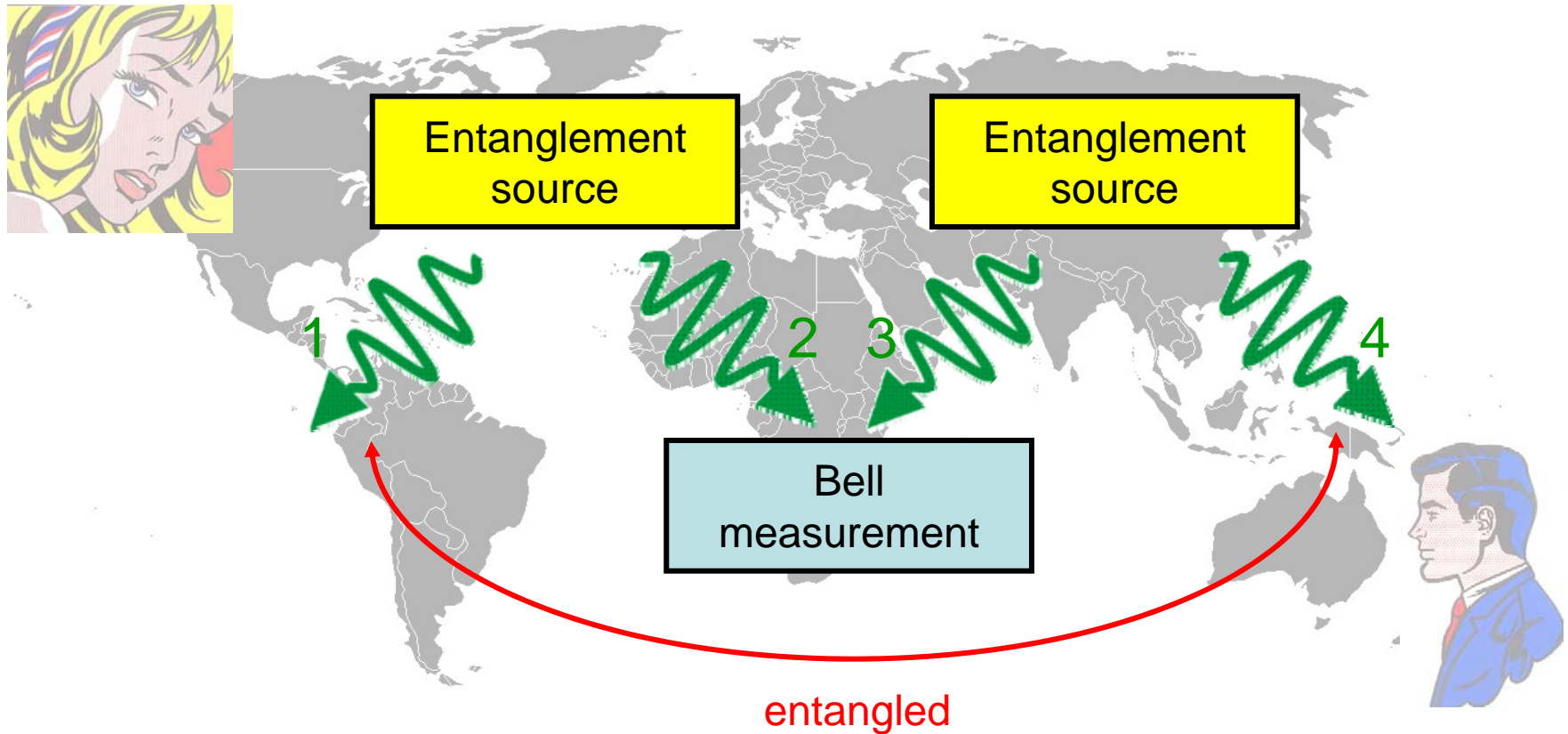


- If Alice and Bob shared an entangled resource,

☹ Alice could *teleport* her photon to Bob

☹ But long-distance entanglement is difficult to create

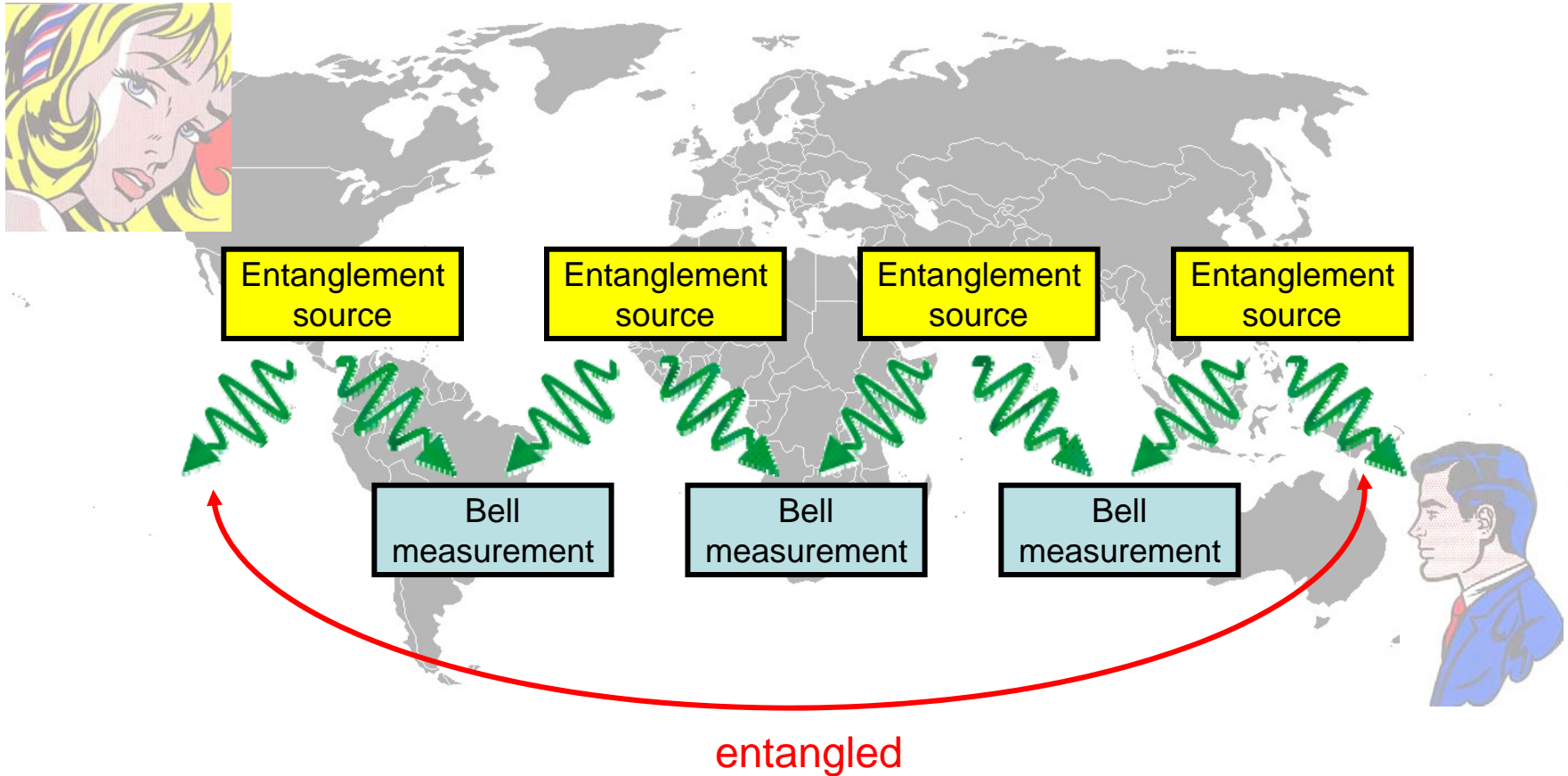
# Quantum relay



Long-distance entanglement can be created by *entanglement swapping*

**A Bell measurements on modes 2 and 4 entangles modes 1 and 4**

# Quantum relay

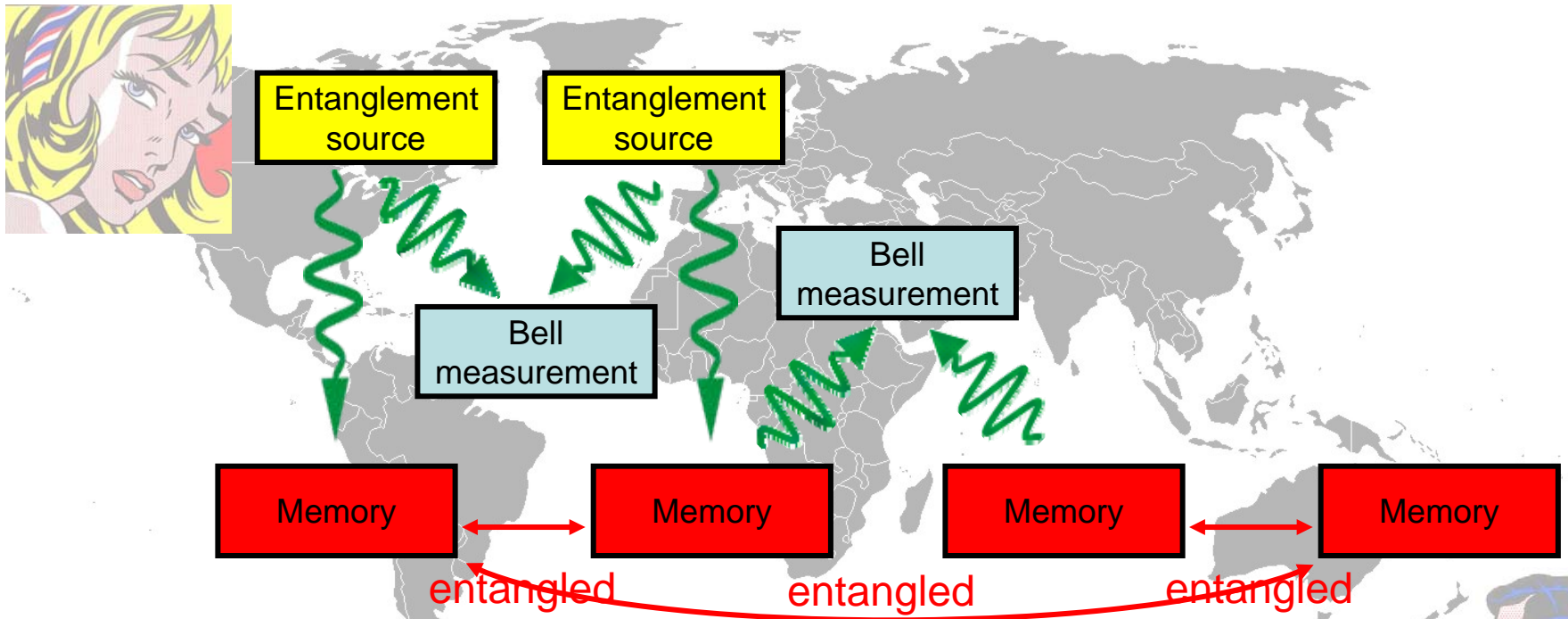


Long-distance entanglement can be created by *entanglement swapping*

☹️ but to succeed, all links must work simultaneously.

→ success probability still decreases exponentially with distance.

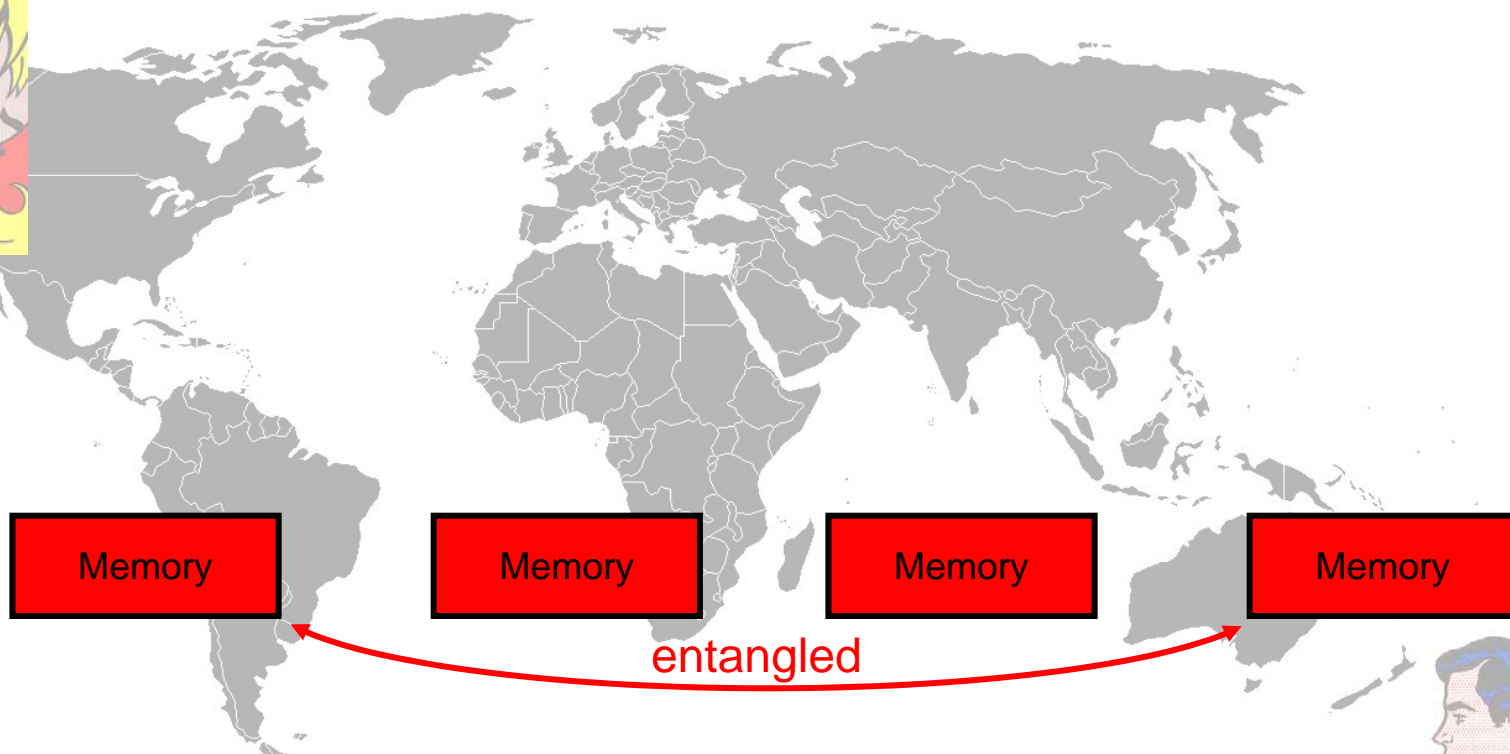
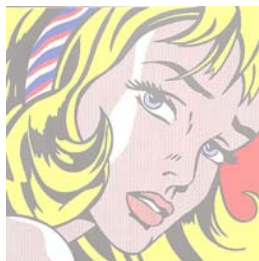
# The role of memory



- **But if we had quantum memory,**
  - entanglement in a link could be stored... until entanglement in other links has been created, too.
