QUANTUM INFORMATION
with light and atoms

Lecture 1

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UNIVERSITY OF CALGARY

Course Structure

1. Measuring quantum states of light
2. Making quantum states of light

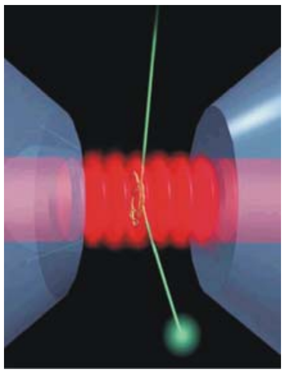
3. Quantum repeaters
4. Quantum memory for light
5. Quantum gates with photons

MOTIVATION

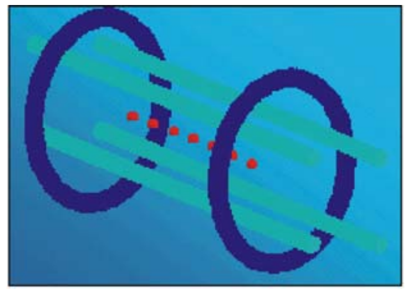
Photon as a qubit

- **Among many physical media suitable for quantum computation…**

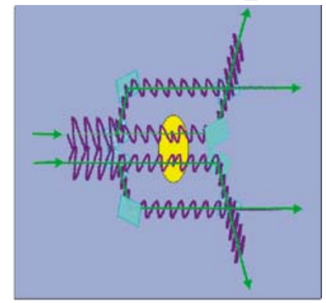
- cavity QED



- trapped ions

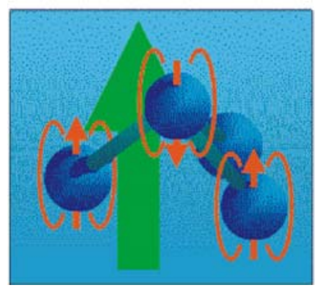


- nonlinear optics

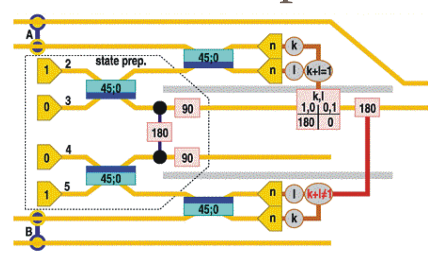


- quantum dots

- NMR



- linear optics



- **…why study the optical one?**

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Photon as a qubit
(…continued)

- **Because:**
  - **A photon makes an intuitive qubit**
  - **A photon is a good carrier of quantum information**
  - **Virtually no decoherence**
  - **Efficient gate operations (Knill-Laflamme-Milburn)**

- **Challenges:**
  - **Synthesis,**
  - **Characterization**
  - **Storage**
  - **Computational gates**

  of quantum optical states

MEASURING QUANTUM STATES OF LIGHT

1. By photon counting
2. By homodyne tomography

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MEASURING THE QUANTUM STATE OF LIGHT

1. **By photon counting**
2. **By homodyne tomography**

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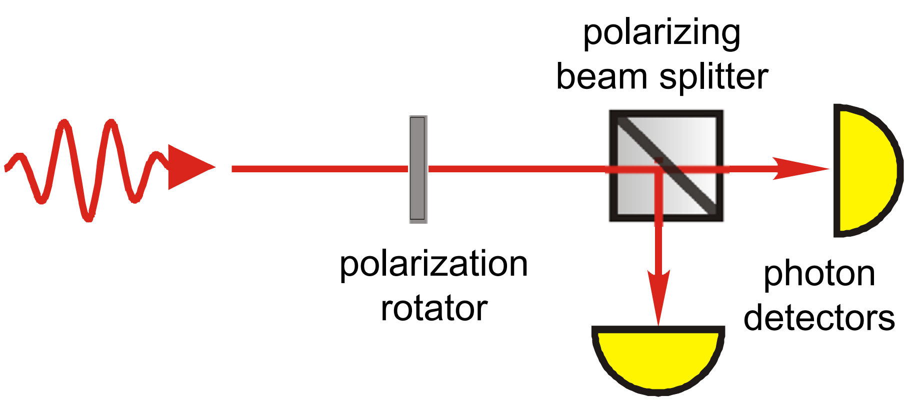
How to characterize a quantum state?

- **A single measurement won't do**
- **Repeated, identical measurements → projection onto only one basis**
- **Need many sets of measurements in different bases (quantum tomography)**

  Generally, $d^2 - 1$ bases are required
  for full tomography of a d-dimensional system

Example: a polarization qubit

(photon in a superposition of horizontal and vertical polarization states)



probability of horizontal polarization $= \langle H|\hat{\rho}|H\rangle$

$$\hat{\rho} = \begin{pmatrix} \rho_{HH} & \rho_{HV} \\ \rho_{VH} & \rho_{VV} \end{pmatrix}$$

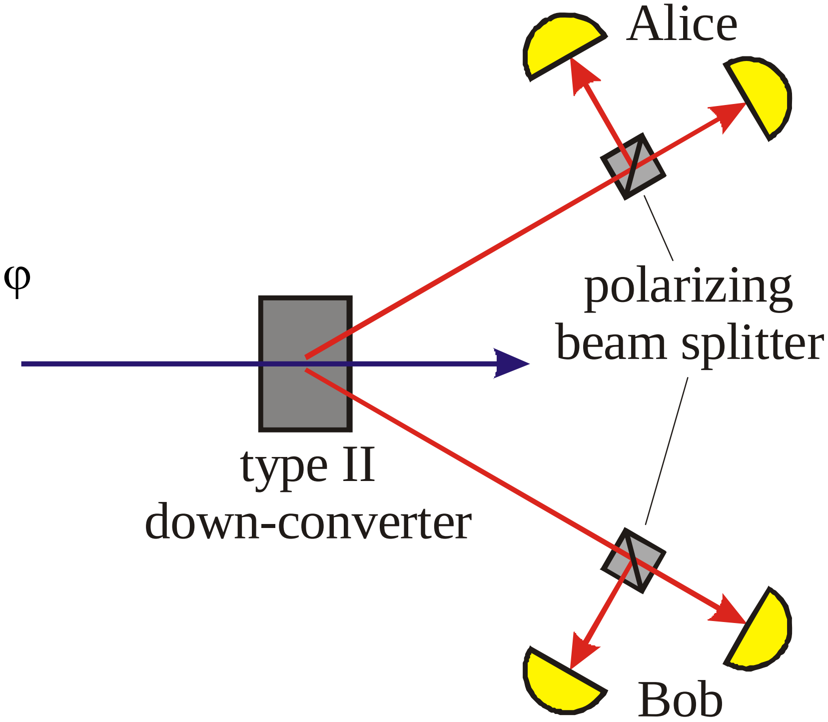
probability of vertical polarization $= \langle V|\hat{\rho}|V\rangle$

probability of 45° polarization $= \frac{1}{2}\left(\langle H| + \langle V|\right)\hat{\rho}\left(|H\rangle + |V\rangle\right)$

prob. of right circular polarization $= \frac{1}{2}\left(\langle H| - i\langle V|\right)\hat{\rho}\left(|H\rangle + i|V\rangle\right)$

Quantum measurement of the Bell state

- **Measuring an entangled state** $|\Psi^-\rangle = |HV\rangle - |VH\rangle$
  - **Perfect anticorrelation:**
    - If Alice observes *H*, Bob observes *V*
    - If Alice observes *V*, Bob observes *H*
  - **This measurement is insufficient.**
    The state can be $|HV\rangle + e^{i\varphi}|VH\rangle$ with any φ
    or even an unentangled mixture
    $$|HV\rangle\langle HV| + |VH\rangle\langle VH|$$
  - **To determine φ, turn polarizers 45°**
    $$|H\rangle \to (|H'\rangle + |V'\rangle)/\sqrt{2}$$
    $$|V\rangle \to (|H'\rangle - |V'\rangle)/\sqrt{2}$$
    Then
    $$|HV\rangle - |VH\rangle \to |H'V'\rangle - |V'H'\rangle$$ remains the same
    $$|HV\rangle + |VH\rangle \to |H'H'\rangle - |V'V'\rangle$$ changes
    $\Rightarrow$ **We can verify that the state is indeed** $|\Psi^-\rangle$



Problem. Verify this.

