Quantum observations
talk 2

Vladimír Bužek
Research Center for Quantum Information, Bratislava, Slovakia

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Optimal state reconstructions

- Quantum states of light and the Wigner function
- Quantum tomography
- Quantum state reconstructions on various observation levels
- Maximum entropy (MaxEnt) principle
- Vibrational motion of neutral atoms
- Reconstruction of a cavity field via measurement of parity operator
- Max-Ent reconstruction of a state of a qubit

Quantum States of Light

Single mode field = quantum harmonic oscillator

$$\hat{E}(r,t) = \sqrt{2}\mathcal{E}_0 \left(\hat{a}e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t}\right) u(r)$$

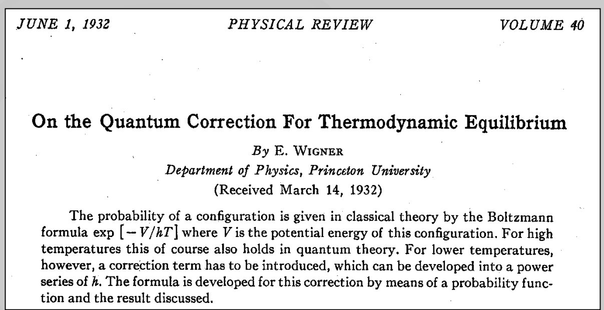
- $u(r)$ the spatial field distribution
- $\mathcal{E}_0 = (\hbar\omega/2\epsilon_0 V)^{1/2}$ electric field per photon
- $[\hat{a}, \hat{a}^\dagger] = 1$

$$\hat{q} = \sqrt{\frac{\hbar}{2}}\left[\hat{a} + \hat{a}^\dagger\right] \quad \hat{p} = \frac{\sqrt{\hbar}}{i\sqrt{2}}\left[\hat{a} - \hat{a}^\dagger\right]$$

Description of states

- State vector $|\Psi\rangle$
- density operator $\hat{\rho}$
- moments of system operators $\langle(\hat{a}^\dagger)^m \hat{a}^n\rangle$
- Wigner function

Wigner function: an insight into a quantum state


JUNE 1, 1932 *PHYSICAL REVIEW* *VOLUME 40*

On the Quantum Correction For Thermodynamic Equilibrium

By E. Wigner
Department of Physics, Princeton University
(Received March 14, 1932)

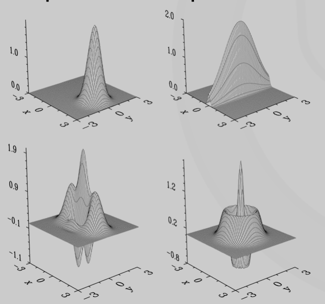
The probability of a configuration is given in classical theory by the Boltzmann formula exp $[-V/hT]$ where V is the potential energy of this configuration. For high temperatures this of course also holds in quantum theory. For lower temperatures, however, a correction term has to be introduced, which can be developed into a power series of h. The formula is developed for this correction by means of a probability function and the result discussed.

Wigner Functions of Light States

$$\boxed{W(q,p) = \frac{1}{2\pi\hbar}\int C(q',p')\exp\left[-\frac{i(qp'-pq')}{\hbar}\right]dq'dp'}$$

characteristic function $C_{\hat{\rho}}^{(W)}(q,p) = \mathrm{Tr}\left[\hat{\rho}\hat{D}(q,p)\right]$

displacement operator $\hat{D}(q,p) = \exp\left[\frac{i}{\hbar}(\hat{q}p - \hat{p}q)\right]$



Marginal distributions

$$P_{\hat{\rho}}(q) \equiv \frac{1}{\sqrt{2\pi\hbar}}\int dp\, W_{\hat{\rho}}(q,p) = \sqrt{2\pi\hbar}\langle q|\hat{\rho}|q\rangle$$

Pauli problem

M.Hillery, R.F.O' Connell, M.O.Scully, and E.P.Wigner, *Phys. Rep.* 106, 121 (1984)

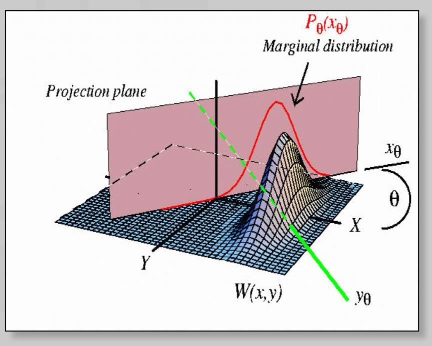
Quantum tomography

- **rotated quadratures**

$$\hat{x}_\theta = \sqrt{\frac{\hbar}{2}}\left[\hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}\right] \qquad \hat{x}_{\theta+\pi/2} = \frac{\sqrt{\hbar}}{i\sqrt{2}}\left[\hat{a}e^{-i\theta} - \hat{a}^\dagger e^{i\theta}\right]$$

- **marginal distribution for**

$$P_\theta(x_\theta)$$



K.Vogel and H.Risken, *Phys. Rev. A* 40, 2847 (1987);

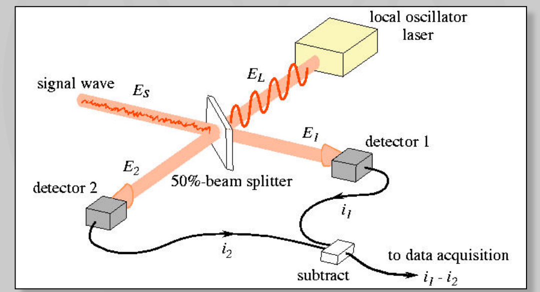
U.Leonhardt: ***Measuring the quantum state of light*** (Cambridge University Press, Cambridge, 1997).

