# Quantum observations
# talk 5

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# Reconstruction of q-channels

# Qubit – encoding orientation

Pure state of a spin -1/2 particle

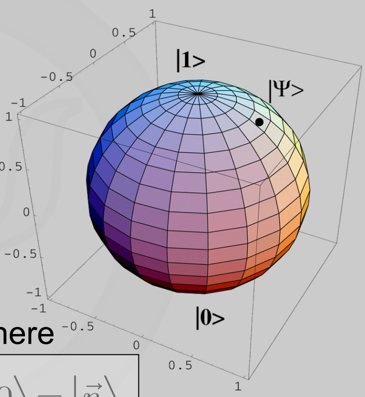
$$|\psi\rangle = \cos\vartheta/2\,|1\rangle + e^{i\varphi}\sin\vartheta/2\,|0\rangle$$

2-d Hilbert space

density operator

$$\hat{\rho} = \frac{1}{2}\left(\hat{I} + \vec{n}.\vec{\sigma}\right) = \frac{1}{2}\left(\hat{I} + n_x\hat{\sigma}_x + n_y\hat{\sigma}_y + n_z\hat{\sigma}_z\right)$$

$$\rho = \frac{1}{2}\left(1 + \vec{n}.\vec{\sigma}\right) = \frac{1}{2}\begin{pmatrix}1\\ n_x\\ n_y\\ n_z\end{pmatrix} \leftrightarrow \vec{n} = \left(n_x, n_y, n_z\right)$$

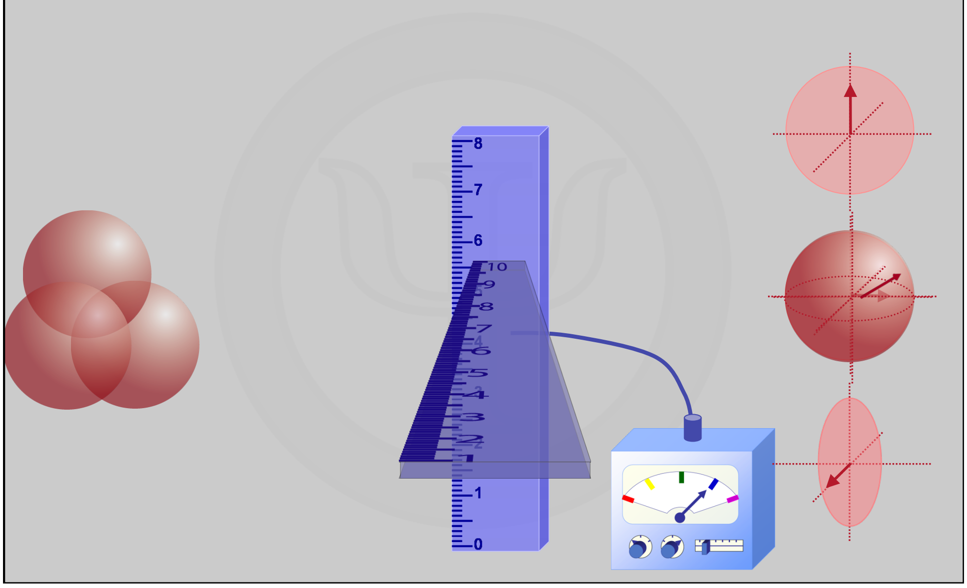



State space – Bloch (Poincare) sphere

$$|\psi\rangle = \cos\vartheta/2\,|1\rangle + e^{i\varphi}\sin\vartheta/2\,|0\rangle = |\vec{n}\rangle$$

$$\vec{n}.\vec{\sigma}\,|\vec{n}\rangle = \vec{n}\,|\vec{n}\rangle$$

# Complete state measurement

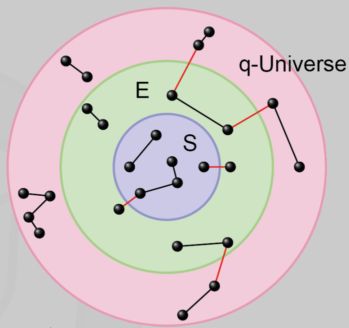


# Black-box problem

- Having a black box (with no memory) processing one qubit in a time, how can we determine this channel?