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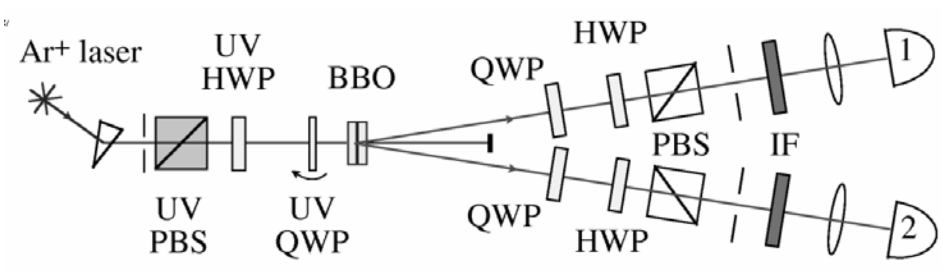
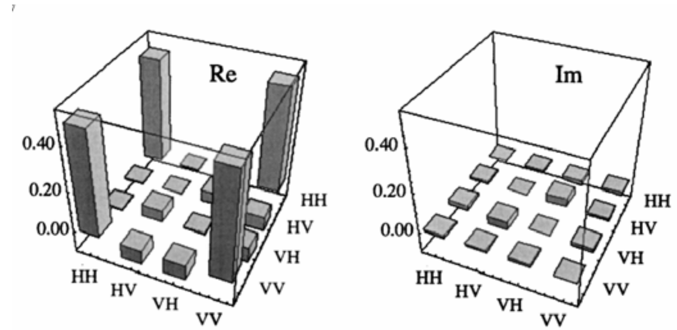
Quantum tomography by photon counting

- **Example:**

A. G. White *et al.*, PRL 83, 3103 (1999)

  - **Tomography of a two-mode, partially entangled state**





- **Measurements complete. What next?**
  - **Need to determine the density matrix from measurement results**
  - **Likelihood function**

$$\mathfrak{L}(\hat{\rho}) = \prod_{\text{measurements}} \text{pr}_i(\hat{\rho})$$

(where i is the number of the measurement, ρ is the density matrix)

  - **Likelihood-maximization algorithm**

Finds, among all possible density matrices, the one that maximized $\mathfrak{L}$

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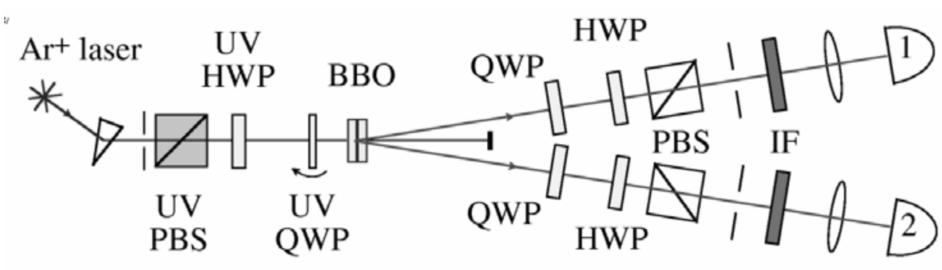

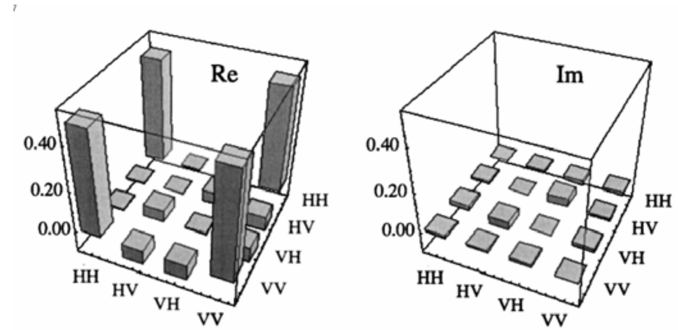

- This looks good, but…
  - **There's implicit assumption there is always a photon on each channel**
  - **Actually, the down-converter does not generate a photon pair "on demand"**
    $\rightarrow$ this characterization is postselected based on detecting a photon pair
    $\rightarrow$ the actual two-mode state is mostly vacuum
  - **the photon-counting based characterization technique and the postselection issue are common in modern experiments**

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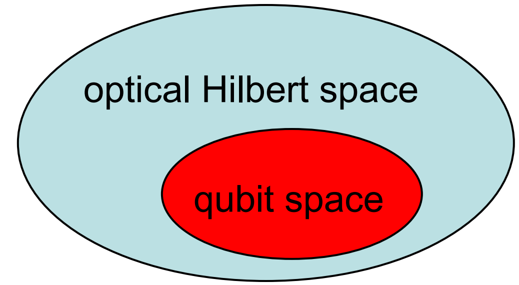
Tomography by photon counting
Drawbacks

- **Polarization qubit**

$$\alpha|H\rangle+\beta|V\rangle=\alpha|1_H,0_V\rangle+\beta|0_H,1_V\rangle\ldots$$



$$+\gamma|0_H,0_V\rangle+\delta|1_H,1_V\rangle+\varepsilon|2_H,3_V\rangle\ldots$$?

- **Traditional approach neglects non-qubit terms**

  $\rightarrow$ incomplete state characterization

  $\rightarrow$ incorrect evaluation of experimental quantum algorithms

  $\rightarrow$ postselection $\Rightarrow$ loss of scalability

- **New technology: number discriminating detector**
  - “Regular” photon detector: “click” or “no click”
  - Number discriminating detector: can determine the number **of photons**
  - Still, no phase information

Problem. suppose you have many highly-efficient “regular” detectors.
Can you use them to construct a discriminating detector?

MEASURING
THE QUANTUM STATE OF LIGHT

1. By photon counting
2. **By homodyne tomography**

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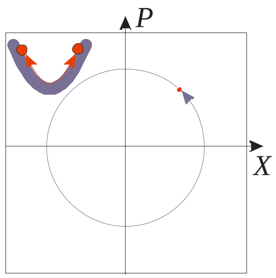
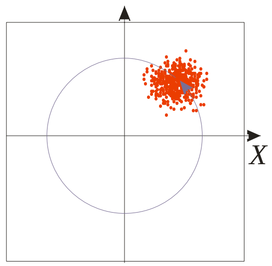
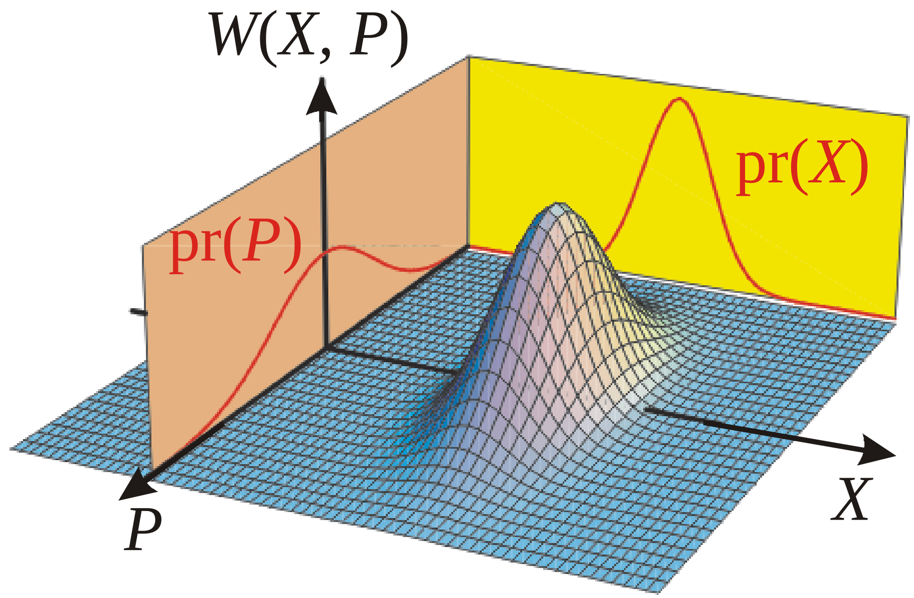
Phase-space probability distribution