  - Bell-measurement on adjacent quantum memories... will create the desired long-distance entanglement.
  - Alice can teleport her photon to Bob



# Quantum repeater



- **This technology is called *quantum repeater***
  - Initial idea: H. Briegel *et al.*, 1998
  - In application to EIT and quantum memory: L.M. Duan *et al.*, 2001
- Quantum memory for light is essential for long-distance quantum communications.

# By the way...

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- **Quantum memory for light is also useful in quantum computing**
  - Photon makes an excellent qubit... but does not like to stay put
  - Any computer, quantum or classical, needs memory

# ELECTROMAGNETICALLY INDUCED TRANSPARENCY

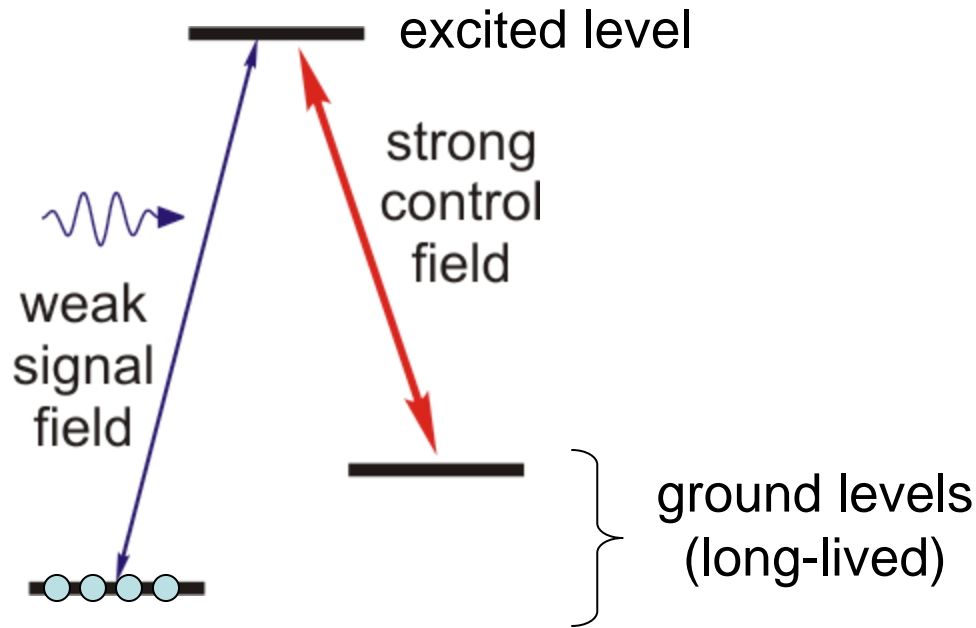
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*and memory for light*