C.W.Holstrom, ***Quantum detection and estimation theory*** **(Academic Press, New York, 1976)**
A.S.Holevo, ***Probabilistic and statistical aspects of quantum theory*** **(North Holland, Amsterdam, 1982)**
**J.F.Poyatos and J.I.Cirac, PRL 78, 390 (1997)**

# Open quantum systems




$$\hat{H}_{SE} = \hat{H}_S \otimes \hat{1}_E + \hat{H}_{int} + \hat{1}_S \otimes \hat{H}_E$$

$$\hat{\rho}_{SE}(t) = \exp[-i(t-t_0)\hat{H}_{SE}]\hat{\rho}_{SE}(t_0)\exp[i(t-t_0)\hat{H}_{SE}],$$

$$\hat{\rho}_{SE}(t_0) = \hat{\rho}_S(t_0) \otimes \hat{\rho}_E(t_0)$$

# Reduced dynamics

$$\hat{\rho}_S(t) := \hat{\mathcal{T}}(t,t_0)\hat{\rho}_S(t_0) = \mathrm{Tr}_E\left[\hat{\rho}_{SE}(t)\right]$$

$\hat{\mathcal{T}}(t,t_0)$ is a linear map which transforms the input state $\hat{\rho}_S(t_0)$ onto the output state $\hat{\rho}_S(t)$.

**The question**: how we can determine (reconstruct) the master equation which governs the time evolution of the reduced density operator $\hat{\rho}_S(t)$

$$\frac{d}{dt}\hat{\rho}(t) = \hat{\mathcal{L}}(t,t_0)\hat{\rho}(t).$$

# Liouvillian superoperator

This master equation can be written in the *convolutionless* form

$$\frac{d}{dt}\hat{\rho}(t) = \hat{\mathcal{L}}(t,t_0)\hat{\rho}(t).$$

due to the fact that in the *finite-dimensional* Hilbert spaces matrix elements of density operators are analytic functions. Consequently, $\hat{\mathcal{T}}(t,t_0)$ are non-singular operators (except may be for a set of *isolated* values of $t$) in which case the inverse operators $\hat{\mathcal{T}}(t,t_0)^{-1}$ exist and the Liouvillian superoperator can be expressed as

$$\hat{\mathcal{L}}(t,t_0) := \left[\frac{d}{dt}\hat{\mathcal{T}}(t,t_0)\right]\hat{\mathcal{T}}^{-1}(t,t_0).$$

$\hat{\mathcal{T}}(t,t_0)$ is uniquelly specified by $\hat{H}_{SE}$ and by the initial state $\hat{\rho}_E(t_0)$ of the environment.

## Reconstruction of the map $\hat{\mathcal{T}}(t, t_0)$

Let us assume that the system $S$ has been initially prepared in a pure state

$$|\Psi(t_0)\rangle = \sum_{i_1=0}^{N} c_{i_1} |i_1\rangle$$

where $|i_1\rangle$ are basis vectors in the $(N+1)$-dimensional Hilbert space $\mathcal{H}_S$ of the system under consideration.

The environment is initially prepared in a state $\hat{\rho}_E(t_0) = \sum_{\alpha_1\alpha_2} d_{\alpha_1\alpha_2} |\alpha_1\rangle_E \langle\alpha_2|$, where $|\alpha_i\rangle_E$ are basis vectors in the Hilbert space $\mathcal{H}_E$ of the environment.

## Reconstruction of the map $\hat{\mathcal{T}}(t, t_0)$

The physical process $\hat{\mathcal{T}}(t_k, t_0)$ is determined by a transformation acting on basis vectors of the system and the environment

$$|i_1\rangle_S |\alpha_1\rangle_E \xrightarrow{\hat{\mathcal{T}}(t_k)} \sum_{j_1=0}^{N} \sum_{\beta_1} E_{(i_1 j_1)(\alpha_1\beta_1)}(t_k) |j_1\rangle_S |\beta_1\rangle_E .$$