- **Classical mechanics**
  - phase space picture of harmonic oscillator



  - many oscillators → probability distribution

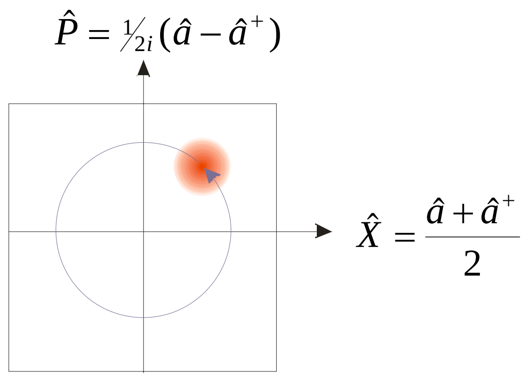




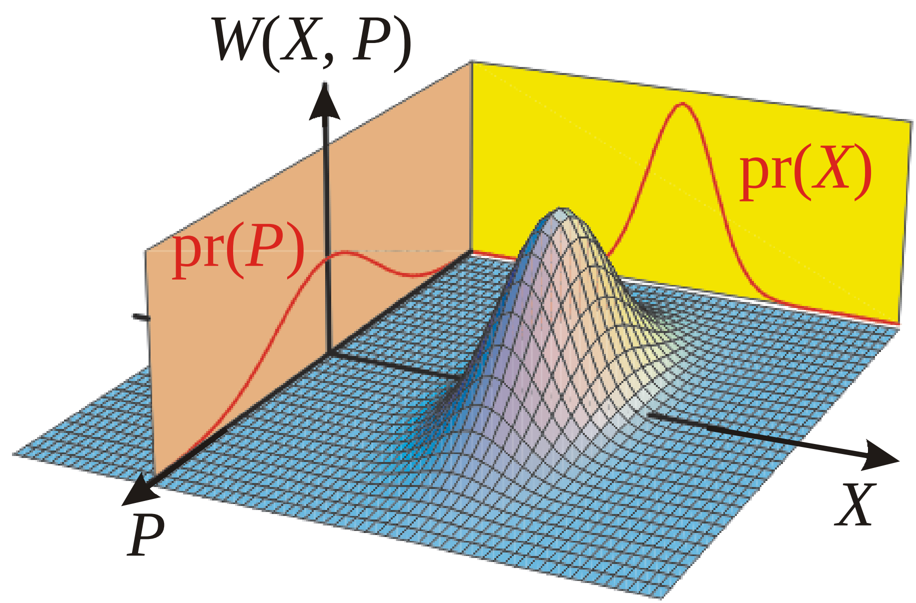
$W(X,P)$ —integration→ $\mathrm{pr}(X)$, $\mathrm{pr}(P)$

Wigner Function

- **Quantum mechanics → Uncertainty principle**
  → phase space probability density cannot be defined
  → only individual quadratures can be measured
- **Phase-space "quasi"probability density (Wigner function)**
  → projection onto each quadrature determines its probability density



$$W(X,P) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp(ipq)\left\langle X - \frac{q}{2}\right|\hat{\rho}\left|X + \frac{q}{2}\right\rangle dq$$

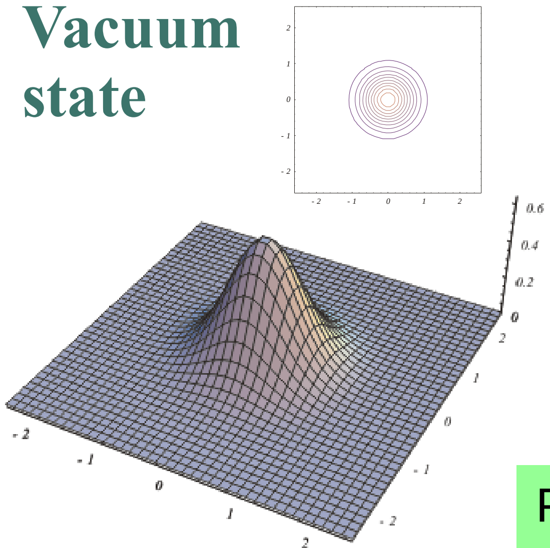


- **Properties**
  → completely describes a quantum state
  → real, normalized
  → not necessarily positive definite

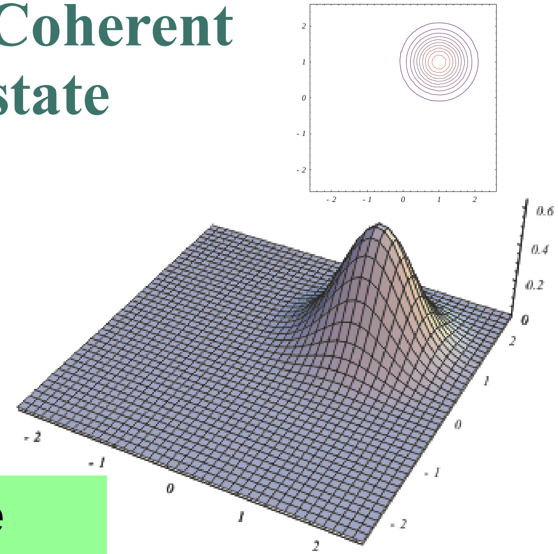
E.P. Wigner, Phys. Rev. **40**, 749 (1932)