# What is EIT?

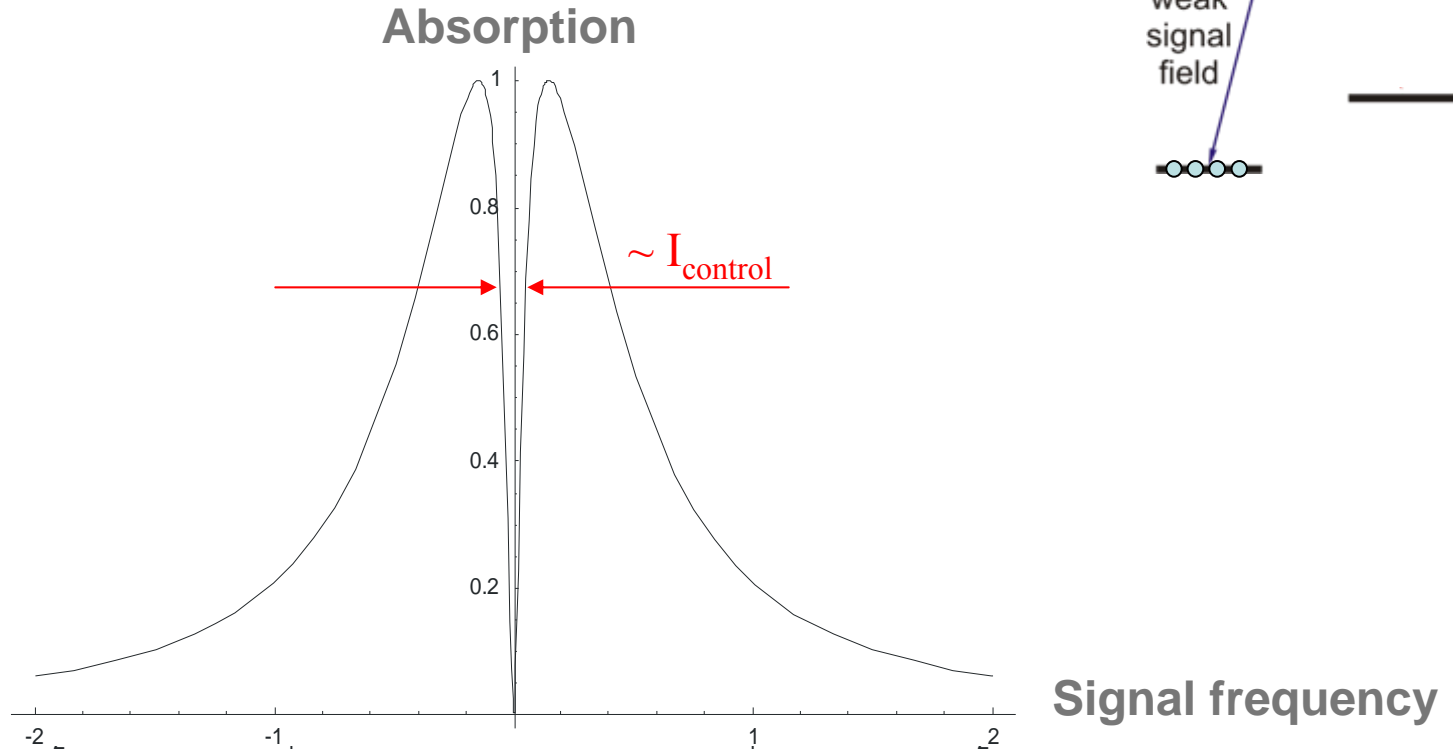
## Quantum interference effect in atoms with $\Lambda$ -shaped level structure



**What will happen to the signal field when we send it through an EIT medium?**

# Absorption of the signal field

- Without control field

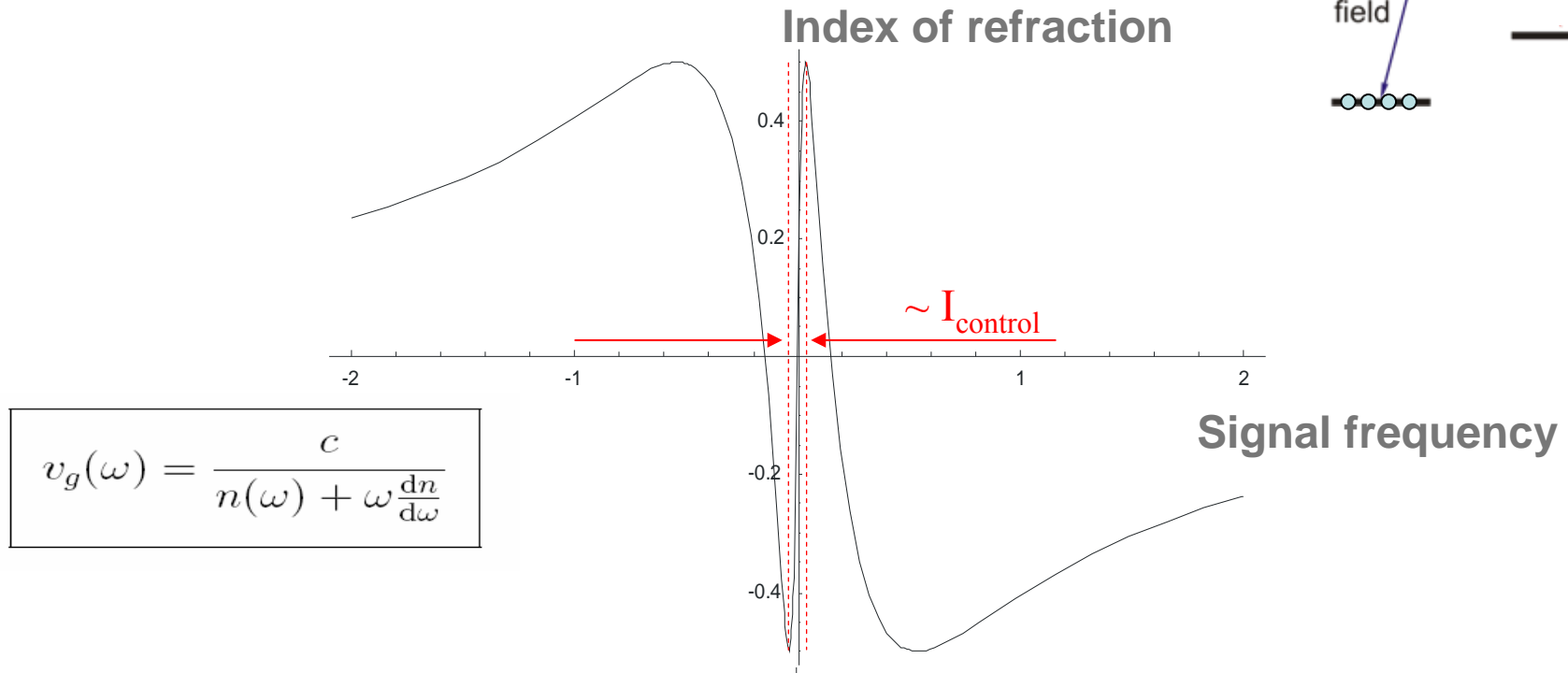


Narrow transparency window on resonance.

- Light propagates through an otherwise opaque medium.

# Dispersion of the signal field

- Without control field



$$v_g(\omega) = \frac{c}{n(\omega) + \omega \frac{dn}{d\omega}}$$

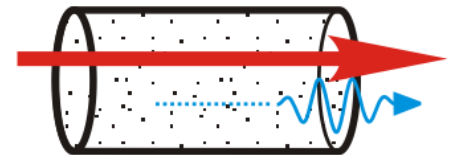
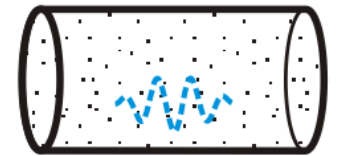
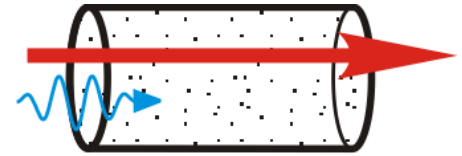
We can enormously reduce the group velocity

- Group velocity is proportional to the control field intensity

# EIT for quantum memory

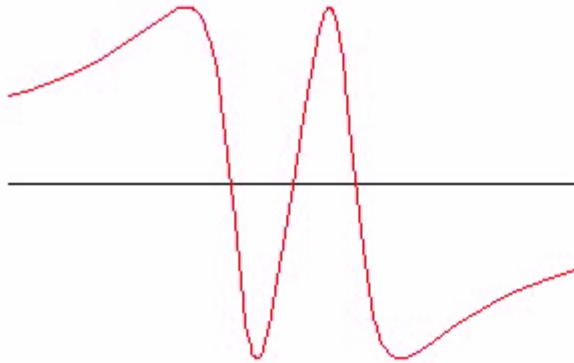
- **The idea**

- **Turning the control field off will reduce the group velocity to zero**
- **Quantum information contained in the pulse is stored in a collective atomic ground state superposition**
- **Turning the control field back on will retrieve the pulse in the original quantum state**