## The output density operator $\hat{\rho}(t_k)$

$$\hat{\rho}(t_k) = \sum_{i_1,i_2=0}^{N} c_{i_1}(c_{i_2})^* \hat{R}_{(i_1,i_2)}(t_k),$$

where $(N+1)^2$ operators $\hat{R}_{(i_1,i_2)}(t_k)$

$$\hat{R}_{(i_1,i_2)}(t_k) = \sum_{j_1,j_2=0}^{N} D_{(i_1,i_2)(j_1,j_2)}(t_k)|j_1\rangle\langle j_2|,$$

with

$$D_{(i_1,i_2)(j_1,j_2)}(t_k) = \sum_{\alpha_1,\alpha_2,\gamma} d_{\alpha_1\alpha_2} \times E_{(i_1 j_1)(\alpha_1\gamma)}(t_k) E^*_{(i_2 j_2)(\alpha_2\gamma)}(t_k).$$

## Properties of $\hat{R}_{(i_1,i_2)}(t_k)$

The process $\hat{\mathcal{T}}(t_k)$ for a given time $t_k$ is *completely* determined by $(N+1)^2$ operators $\hat{R}_{(i_1,i_2)}(t_k)$, which in turn are specified by the $(N+1)^2 \times (N+1)^2$ matrix elements $D_{(i_1,i_2)(j_1,j_2)}(t_k)$.

$\hat{R}_{(i_1,i_2)}(t_k)$ have the properties

$$\mathrm{Tr}\hat{R}_{(i_1,i_2)}(t_k) = \delta_{i_1,i_2}; \qquad (\hat{R}_{(i_1,i_2)}(t_k))^\dagger = \hat{R}_{(i_2,i_1)}(t_k),$$

or, equivalently,

$$\sum_{j=0}^{N} D_{(i_1,i_2)(j,j)}(t_k) = \delta_{i_1,i_2}; \qquad D^*_{(i_1,i_2)(j_1,j_2)}(t_k) = D_{(i_2,i_1)(j_2,j_1)}(t_k).$$

# Determination of $\hat{R}_{(i_1,i_2)}(t_k)$

In order to specify the $(N+1)^2$ operators $\hat{R}_{(i_1,i_2)}(t_k)$ one has to consider $(N+1)^2$ specific initial conditions $|\Psi^{(k_1,k_2)}\rangle_{in} = \sum_{i_1=0}^{N} c_{i_1}^{(k_1,k_2)}|i_1\rangle$ where $k_1, k_2 = 0, 1, ..., N$ and to measure the corresponding $(N+1)^2$ output density operators $\hat{\rho}^{(k_1,k_2)}(t_k)$ which can be expressed as

$$\hat{\rho}^{(k_1,k_2)}(t_k) = \sum_{i_1,i_2=0}^{N} M_{(k_1,k_2)(i_1,i_2)}\hat{R}_{(i_1,i_2)}(t_k),$$

where

$$M_{(k_1,k_2)(i_1,i_2)} = c_{i_1}^{(k_1,k_2)}(c_{i_2}^{(k_1,k_2)})^*.$$

If the $(N+1)^2$ initial conditions $|\Psi^{(k_1,k_2)}\rangle_{in}$ are chosen so, that the matrix $M_{(k_1,k_2)(i_1,i_2)}$ is invertible, then the set of equations can be solved with respect of the operators $\hat{R}_{(i_1,i_2)}(t_k)$.

# Determination of $\hat{R}_{(i_1,i_2)}(t_k)$

To make the reconstruction possible the matrix $M$ has to be invertible. Obviously, there are many choices of such matrix. For instance $M$ is given by complex amplitudes $c_i^{(k_1,k_2)}$ specified as

$$c_i^{(k_1,k_2)} = \begin{cases} (\delta_{i,k_1} + \delta_{i,k_2})/\sqrt{2} & \text{if} \quad k_1 > k_2 \\ \delta_{i,k_1} & \text{if} \quad k_1 = k_2 \\ (\delta_{i,k_1} + i\delta_{i,k_2})/\sqrt{2} & \text{if} \quad k_1 < k_2 \end{cases} . \tag{1}$$

The reconstruction process described above gives us a set of operators $\hat{R}_{(i_1,i_2)}(t_k)$ which describe the transition of the system from the state $\hat{\rho}(t_0)$ to the state $\hat{\rho}(t_k)$ at a given time $t_k$. In principle, one can perform a whole sequence of such reconstructions at different times $t_1, t_2, ....t_K$ so that the *reduced dynamics* of the studied system can be reconstructed from the measured data.