Examples of Wigner functions

- Vacuum state

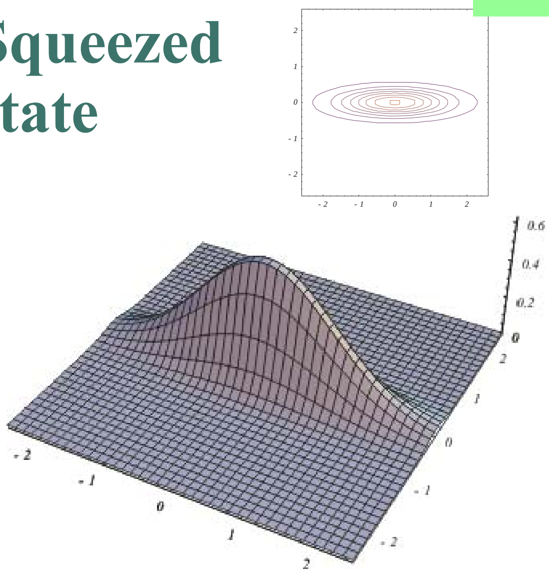


- Coherent state

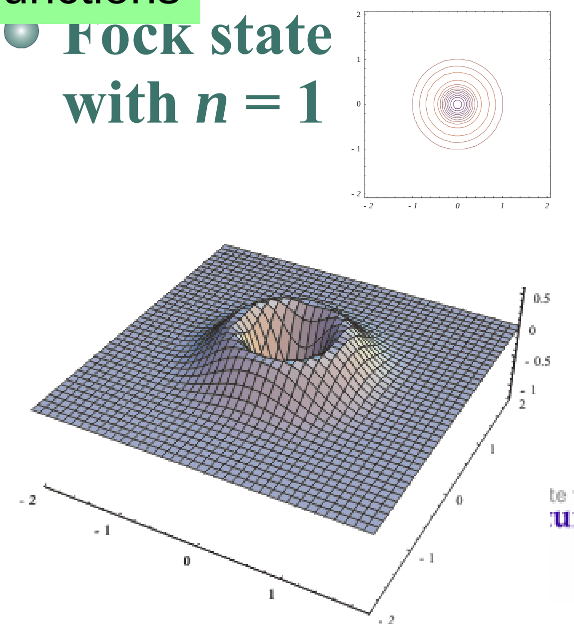


Problem. Calculate these Wigner functions

- Squeezed state



- Fock state with $n = 1$



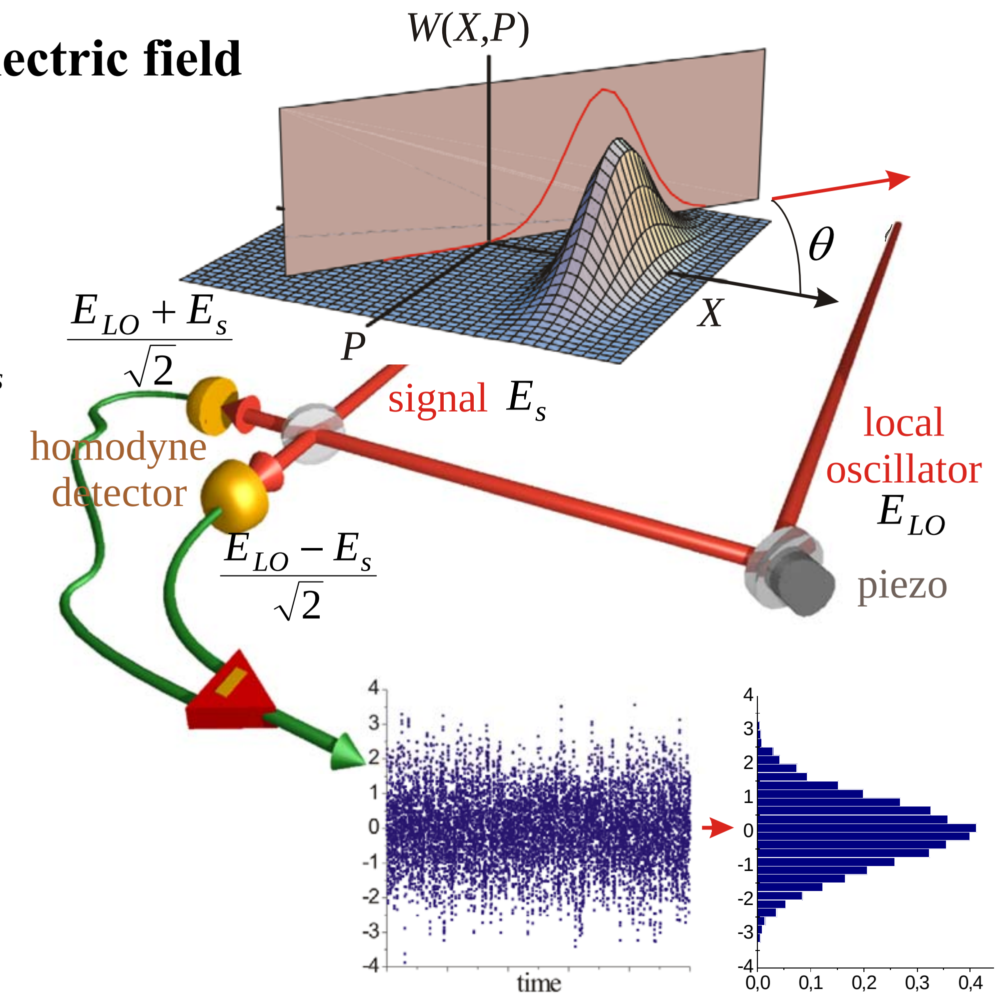
Homodyne tomography

- **Phase-sensitive measurements of electric field**
  $\rightarrow$ cannot be done directly
  $\rightarrow$ use interference with local oscillator
- **Measure subtraction photocurrent**

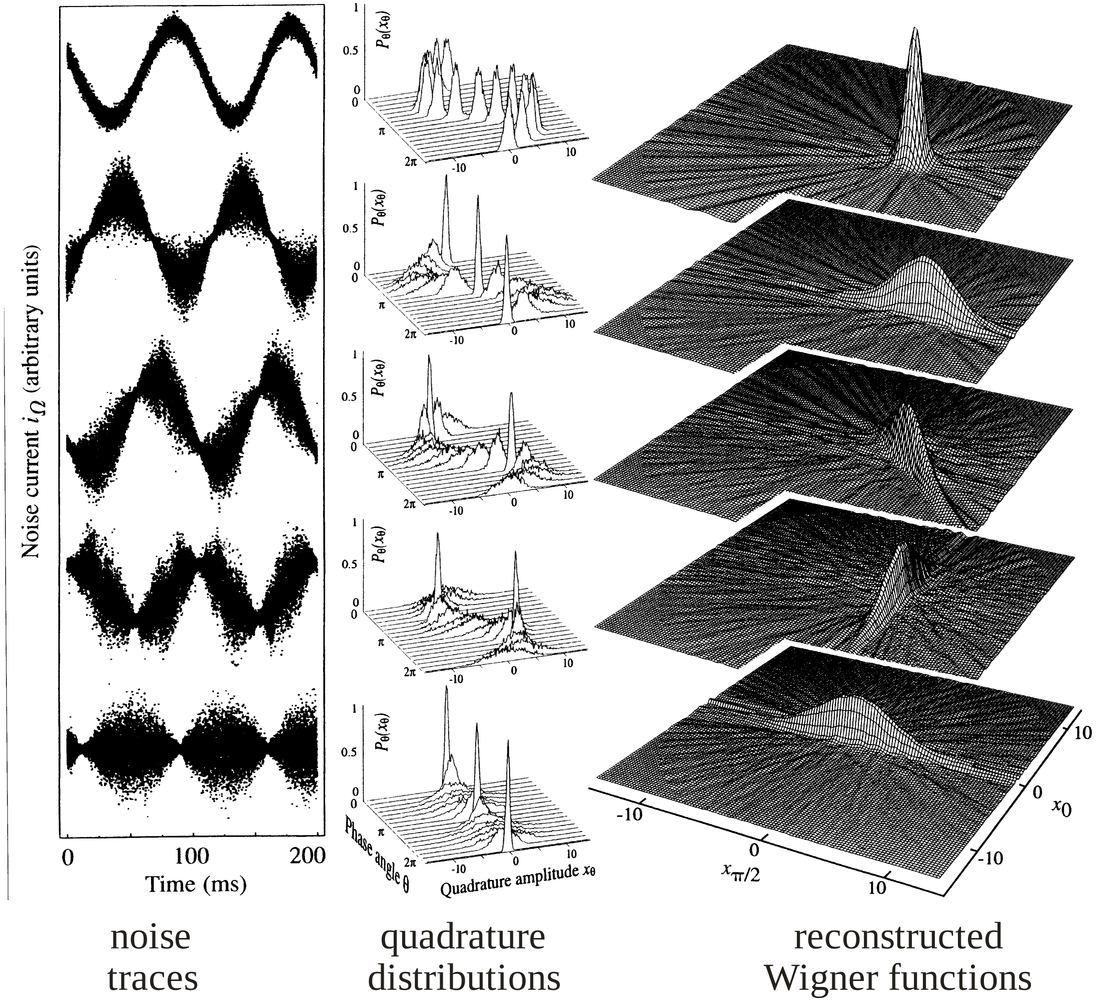
$$I_- = \left|\frac{E_{LO}+E_s}{\sqrt{2}}\right|^2 - \left|\frac{E_{LO}-E_s}{\sqrt{2}}\right|^2 = 2E_{LO}E_s$$