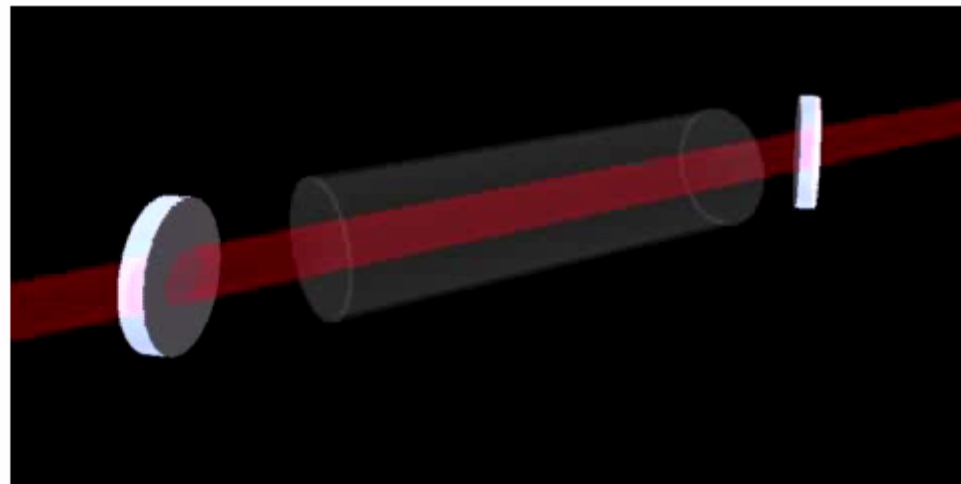
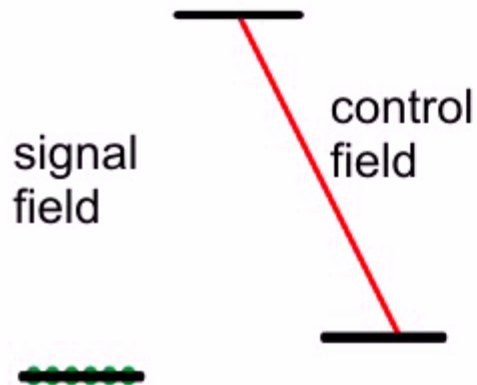
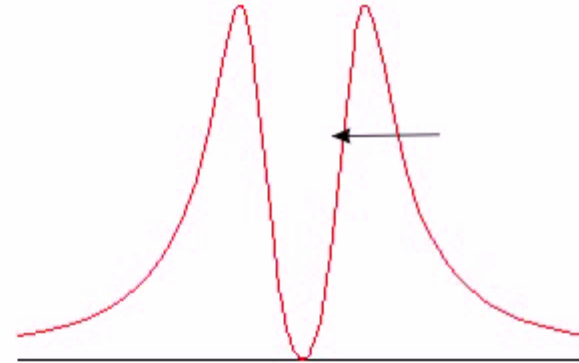


# EIT for quantum memory

EIT dispersion



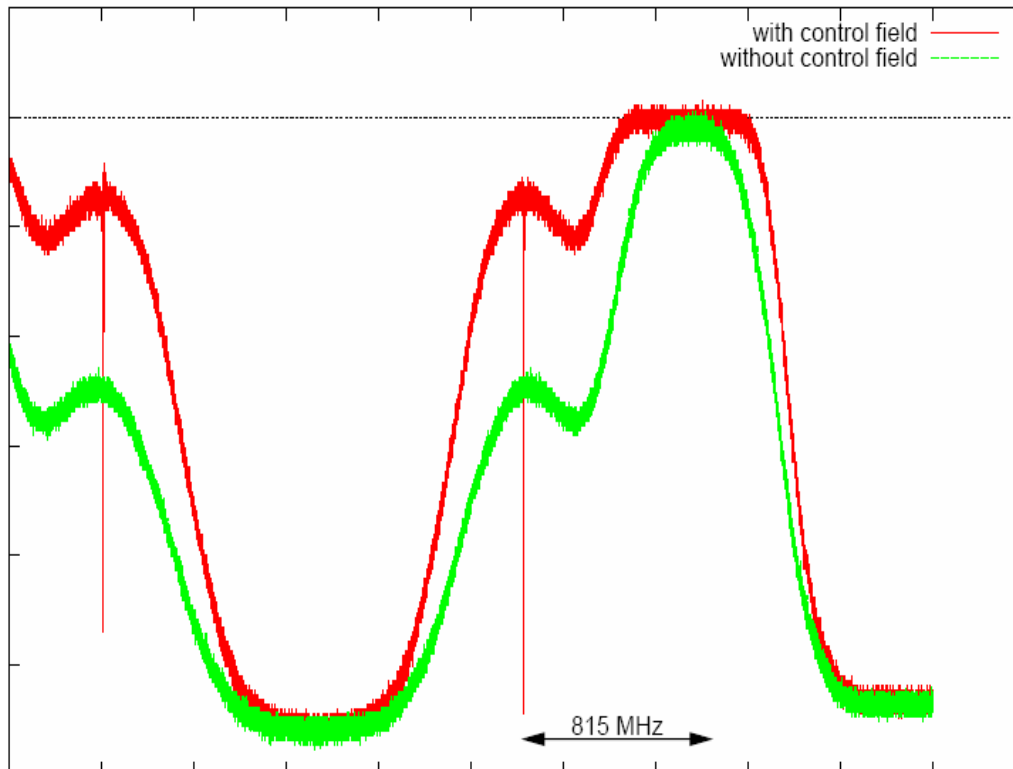
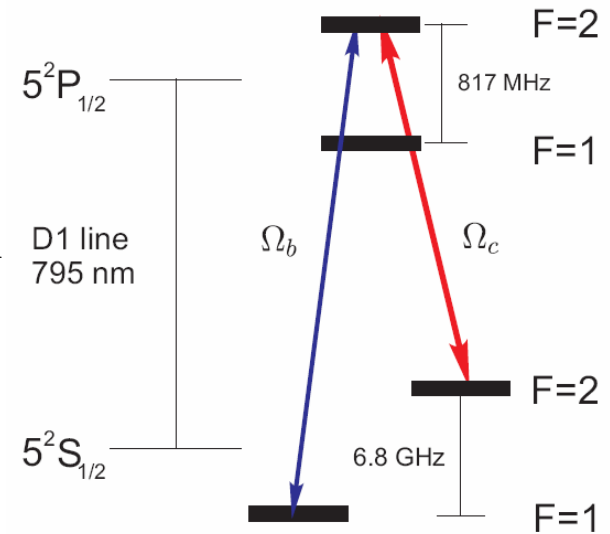
EIT absorption



# EIT in our lab

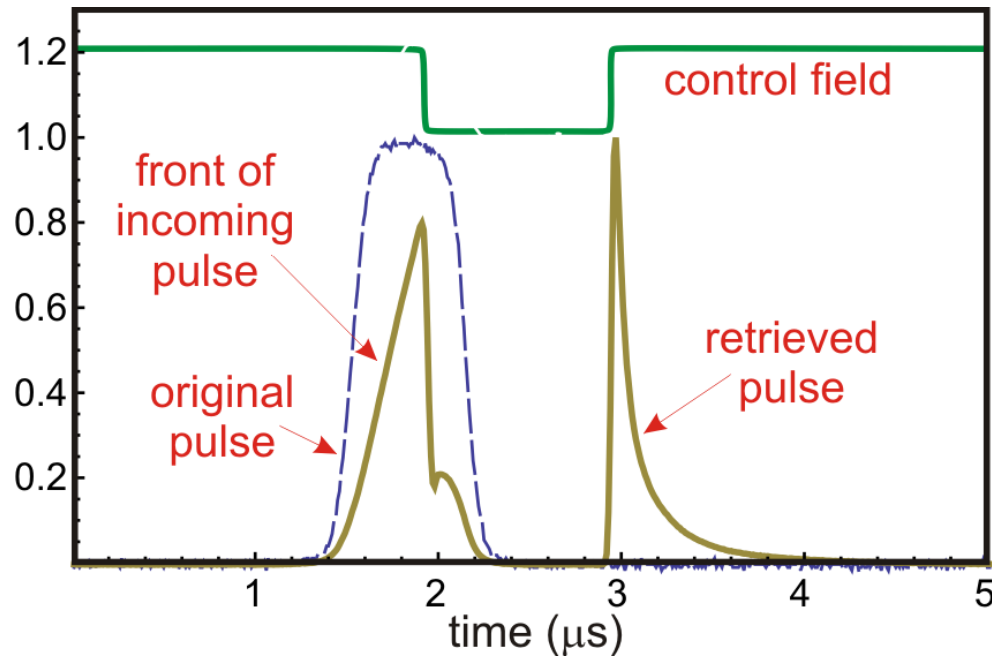
- **Implementation in atomic rubidium**

- **Ground level split into two hyperfine sublevels**  
→ a perfect  $\Lambda$  system
- **Control and signal lasers must be phase locked to each other at 6.8 GHz**