## Reconstruction of $\hat{\mathcal{L}}(t)$

Now our task is to determine (reconstruct) from a set of measurements of the output states $\hat{\rho}^{(k_1,k_2)}(t)$ for given input states $\hat{\rho}^{(k_1,k_2)}(t_0)$, the form of the Liouvillian superoperator $\hat{\mathcal{L}}(t)$.

The operators $\hat{R}_{(i_1,i_2)}(t)$ are also governed by the same master equation

$$\frac{d}{dt}\hat{R}_{(i_1,i_2)}(t) = \hat{\mathcal{L}}(t)\hat{R}_{(i_1,i_2)}(t),$$

Alternatively, for matrix elements $D_{(i_1,i_2)(k_1,k_2)}(t)$

$$\frac{d}{dt}D_{(i_1,i_2)(k_1,k_2)}(t) \sum_{j_1,j_2=0}^{N} D_{(i_1,i_2)(j_1,j_2)}(t)G_{(j_1,j_2)(k_1,k_2)}(t),$$

Here the matrix $G_{(j_1,j_2)(k_1,k_2)}(t)$ is defined as

$$G_{(j_1,j_2)(k_1,k_2)}(t) = \langle k_1| \left(\hat{\mathcal{L}}(t)|j_1\rangle\langle j_2|\right) |k_2\rangle,$$

and it *uniquely* determines the Liouvillian superoperator $\hat{\mathcal{L}}(t)$.

## Reconstruction of $\hat{\mathcal{L}}(t)$

We already know how to reconstruct matrices $D$ from the measured data for arbitrary time $t$ (from these data we can also evaluate the corresponding time derivatives). Providing the matrix $D_{(i_1,i_2)(j_1,j_2)}(t)$ is not singular its inverse $\tilde{D}_{(j_1,j_2)(i_1,i_2)}(t)$ can be found and then the reconstructed matrix $G_{(j_1,j_2)(k_1,k_2)}(t)$ is given by a simple expression

$$G_{(j_1,j_2)(k_1,k_2)}(t) = \sum_{i_1,i_2=0}^{N} \tilde{D}_{(j_1,j_2)(i_1,i_2)}(t)\frac{d}{dt}D_{(i_1,i_2)(k_1,k_2)}(t)$$

from which the superoperator $\hat{\mathcal{L}}(t)$ at time $t$ can be determined.

**The end**

# Example 1: Decay of 2-leve atom

Let us consider a two-level system (a two-level atom, a spin-1/2, or a qubit) with a two-dimensional Hilbert space $\mathcal{H}_S$ spanned by two vectors $|1\rangle$ and $|0\rangle$. In order to specify the Liouvillian superoperator $\hat{\mathcal{L}}(t)$ for the two-level atom we have to know the time evolution of four initial states. Let us assume that from the measured data it is found that these states evolve as

$$\begin{aligned}
\hat{\rho}^{(0,0)}(t) &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \qquad \hat{\rho}^{(1,1)}(t) = \begin{pmatrix} e^{-\Gamma t} & 0 \\ 0 & 1-e^{-\Gamma t} \end{pmatrix}. \\
\hat{\rho}^{(0,1)}(t) &= \frac{1}{2}\begin{pmatrix} e^{-\Gamma t} & ie^{-\Gamma t/2} \\ -ie^{-\Gamma t/2} & 2-e^{-\Gamma t} \end{pmatrix}; \\
\hat{\rho}^{(1,0)}(t) &= \frac{1}{2}\begin{pmatrix} e^{-\Gamma t} & e^{-\Gamma t/2} \\ e^{-\Gamma t/2} & 2-e^{-\Gamma t} \end{pmatrix}.
\end{aligned}$$