  Assume $|E_{\mathrm{LO}}| >> |E_s|$ so the local oscillator can be treated classically

  $\Rightarrow$ Subtraction photocurrent
  $\propto$ signal field (= X_θ)
- **Many measurements**
  $\rightarrow$ histogram pr(X_θ)
  ("marginal distribution")
- **Set of pr(X_θ) for all θ**
  $\rightarrow$ Wigner function W(X, P) (via inverse Radon transform)
  $\rightarrow$ Density matrix $\hat{\rho}$ (via likelihood maximization)



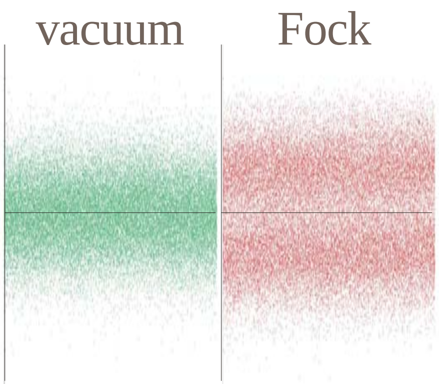
Example 1: squeezed states



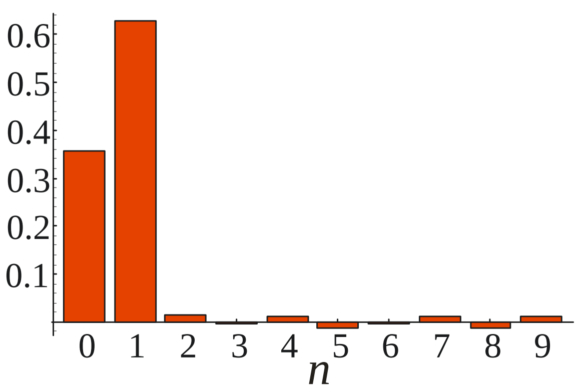
[G. Breitenbach, S. Schiller & J. Mlynek, Nature **387**, 471 (1997)]

Example 2:
Single-photon Fock state tomography

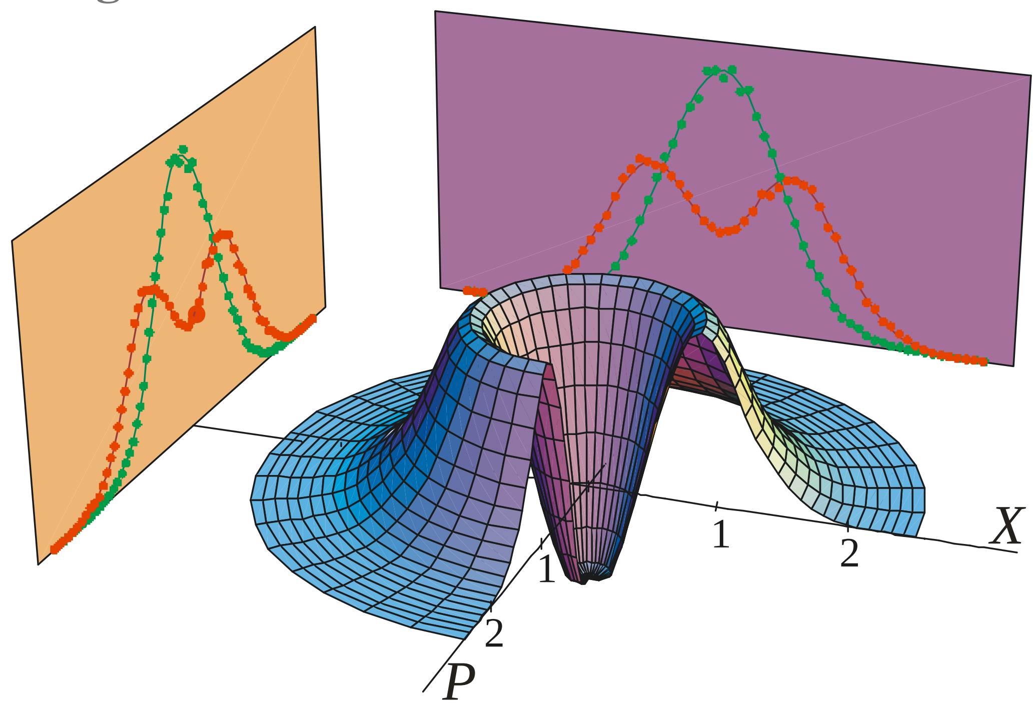
- Quadrature noise: raw data, 45000 pts



- Density matrix (diagonal elements)



- Wigner function reconstruction



Efficiency: 62%

Wigner function is negative in the origin of the phase space

A. I. Lvovsky et al., PRL **87**, 050402 (2001)

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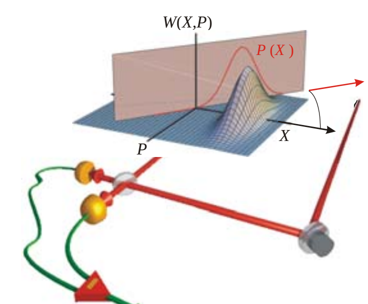
Summary to Part 1
2 methods of measuring quantum states of light

- **Photon counting**
  - Useful for characterizing multiple dual-rate qubits
  - Drawbacks of existing detectors
    - poor quantum efficiency
    - poor photon number resolving capability
  - Projects optical Hilbert space onto the qubit space

    $\Rightarrow$ Incomplete reconstruction



- **Homodyne tomography**
  - No photon number sensitivity required
  - High quantum efficiency
  - Complete characterization of the optical state in the local oscillator mode(s)
  - More difficult
    - Requires mode matching with the local oscillator
    - Requires more measurements

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MAKING QUANTUM STATES OF LIGHT

1. Photons
2. Biphotons
3. Squeezed states
4. Beam splitter
5. Conditional measurements

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MAKING QUANTUM STATES OF LIGHT

1. **Photons**
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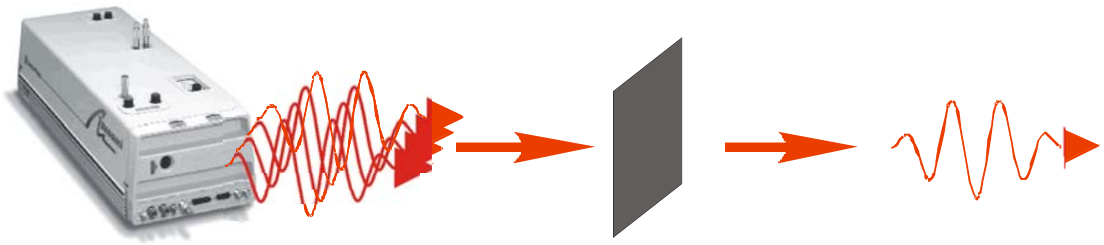
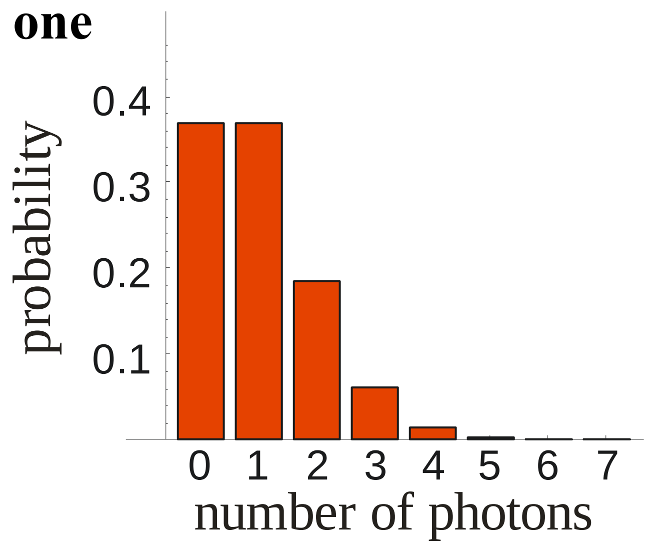
How to generate a photon?