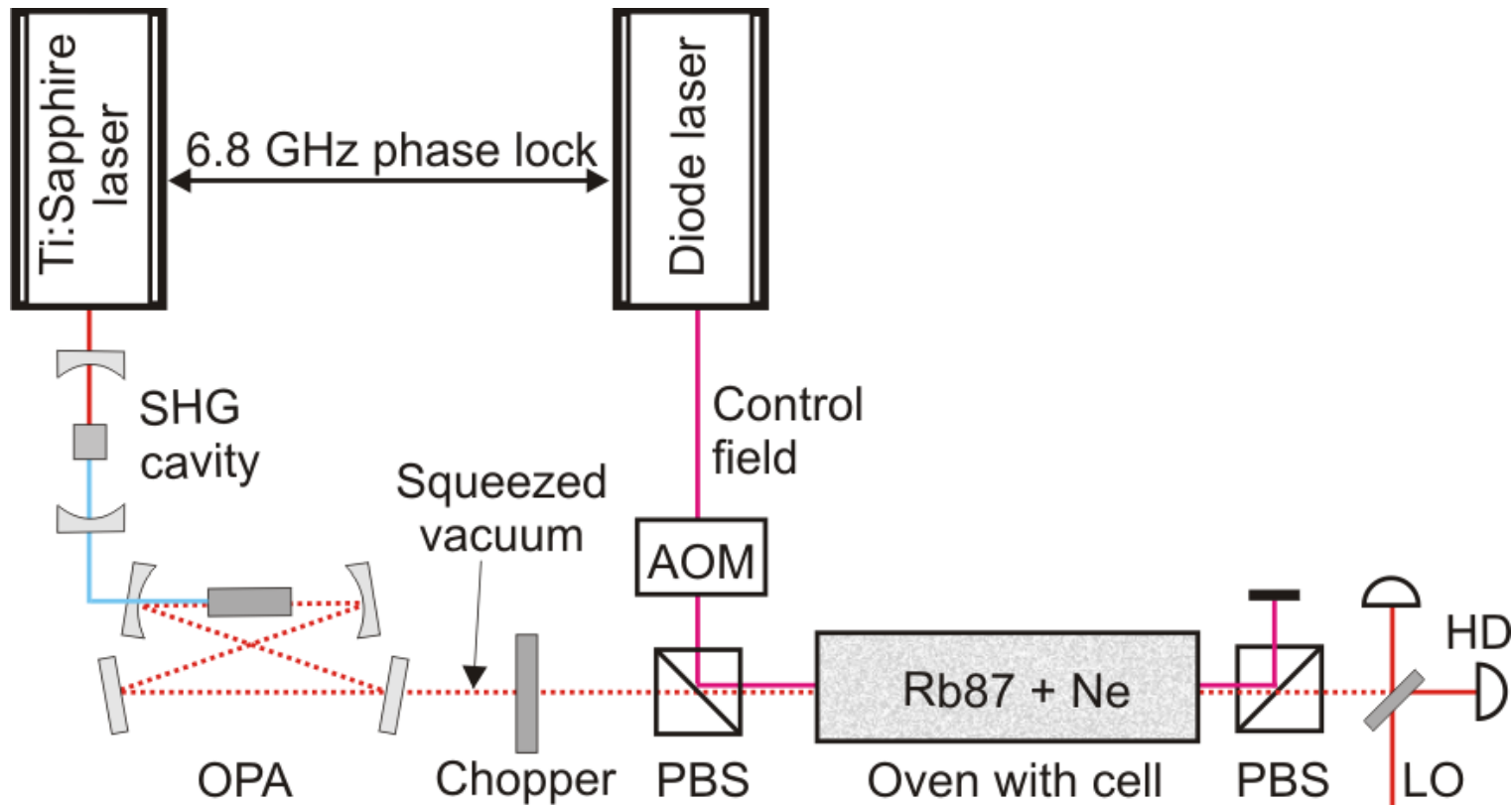
# EIT-based memory: practical limitations

- EIT window not perfectly transparent  
→ part of the pulse will be absorbed
- Memory lifetime limited by atoms colliding, drifting in and out the interaction region



# Storage of squeezed vacuum

## The setup

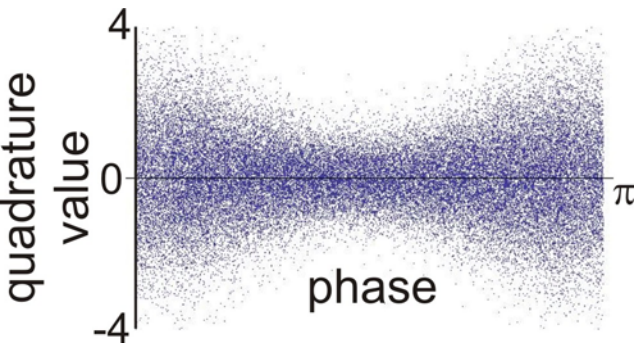




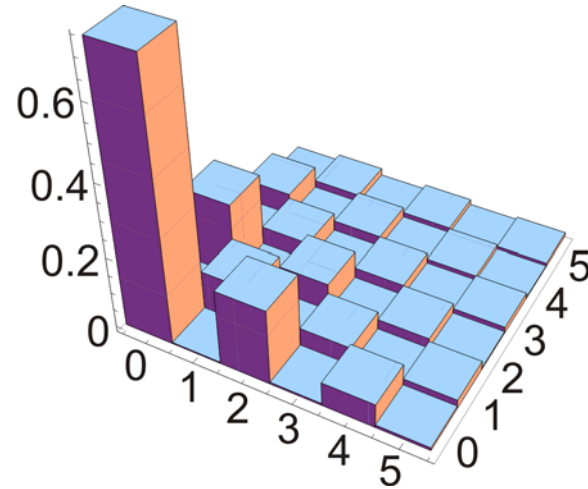
# Storage of squeezed vacuum

## The initial state

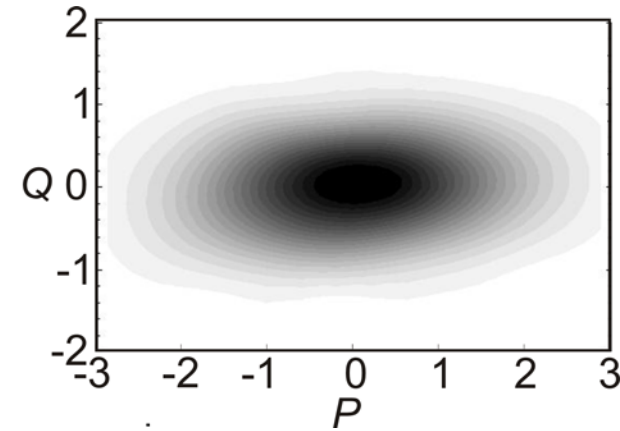
Quadrature data



Density matrix



Wigner function

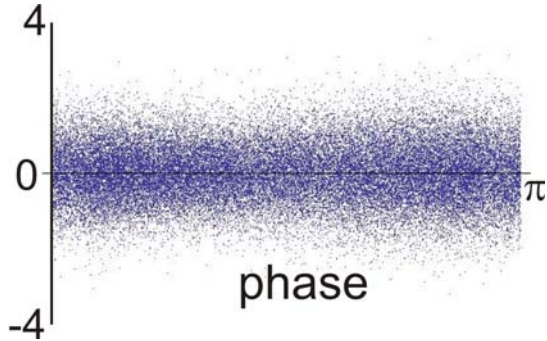


# Storage of squeezed vacuum

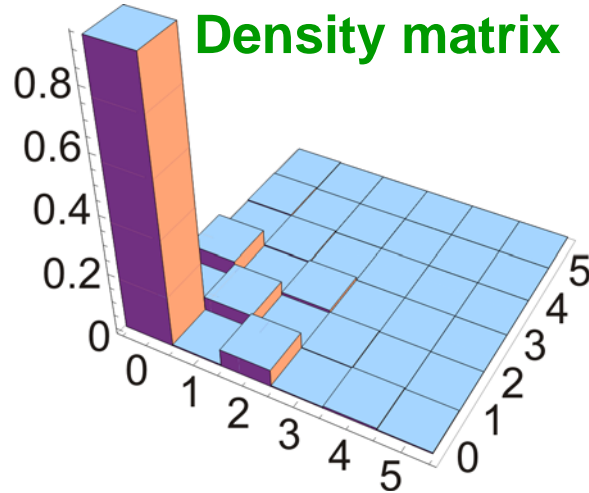
## The retrieved state

J. Appel et al., PRL **100**, 093602 (2008)

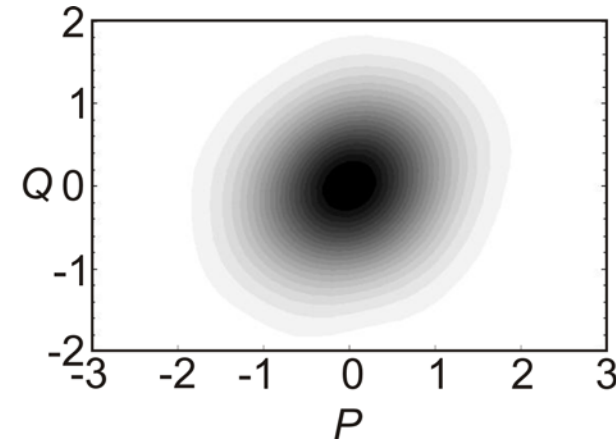
### Quadrature data



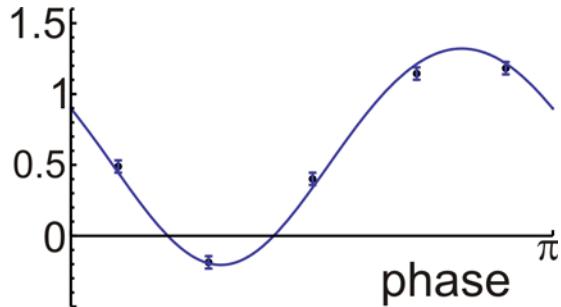
### Density matrix



### Wigner function



### Quadrature noise



- **Maximum squeezing:  $0.21 \pm 0.04$  dB**
- **Squeezing observed in the retrieved state!**

# EIT for quantum memory: state of the art

---

## The “holy grail”

- Store and retrieve arbitrary states of light for unlimited time
- State after retrieval must be identical to initial

## Existing work

- L. Hau, 1999: slow light
- M. Fleischauer, M. Lukin, 2000: original theoretical idea for light storage
- M. Lukin, D. Wadsworth *et al.*, 2001: storage and retrieval of a classical state
- A. Kuzmich *et al.*, M. Lukin *et al.*, 2005: storage and retrieval of single photons
- M. Kozuma *et al.*, A. Lvovsky *et al.*, 2007: memory for squeezed vacuum

## Existing benchmarks

- Memory lifetime: up to milliseconds in rubidium, up to seconds in solids
- Memory efficiency: up to 50 % in rubidium, lower for solids
- Things get much worse when we attempt to store nonclassical states of light