# Example 1: Decay of 2-leve atom

Now we can apply our reconstruction scheme and we find for the matrix $G_{(j_1,j_2)(k_1,k_2)}(t)$ the expression

$$G_{(j_1,j_2)(k_1,k_2)}(t) = \begin{pmatrix} -\Gamma & 0 & 0 & \Gamma \\ 0 & -\Gamma/2 & 0 & 0 \\ 0 & 0 & -\Gamma/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

This matrix corresponds to the Liouvillian which defines the master equation

$$\frac{d}{dt}\hat{\rho} = \hat{\mathcal{L}}\hat{\rho} = \frac{\Gamma}{2}\left[2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-\right],$$

describing the decay of a two-level atom into a zero-temperature reservoir. The Liouvillian is time independent which reflects the fact that the state of the reservoir does not change in time under the influence of the system.

# Example 2: Decay of 2-leve atom

Let us consider a single two-level atom coupled to $K$ modes of the electromagnetic field in a one-dimensional cavity of the length $L$. The spectrum of modes is discrete with frequencies $\omega_k = k\pi c/L$. The corresponding total Hamiltonian in the dipole and rotating-wave approximations reads

$$\hat{H} = \omega_A \hat{\sigma}_z + \sum_{k=1}^{K} \omega_k \hat{a}_k^\dagger \hat{a}_k + \sum_{k=1}^{K} \lambda_k (\hat{\sigma}_+ \hat{a}_k + \hat{\sigma}_- \hat{a}_k^\dagger).$$

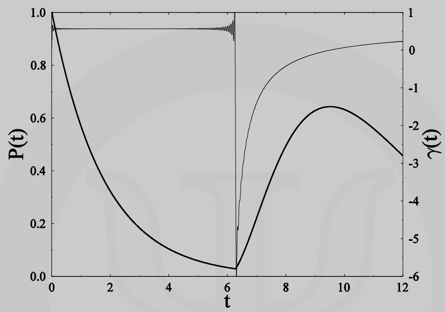
The field is assumed to be initially in the *vacuum* state. By applying our algorithm we find the master equation for the atom

$$\frac{d}{dt}\hat{\rho} = \hat{\mathcal{L}}\hat{\rho} = \frac{\gamma(t)}{2}\left[2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-\right],$$

except the "decay" rate $\Gamma \to \gamma(t)$ is now explicitly time dependent. It can be expressed in terms of the "measured" probability $P(t) = \langle 1|\hat{\rho}_A(t)|1\rangle$ that the upper atomic level is excited:

$$\gamma(t) = -\left(\frac{dP(t)}{dt}\right) P(t)^{-1}.$$

# Example 2: Decay of 2-leve atom




The time evolution of the decay rate $\gamma(t)$ (thin line) and the population of the excited atomic level $P(t)$ (thick line). We assume the atom to be in the center of the 1-D cavity, so it is coupled only to the odd modes (i.e. $\lambda_{2k} = 0$). We assume $L = 2\pi$ and $c = 1$ so that $\omega_{2k+1} = k+1/2$, and $\lambda_{2k+1} = \lambda = 0.3$. The effective density of modes which interact with the atom is $d_{eff}(\omega) = L/2c\pi = 1$. Therefore the decay rate $\Gamma = 2\pi\lambda^2 d_{eff}(\omega) \simeq 0.564$. We consider $K = 400$ modes of the field initially in the vacuum state and the atom (with $\omega_A = 101$) in its upper state $|1\rangle$.

# General operations (maps, channels)

- The density operator $\rho = \frac{1}{2}\left(1+\vec{r}.\vec{\sigma}\right) = \frac{1}{2}\begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} \leftrightarrow \vec{r} = \left(x,y,z\right)$

- The general operation is an affine transformation of the Bloch sphere

$$\vec{r} \to \vec{r}' = T\vec{r} + \vec{t}$$

$$\varepsilon = \begin{pmatrix} 1 & 0 \\ \vec{t} & \vec{T} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ x & \alpha_1 & \alpha_2 & \alpha_3 \\ y & \beta_1 & \beta_2 & \beta_3 \\ z & \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$$

$$\rho' = \varepsilon\left[\rho\right] = \sum_l A_l \rho \, A_l^\dagger$$

- With $x^2 + y^2 + z^2 \le 1$ and $\forall j \quad \left(x-\alpha_j\right) + \left(y-\beta_j\right) + \left(z-\gamma_j\right) \le 1$