- **Attenuate a laser beam?**
  - **Use a pulsed laser → attenuate to the one-photon level**



Output will be stochastic (Poissonian statistics): sometimes zero photons, sometimes more than one



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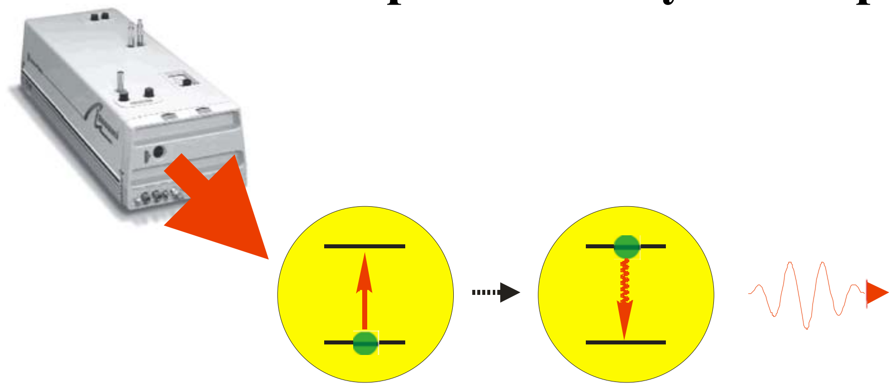
How to generate a photon? (…continued)

- Microscopic system (e.g. atom)
  - Excite using a laser
  - After a while, the svstem will spontaneously emit a photon



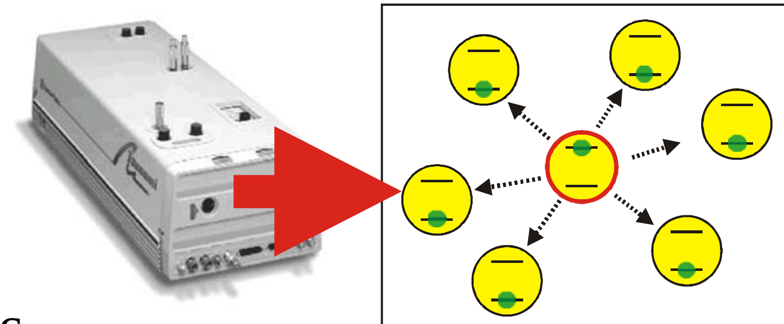
☺ Only one photon emitted at a time

☹ System is hard to prepare and keep stable

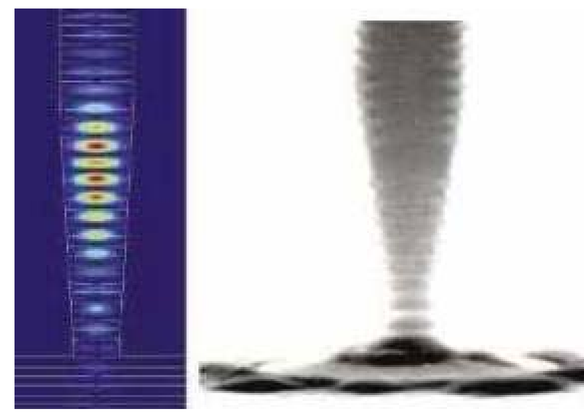
- Nitrogen vacancies in diamond
  - A single structure defect in a crystal
  - Similar to a single atom
  - When excited, cannot emit more than one photon at a time

How to generate a photon? (…continued)

- **Mesoscopic system (e.g. a quantum dot)**
  - Microscopic elements “talk” to each other
    → One excited element
    will prevent excitation of the others
  - ☺ Only one photon emitted at a time
  - ☺ System is easier to handle than microscopic



- **Quantum dot photon sources**
  - Self-assembled
    ⇒ need to pick a good dot to work with
  - Operate at cryogenic temperatures
  - Excited electrically or optically
  - Pico-or femtosecond pulse width
  - Difficult to make transform-limited
    → verification by the Hong-Ou-Mandel dip
  - Difficult to collect
    → microcavities



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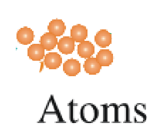
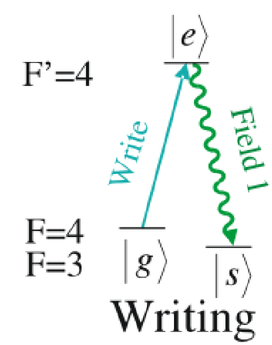
How to generate a photon? (…continued)

- ## Duan-Lukin-Cirac-Zoller method [Nature **414**, 413 (2001)]
  - **Step 1: “Writing”**
    - Prepare all atoms in the $|g\rangle$ state
    - Excite the $|g\rangle \to |e\rangle$ transition with a weak laser pulse
    - Observe Raman scattering of a single photon on $|e\rangle \to |s\rangle$ transition
    - “Single atom” (spin wave) now stored in $|s\rangle$
  - **Step 2: “Reading”**
    - Excite the $|s\rangle \to |e\rangle$ transition
    - Single Photon will be emitted on the $|e\rangle \to |g\rangle$
- ## Comments
  - Up to 50 % efficiency achieved
  - Narrowband photon $\to$ suitable for experiments with atoms





[Reproduced from J. Laurat *et al.*, Optics Express **14**, 6912 (2006)]

MAKING QUANTUM STATES OF LIGHT

1. Photons
2. **Biphotons**
3. Squeezed states
4. Beam splitter
5. Conditional measurements

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Elements of nonlinear optics

- **Linear medium:**

  polarization is proportional to the EM field

- **Nonlinear medium:**

  $$P_i \propto \chi_{ij}^{(1)} E_j + \chi_{ijk}^{(2)} E_j E_k + \ldots$$

  - **If** $E \propto e^{i\omega t}$ **then** $P_i \propto \ldots + e^{2i\omega t} + \ldots$

    → second harmonic generation

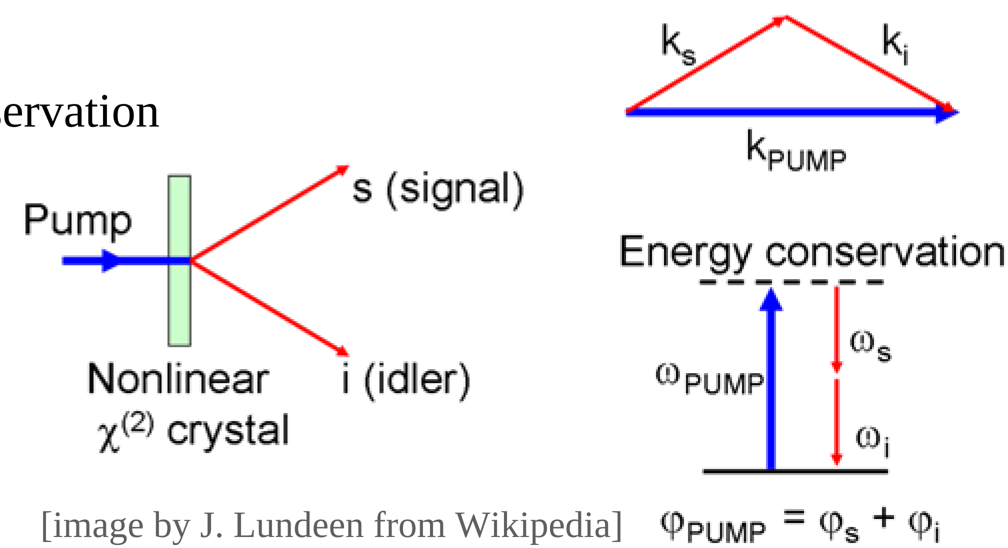
  - **If two fields are present (ω_1 and ω_2) then** $P_i \propto \ldots + e^{i(\omega_1+\omega_2)t} + e^{i(\omega_1-\omega_2)t} + \ldots$