# QUANTUM COMPUTATION GATES

---

**1. With EIT**

**2. Using conditional measurements**

# An optical C-NOT gate

- What we need

$|H\rangle$  or  $|V\rangle$



$|H\rangle$  or  $|V\rangle$



$$|H\rangle|H\rangle \rightarrow |H\rangle|H\rangle$$

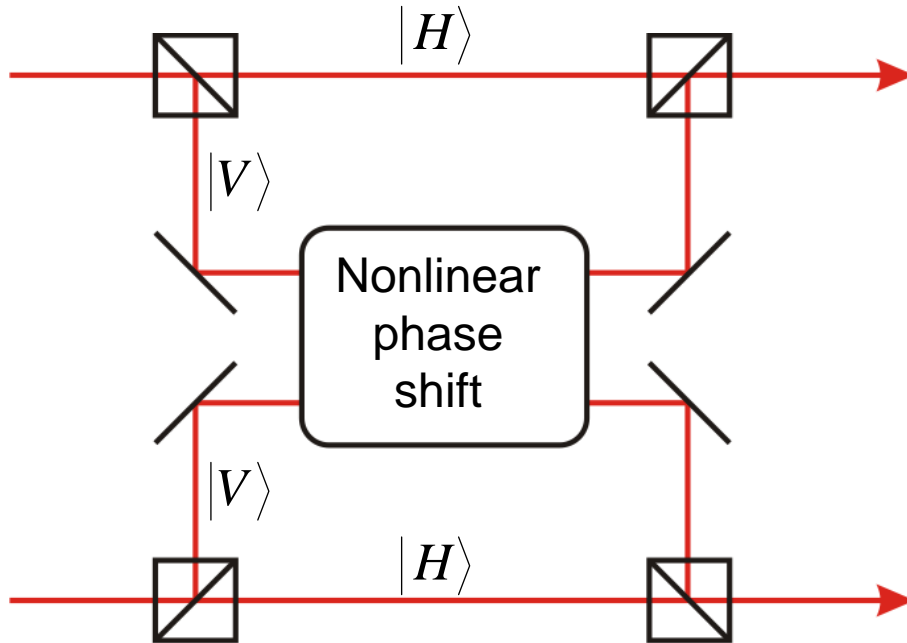
$$|H\rangle|V\rangle \rightarrow |H\rangle|V\rangle$$

$$|V\rangle|H\rangle \rightarrow |V\rangle|H\rangle$$

$$|V\rangle|V\rangle \rightarrow -|V\rangle|V\rangle$$

# An optical C-NOT gate

- **How to implement this**



- **Nonlinear phase shift**

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$

$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$

$$|1\rangle|0\rangle \rightarrow |1\rangle|0\rangle$$

$$|1\rangle|1\rangle \rightarrow -|1\rangle|1\rangle$$

- **Problem**

- **No materials that exhibit optical nonlinearity at the single-photon intensity level**

# QUANTUM COMPUTATION GATES

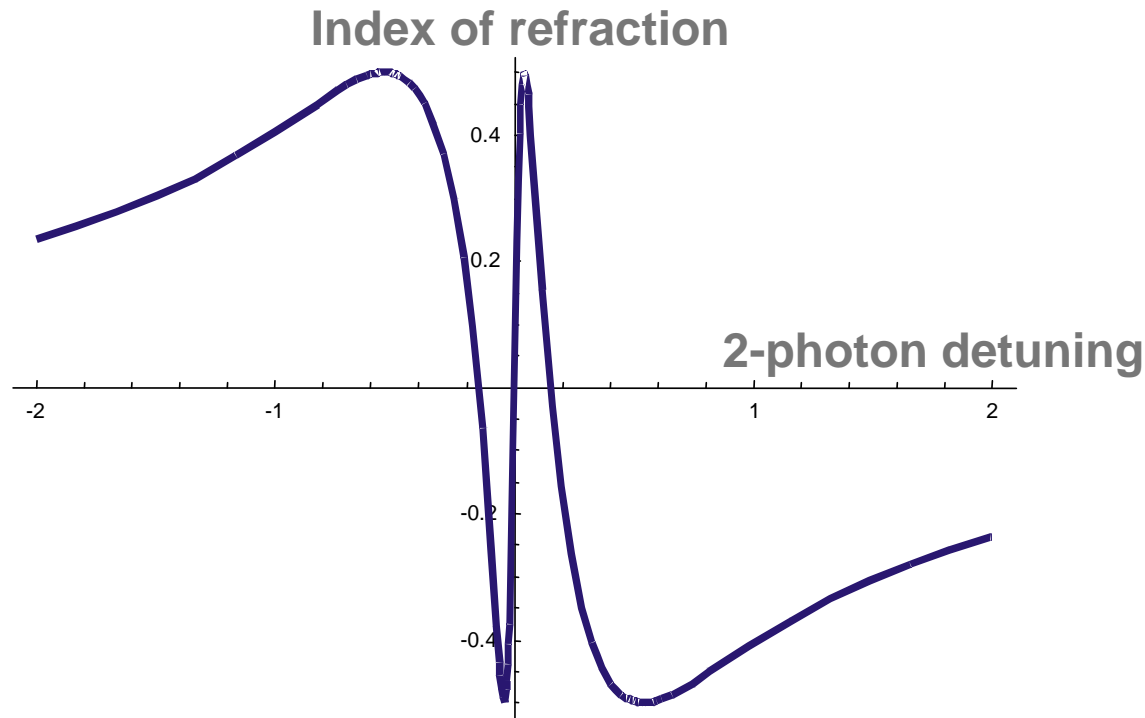
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**1. With EIT**

**2. Using conditional measurements**

# Nonlinear optics with EIT

- **Basic idea: exploit steep dispersion curve to produce large cross-phase modulation**

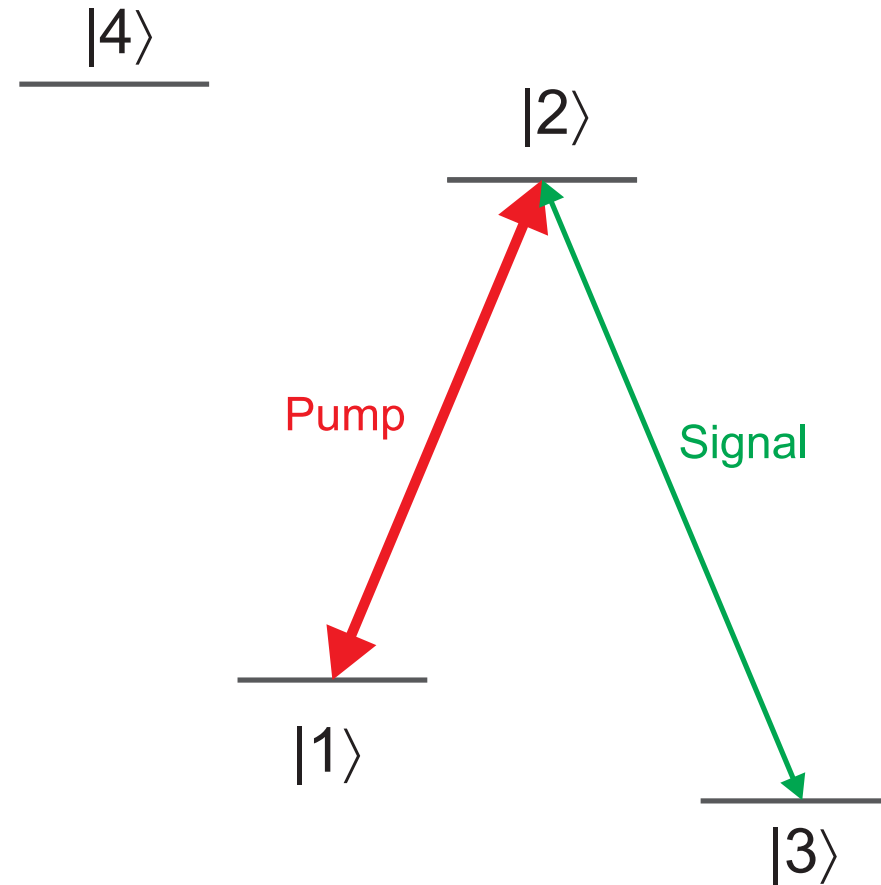


- **Small change in 2-photon detuning**  
→ **Large change in transmitted signal phase**