# Complete Positivity

- **Every guess must be completely positive – in general it is hard to achieve analytically**
- **Check is done by applying an operation in the form**

$$\Omega = I \otimes \varepsilon$$

**on to the maximally entangled state**

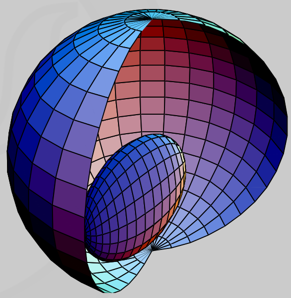
$$\left|\phi_-\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle \otimes \left|1\right\rangle - \left|1\right\rangle \otimes \left|0\right\rangle\right)$$

# Example: Specific channel

- Let us assume a specific transformation – map, channel

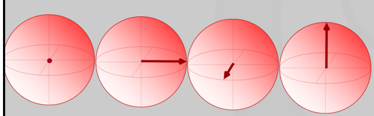
$$\rho' = \varepsilon\left[\rho\right] = \sum_l A_l \rho \; A_l^\dagger$$

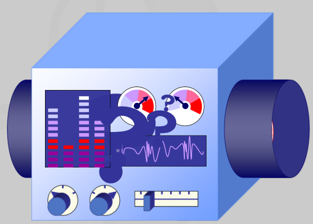
$$\varepsilon = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.2 & -0.1 & 0.1 \\ 0 & 0.2 & 0 & -0.3 \\ 0 & 0 & 0.3 & 0.3 \end{pmatrix}$$



# Complete reconstruction

- For a complete estimation one needs four different states, which are linearly independent.





- Limited resources?

# Reconstruction of processes

- **Data from the Wunderlich exp.**

$$\mathcal{E}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -0.015955 & 0.526585 & 0.069145 & 0.069145 \\ 0.042555 & -0.117015 & 0.585105 & -0.106385 \\ 0.0212745 & -0.1170145 & -0.0319145 & 0.8191495 \end{pmatrix} \quad (3)$$

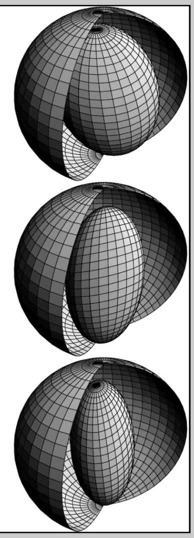
**complete positivity:** OK
**closer phase damping channel:** $\lambda = 0.53$ with $d = 0.166$
**phase damping diagonal:** $\lambda = 0.55$
**phase damping sv:** $\lambda = 0.56$
**singular values:** $0.84285, 0.596037, 0.523654$
**unitality:** $d(\mathcal{E}[I], I) = 0.05$

$$\mathcal{E}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -0.12234 & 0.42021 & -0.0266 & -0.1968 \\ 0.050335 & -0.029055 & 0.428395 & -0.071605 \\ -0.026142 & 0.153802 & 0.153802 & 0.802732 \end{pmatrix} \quad (4)$$

**complete positivity:** OK
**closer phase damping channel:** $\lambda = 0.45$ with $d = 0.25$
**phase damping diagonal** $\lambda = 0.42$
**phase damping sv:** $\lambda = 0.44$
**singular values:** $0.843171, 0.449961, 0.428712$
**unitality:** $d(\mathcal{E}[I], I) = 0.12$

$$\mathcal{E}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -0.03226 & 0.46237 & 0.2043 & -0.0215 \\ -0.021505 & -0.086025 & 0.376345 & -0.075275 \\ -0.0161285 & -0.1021515 & 0.0161285 & 0.7580685 \end{pmatrix} \quad (5)$$

**complete positivity:** OK
**closer phase damping channel:** $\lambda = 0.41$ with $d = 0.24$
**phase damping diagonal** $\lambda = 0.42$
**phase damping sv:** $\lambda = 0.44$
**singular values:** $0.77543, 0.504205, 0.374885$
**unitality:** $d(\mathcal{E}[I], I) = 0.04$