    → sum, difference frequency generation

- **These are classical effects**
- **Quantum interpretation of second harmonic generation:**
  - **Two photons "unite" to form a single photon of higher energy**

Parametric down-conversion

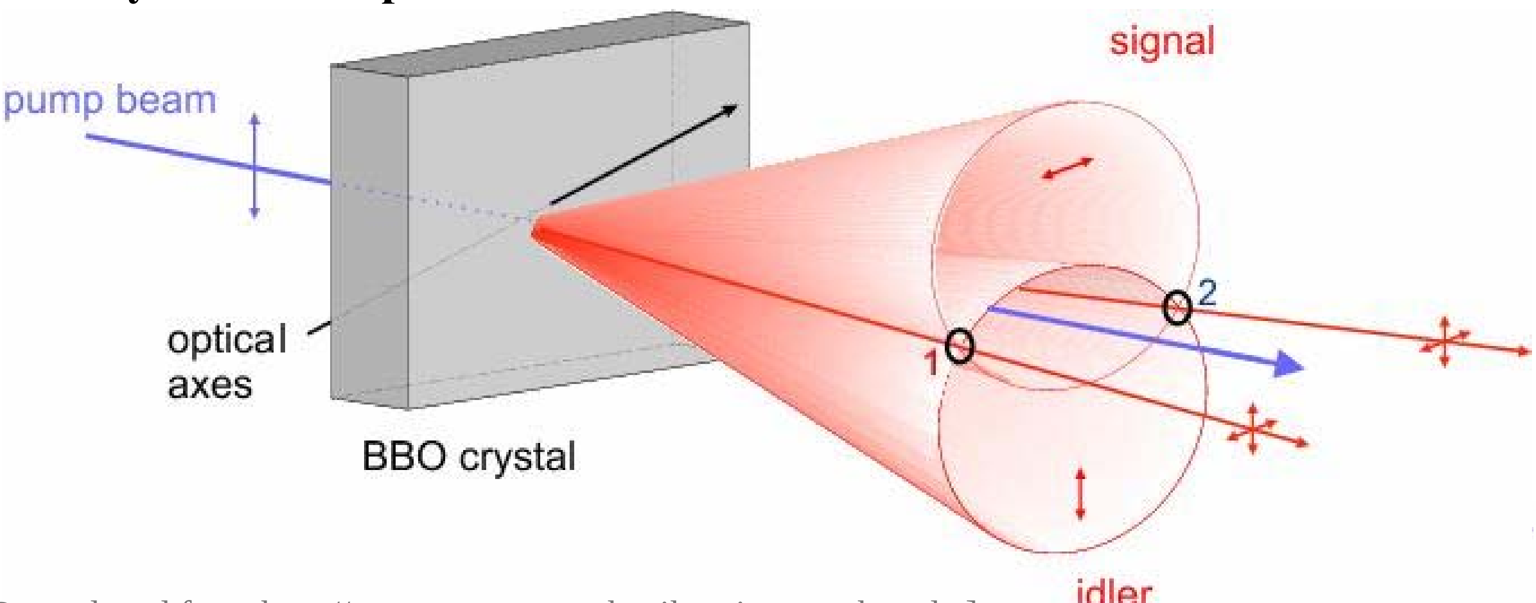
- **Quantum description**
  - Interaction energy/Hamiltonian: $H \propto \vec{E}\,\vec{P} \propto \sum E_i E_j E_k$
  - In the quantum form: $\hat{H} \propto \ldots + \hat{a}_1 \hat{a}_2^\dagger \hat{a}_3^\dagger + \text{H.c.} + \ldots$
  - Evolution (assume weak perturbation):
    $$|\Psi(t)\rangle = e^{i\hat{H}t}|\Psi(0)\rangle \approx |0\rangle + i\hat{H}t|0\rangle = |0\rangle + ig\hat{a}_1\hat{a}_2^\dagger\hat{a}_3^\dagger t|0\rangle$$
- **Interpretation:**
  - a photon of wave 1 ("pump") can split into two photons of waves 2 and 3.
  - may occur spontaneously:
    waves 2 and 3 need not be present
  - Purely quantum effect
  - Energy and momentum conservation (phase matching) must hold.
  - Main property: photons are always born in pairs.



[image by J. Lundeen from Wikipedia]

Type I and Type II down-conversion

- **Type I**
  - **Generated photons are of the same polarization**
  - **Useful for squeezing, preparation of heralded single photons, *etc*.**
- **Type II**
  - **Photons have different polarizations**
  - **Emitted along two cones**
  - **Polarization-entangled biphoton** $|HV\rangle + e^{i\varphi}|VH\rangle$ **at the intersection of cones**
  - **Basis for many modern experiments**



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MAKING QUANTUM STATES OF LIGHT

1. Photons
2. Biphotons
3. **Squeezed states**
4. Beam splitter
5. Conditional measurements

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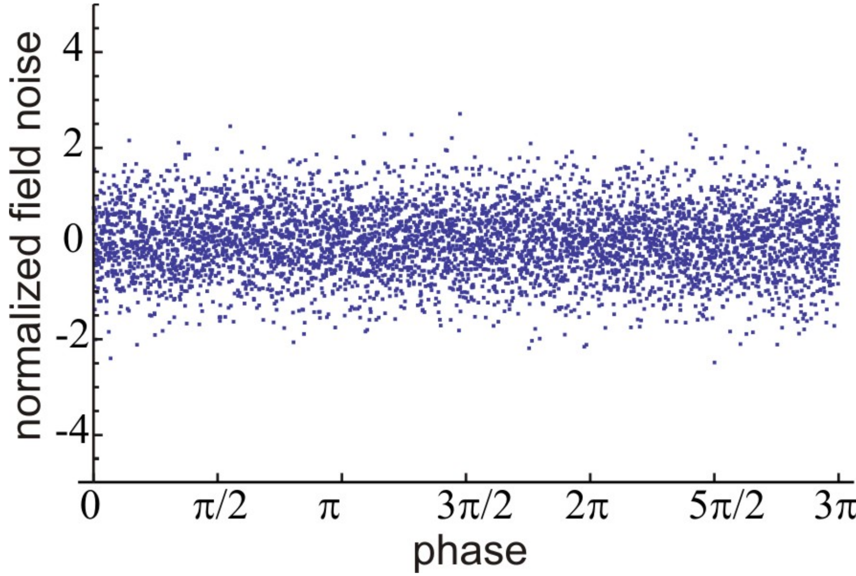
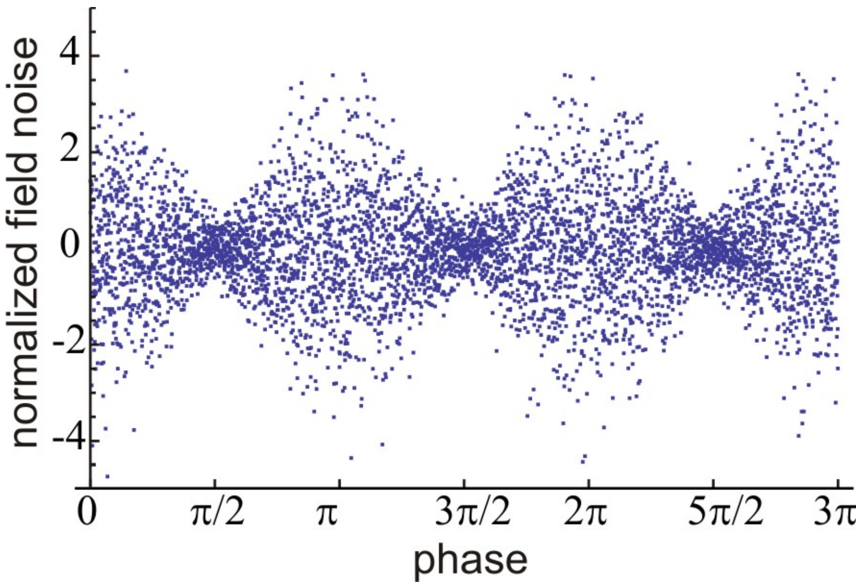
What is squeezed light?