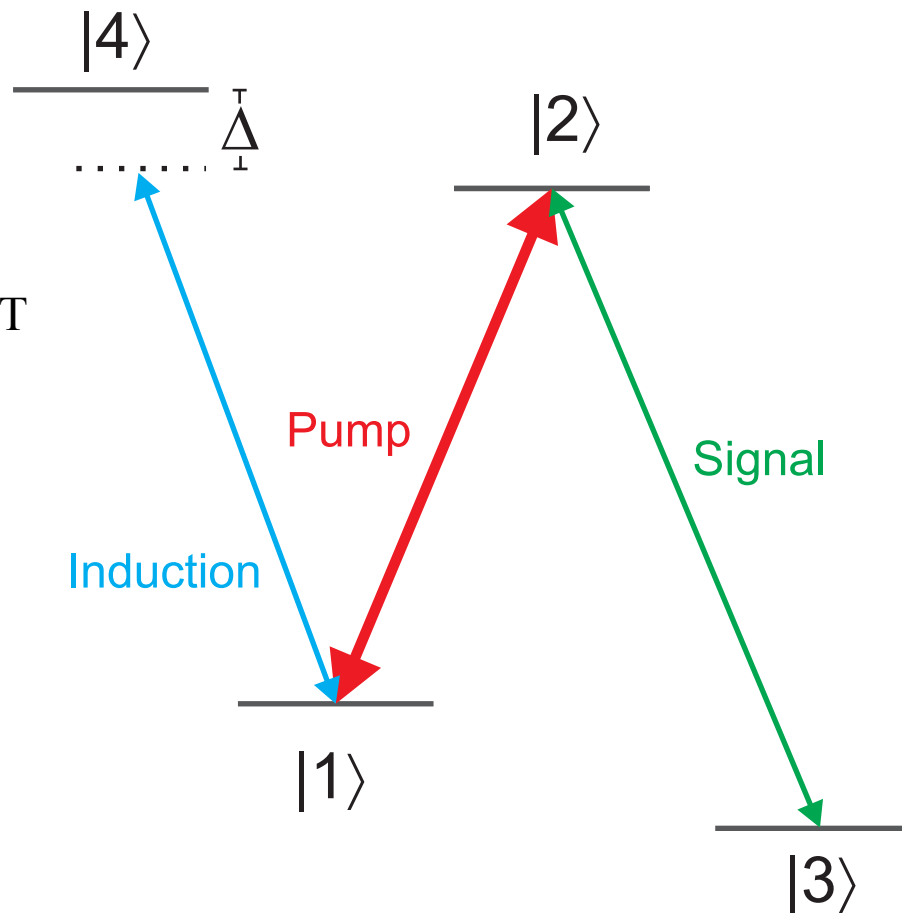
# N-type scheme

- EIT on **signal field** due to  $|1\rangle|2\rangle|3\rangle$



# N-scheme

- EIT on **signal field** due to  $|1\rangle|2\rangle|3\rangle$
- Off-resonant coupling of weak **induction field** produces Stark shift on  $|1\rangle$ 
  - Changes 2-photon detuning of signal EIT
  - Affects phase of transmitted signal



# N-scheme

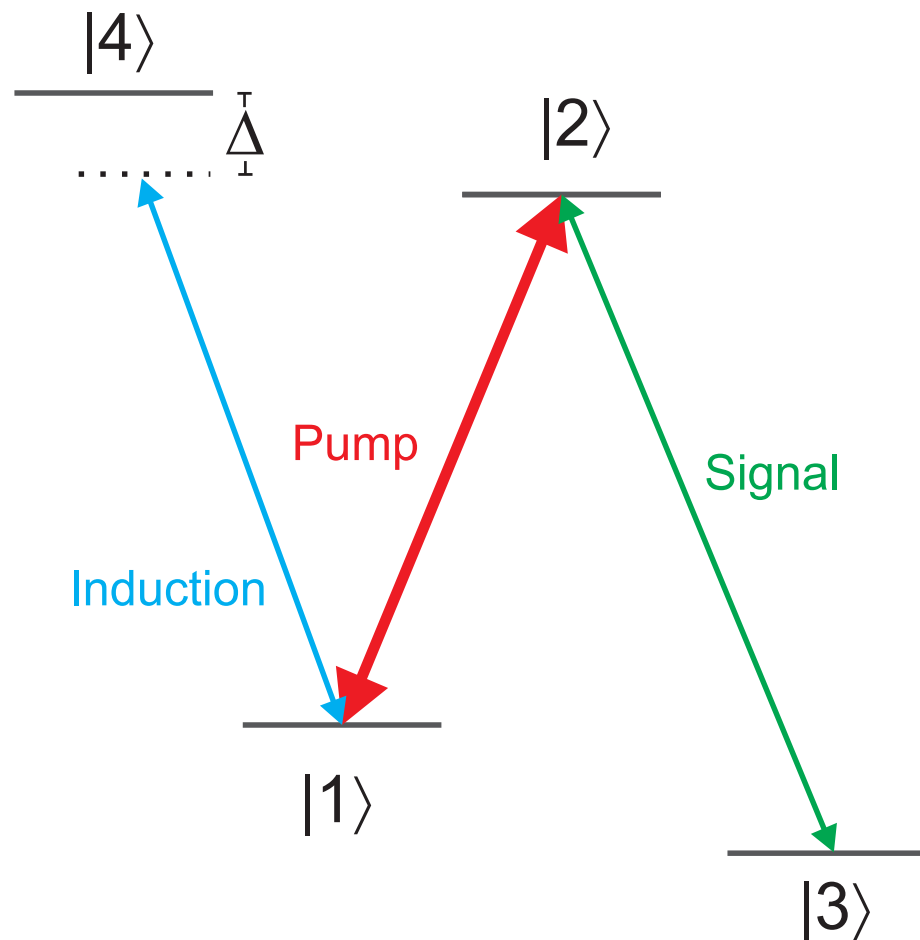
[continued]

- **Problem with N-scheme:**

- Only signal field experiences slowdown.
- For pulses, this is a severe limitation.

- **Solution:**

- Slow down **induction pulse** via another EIT system
- Lukin, Imamoglu (2001): use another atomic species (e.g.  $^{85}\text{Rb}$ )
- Wang, Marzlin, Sanders (2006): use double EIT in the same atom



# QUANTUM COMPUTATION GATES

---

1. With EIT

2. Using conditional measurements

# Non-deterministic phase gate: implementation with the beam splitter

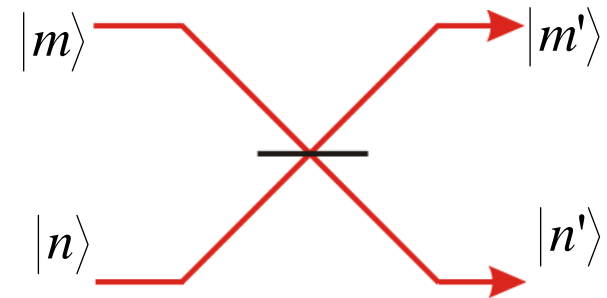
- **General beam splitter transformation**

$$|m, n\rangle \rightarrow \sum_{j,k=0}^{m,n} \sqrt{\frac{(j+k)!(m+n-j-k)!}{m!n!}} \binom{m}{j} \binom{n}{k} (-1)^k t^{n+j-k} r^{m-j+k} |m+n-j-k, j+k\rangle$$

**Entangles input modes**

**Entanglement very complicated**

**Conditional measurement and/or  
postselection are required to implement  
computation gates**

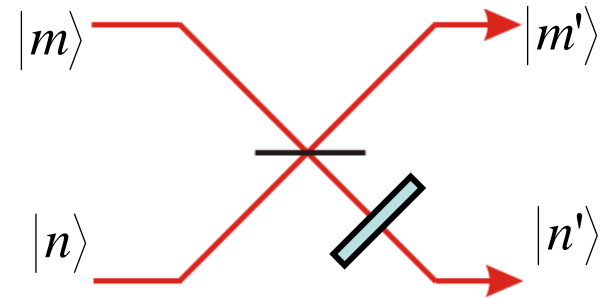


**$\Rightarrow$  Linear-optical quantum  
computing is non-deterministic**

# Non-deterministic phase gate: implementation with the beam splitter

- Beam splitter with reflectivity 1/3

$$(r = \sqrt{1/3}, t = \sqrt{2/3})$$



- Postselect on events in which the number of photons in the reflected channel is the same as that in the corresponding incident channel
- Neglect all other events

$$|0,0\rangle \rightarrow |0,0\rangle$$

$$|1,0\rangle \rightarrow \sqrt{1/3}|1,0\rangle$$

$$|0,1\rangle \rightarrow -\sqrt{1/3}|0,1\rangle$$

$$|1,1\rangle \rightarrow 1/3|1,1\rangle$$

Insert  $\pi$  phase shift  
into the right channel



$$|0,0\rangle \rightarrow |0,0\rangle$$

$$|1,0\rangle \rightarrow \sqrt{1/3}|1,0\rangle$$

$$|0,1\rangle \rightarrow \sqrt{1/3}|0,1\rangle$$

$$|1,1\rangle \rightarrow -1/3|1,1\rangle$$

😊 Phase gate implemented!