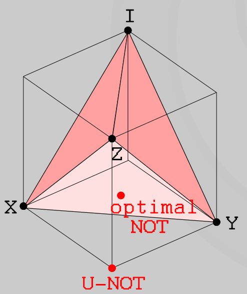
# Clickology: Maximum likelihood

- ML works with finite sets of data, not with infinite ensembles
- In case of quantum operations, the relevant data are
  - Input state specification $\rho_i$
  - Measurement direction $|\psi\rangle_i$
  - Measurement outcome (binary) $p_i$
- We build a functional $F = \prod_i \left[ \langle\psi|_i\, \varepsilon(\rho_i) |\psi\rangle_i\, p_i + \left(1 - \langle\psi|_i\, \varepsilon(\rho_i) |\psi\rangle_i\right)(1 - p_i) \right]$
- The numerical task is to find the $\varepsilon$, for which this functional reaches the maximum (using the logarithm of functional)
- Trace-preservation is obtained automatically from the parameterization, CP has to be checked in the algorithm

# Unital operations

- Displacement $\vec{t} = 0$
- Affine transformation specified as
- Positivity $\forall j \quad |\lambda_j| \leq 1$
- Complete positivity $|\lambda_1 \pm \lambda_2| \leq |1 \pm \lambda_3|$

$$\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & \vec{T} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_3 \end{pmatrix}$$




**Unital CP maps are embeded in the set of all positive unital maps (cube). The CP maps form a tetrahedron with four unitary transformations in its corners (extremal points) I,x,y,z corresponding to the Pauli sigma-matrices.**

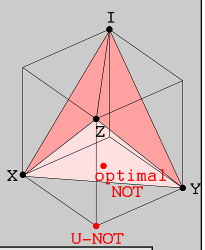
**The unphysical U-NOT operation** $\lambda_1 = \lambda_2 = \lambda_3 = -1$ **and its optimal completely positive approximation quantum universal NOT gate** $\lambda_1 = \lambda_2 = \lambda_3 = -1/3$ **are shown.**

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow |\psi^{\perp}\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$$

**M.Ziman & V.Buzek, PRA 72, 022110 (2005).**

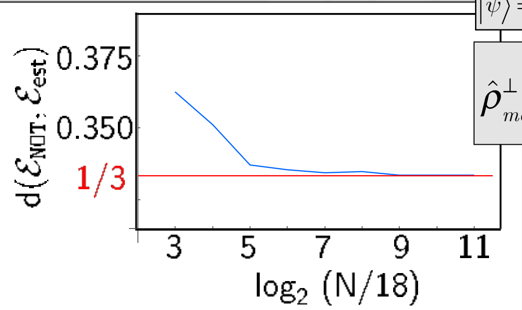
# Non-physical maps: U-NOT gate

- **Universal NOT gate** $\varepsilon = \text{diag}\ (1,-1,-1,-1)$
- **Best approximation** $\varepsilon = \text{diag}\ (1,-1/3,-1/3,-1/3)$

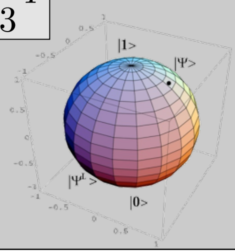



- **6 input states – eigenstates of** $\sigma_j$
- **3 measurements** $\sigma_j$
- **N=100 x 18 clicks**

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow |\psi^{\perp}\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$$




$$\hat{\rho}^{\perp}_{meas} = \frac{1}{3}\hat{\rho}^{\perp} + \frac{1}{3}\hat{I}$$



# Approximation of non-physical maps

- **Nonlinear polarization rotation**
- 1800 input states
- 3 measurements

$$\varepsilon[\rho] = \exp\left(i\frac{\Theta}{2}\langle\sigma_z\rangle_\rho \sigma_z\right)\rho\exp\left(-i\frac{\Theta}{2}\langle\sigma_z\rangle_\rho \sigma_z\right)$$

$$\varepsilon = \mathrm{diag}\ (1,\lambda,\lambda,1)$$