- **Vacuum state: light is off**
  - Quantum noise phase-independent
  - Related to shot noise in electronics
- **Squeezed vacuum state**
  - Quantum noise phase-dependent
  - At some phases, noise *below* the vacuum level
  - At other phases, excessive noise (uncertainty principle!)
- **Applications**
  - Precision interferometric meaurements (e.g. gravitation wave detection)
  - Major quantum information primitive





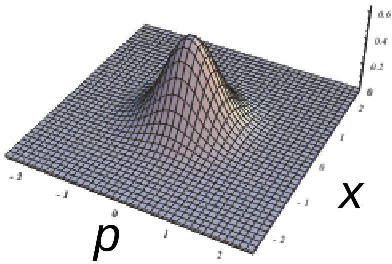
What is squeezed light?

- **Vacuum state: light is off**
  - Quantum noise phase-independent
  - Related to shot noise in electronics
- **Squeezed vacuum state**
  - Quantum noise phase-dependent
  - At some phases, noise *below* the vacuum level
  - At other phases, excessive noise (uncertainty principle!)
- **Applications**
  - Precision interferometric meaurements (e.g. gravitation wave detection)
  - Major quantum information primitive

Vacuum state wave function

$$\psi_o(x) = e^{-x^2/2}$$

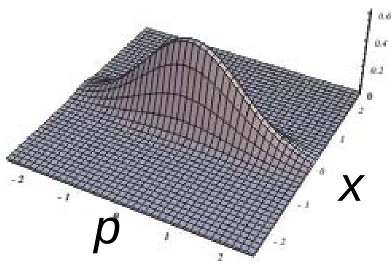
$$\psi_o(p) = e^{-p^2/2}$$



X-squeezed state wave function

$$\psi_s(x) = e^{-(sx)^2/2}$$

$$\psi_s(p) = e^{-(p/s)^2/2}$$



Problem. Normalize the above wave functions

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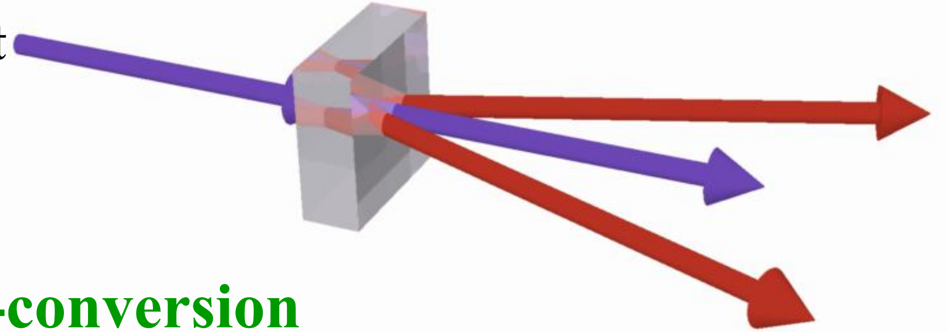
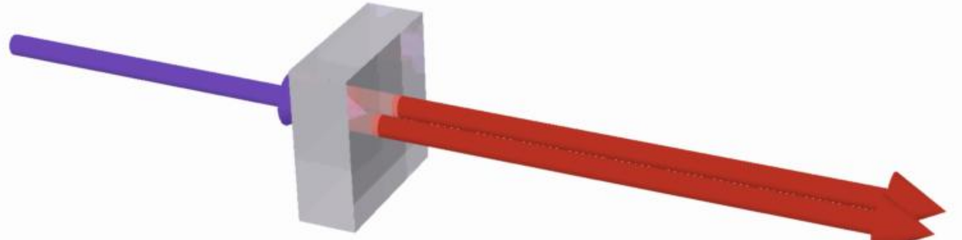
How to produce squeezing?

- **Non-degenerate parametric down-conversion**
  - **Photons are different in direction, frequency, polarization**
  - **Used e.g. to create entanglement**



- **Degenerate parametric down-conversion**
  - **Photons are identical**
  - **If we can generate enough pairs, output will be squeezed**
  - **Use optical cavity to enhance nonlinearity**

Problem. Show that the state $|0\rangle + \beta|2\rangle$ is squeezed for some values of β



Generation of squeezed states

- **Fully degenerate down-conversion**
  - ⇒ **Generated photons are identical:** $\hat{a}_2 = \hat{a}_3$
  - ⇒ **Hamiltonian becomes** $\hat{H} \propto \hat{a}_1 \hat{a}_2^\dagger \hat{a}_3^\dagger + \text{H.c.} \quad \rightarrow \quad \hat{a}_1 \left(\hat{a}_2^\dagger\right)^2 + \text{H.c.}$
- **Strong pump**
  - ⇒ **Can assume classical:** $\hat{a}_1 \rightarrow i\alpha$. **Assume** α **real.**
  - ⇒ **Cannot use one-pair approximation**
- **Heisenberg evolution**
  - **For field operators:**

    $$\dot{\hat{a}}_2 = i[\hat{H}, \hat{a}_2] = 2\alpha \hat{a}_2^\dagger$$

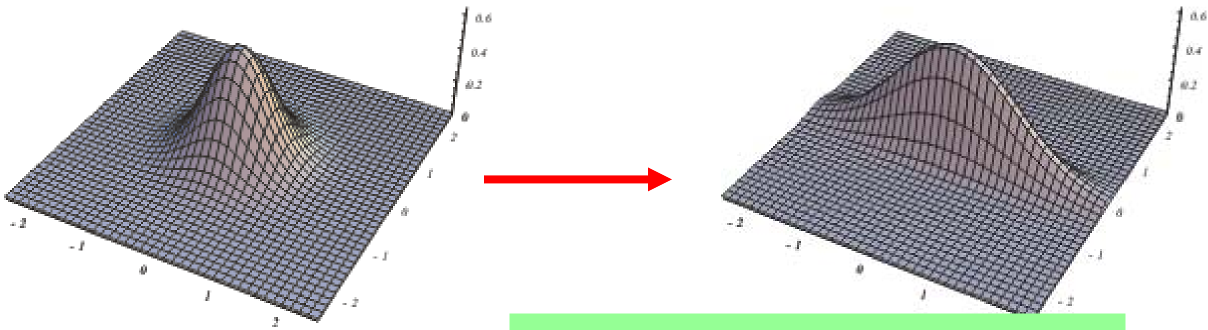
    $$\dot{\hat{a}}_2^\dagger = 2\alpha \hat{a}_2$$

  - **For field quadratures:**

    $$\dot{\hat{X}} = 2\alpha \hat{X} \quad \Rightarrow \quad \hat{X}(t) = e^{2\alpha t} \hat{X}(0)$$

    $$\dot{\hat{P}} = -2\alpha \hat{P} \quad \Rightarrow \quad \hat{P}(t) = e^{-2\alpha t} \hat{P}(0)$$

  - ***P*** **shrinks,** ***X*** **expands → squeezed vacuum!**
  - **Unlike biphotons, squeezed states are "on demand"**



Problem. Repeat this calculation for a complex α

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THE END

http://iqis.org/quantech
PhD student positions available!

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