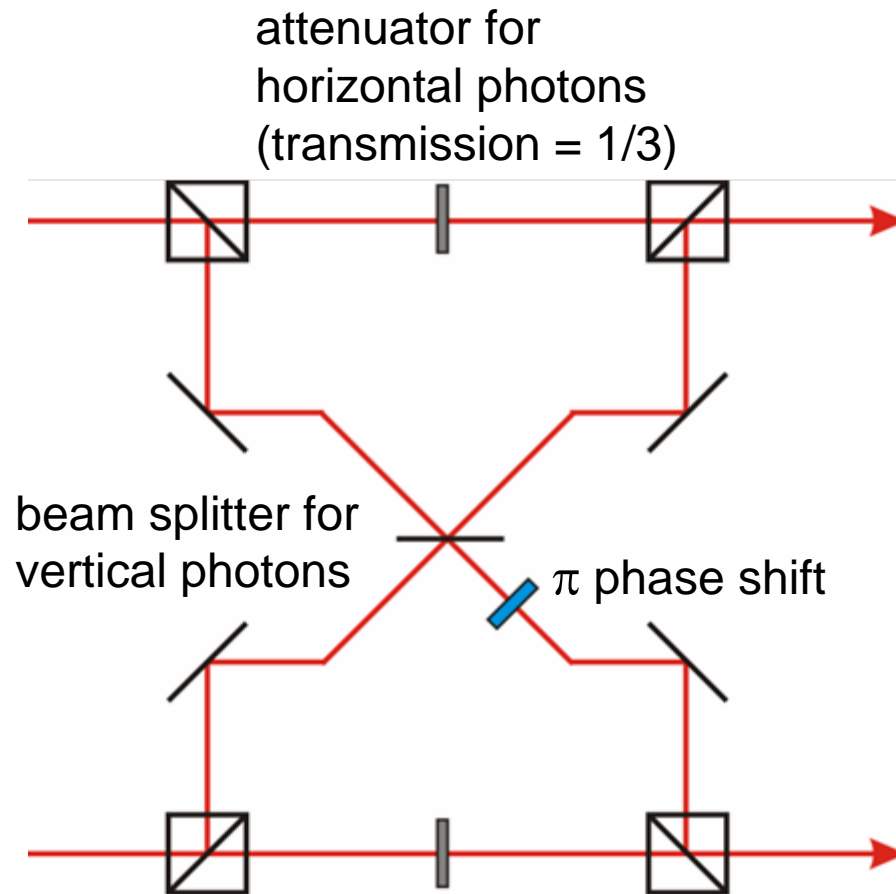
☹️ Non-deterministic (probability = 1/3 per photon)

→ Need to attenuate horizontal photons, too

# Non-deterministic phase gate

[continued]

- **Full scheme**



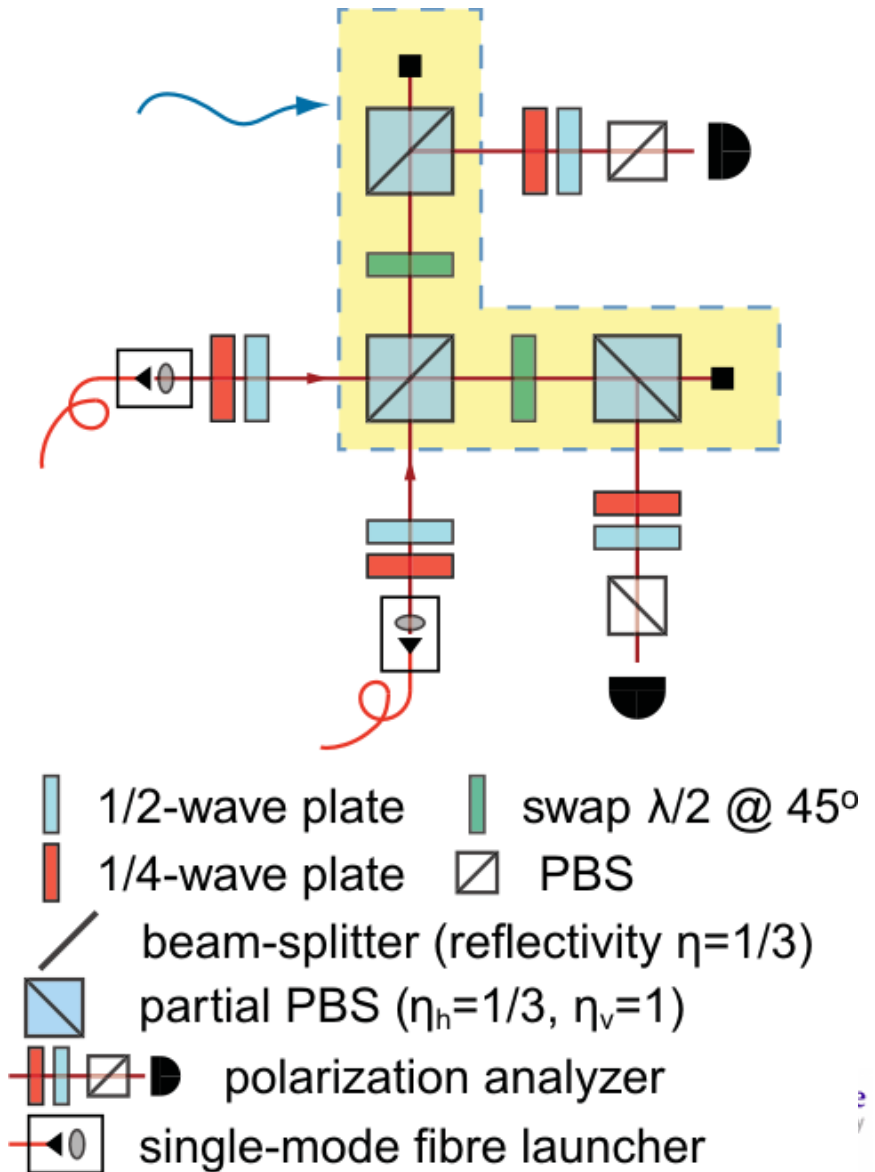
- **Properties**

- **Works conditioned on detecting 1 photon in each output**
- **Works with probability 1/9**
- **Would be useful for quantum computing if we had non-demolition detection of photons**

# Non-deterministic phase gate

## Experimental implementation

- **The setup**
  - **Partially-polarizing beam splitters used to simplify mode-matching**
  - **Operation of the gate as a Bell-state analyzer verified**



N. K. Langford et al., Phys. Rev. Lett. **95**, 210504 (2005)

N. Kiesel et al., Phys. Rev. Lett. **95**, 210505 (2005)

R. Okamoto et al., Phys. Rev. Lett. **95**, 210505 (2005)



# Another example: Conditional preparation of multi-photon entanglement

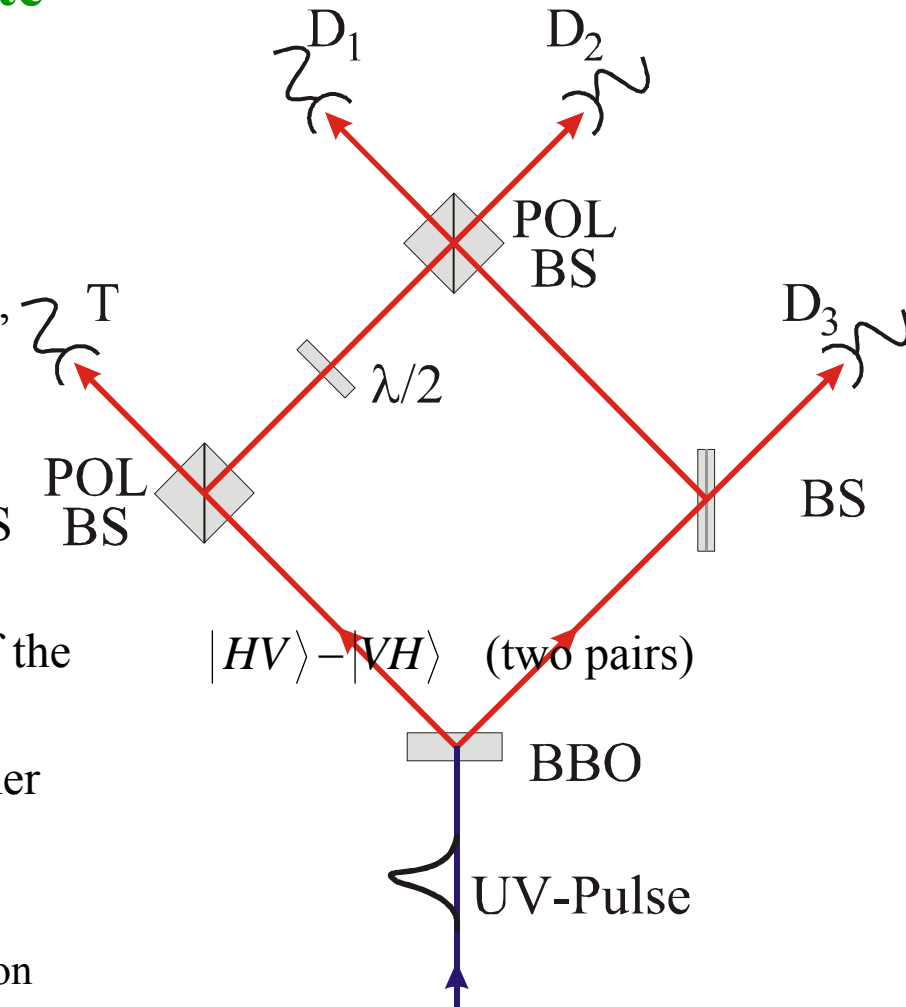
D. Bouwmeester *et al.*,  
PRL 82, 1345 (1999)

- Greenberger-Horne-Zeilinger state

$$|HHV\rangle + |VVH\rangle$$

- Conditioned on 4-photon coincidence (postselected preparation)

- Start from 2 pairs  $|HV\rangle - |VH\rangle$
- Photon that fires T comes from “first pair”
  - $\Rightarrow$  first pair must be  $|HV\rangle$
  - $\Rightarrow$  second pair must be  $|VH\rangle$
- Photons transmitted and reflected from BS must be of opposite polarizations
- Photons detected by  $D_1$  and  $D_2$  must be of the same polarization
- The state incident on  $D_1$ ,  $D_2$  and  $D_3$  is either  $|HHV\rangle$  or  $|VVH\rangle$
- These possibilities are indistinguishable
  - $\Rightarrow$  The output state is a coherent superposition



**☹️ We know the state has been generated only after it's detected**

# Thanks!



- **Funding:**
  - **CIAR**
  - **NSERC**
  - **AIF**
  - **CFI**
  - **QuantumWorks**

**Ph.D. positions available**

<http://qis.ucalgary.ca/quantech/>