Inverse transformations

$$\boxed{P_\theta(x_\theta)\forall\{-\infty \le x_\theta \le \infty; 0 \le \theta \le \pi\} \longrightarrow W(q,p)}$$

- Inverse Radon transformation
- Transformation via sampling functions
- Pauli problem

$$\rho_{mn} = \int_0^\pi \int_{-\infty}^{\infty} P_\theta(x_\theta) F_{mn}(x_\theta, \theta)\, dx_\theta\, d\theta$$



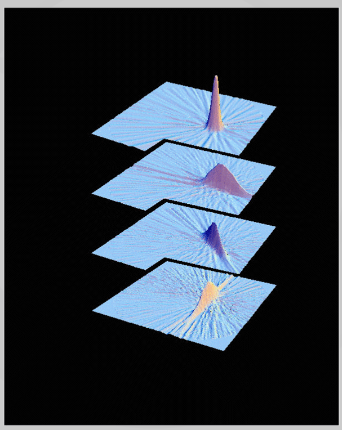
K.Vogel and H.Risken, *Phys. Rev. A* 40, 2847 (1987);

Th.Richter, *Phys. Lett. A* 211, 327 (1996);

G.M.D'Ariano, C.Macchiavelo, and M.G.A.Paris, *Phys. Rev. A* 50, 4298 (1994).

Experiments

- M.G.Raymer – first tomographic reconstruction 1993
- J.Mlynek – WF of squeezed light 1996, 1997



G.Breitenbach

D.T.Smithy, M.Beck, M.Besley, M.G.Raymer: *Phys. Rev. Lett.* 70, 1244 (1993)
G.Breitenbach, S.Schiller, J.Mlynek: *Nature* 387, 471 (1997)

Knowledge about physical situation

...existing quantum theory must be supplemented with some principle that tells us how to translate, or encode, the results of measurements into a definite state description $\hat{\rho}$. Note that the problem is not to find $\hat{\rho}$ which correctly describes "true physical situation". That is unknown, and always remains so, because of incomplete information. In order to have a usable theory we must ask the much more modest question:**What $\hat{\rho}$ best describes our state of knowledge about the physical situation?**

E.T.Jaynes

E.T.Jaynes: "Information theory and statistical mechanics" in *1962 Brandeis Lectures*, p 181

Incomplete observation levels

When instead of the density operator $\hat{\rho}$, expectation values G_ν of a set $\mathcal{O}$ of operators $\hat{G}_\nu$ $(\nu = 1, \ldots, n)$ are measured then a large number of density operators which fulfill the conditions

$$\mathrm{Tr}\,\hat{\rho}_{\{\hat{G}\}} = 1,$$

$$\mathrm{Tr}\,(\hat{\rho}_{\{\hat{G}\}}\hat{G}_\nu) = G_\nu\,, \quad \nu = 1, 2, ..., n;$$

can be found for a given set of expectation values $G_\nu = \langle \hat{G}_\nu \rangle$. That is, the conditions specify a set $\mathcal{C}$ of density operators which has to be considered.

MaxEnt principle

Each of these density operators $\hat{\rho}_{\{\hat{G}\}}$ can posses a different value of the uncertainty measure $\eta[\hat{\rho}_{\{\hat{G}\}}]$. If we wish to use only the expectation values G_ν of the chosen observation level for determining the density operator, we must select a particular density operator $\hat{\rho}_{\{\hat{G}\}} = \hat{\sigma}_{\{\hat{G}\}}$ in an unbiased manner. According to the Jaynes principle of the Maximum Entropy this density operator $\hat{\sigma}_{\{\hat{G}\}}$ must be the one which has the largest uncertainty measure

$$\eta_{\max} \equiv \max \left\{ \eta[\hat{\sigma}_{\{\hat{G}\}}] \right\}$$

and simultaneously fulfills constraints

$$\mathrm{Tr}\,(\hat{\rho}_{\{\hat{G}\}} \hat{G}_\nu) = G_\nu\,, \quad \nu = 1, 2, ..., n;$$

The MaxEnt principle is the most conservative assignment in the sense that it does not permit one to draw any conclusions not warranted by the data.

Generalized canonical DO

$$\hat{\sigma}_{\{\hat{G}\}} = \frac{1}{Z_{\{\hat{G}\}}} \exp\,(-\sum_\nu \lambda_\nu \hat{G}_\nu);$$

$$Z_{\{\hat{G}\}}(\lambda_1, ..., \lambda_n) = \mathrm{Tr}[\exp(-\sum_\nu \lambda_\nu \hat{G}_\nu)],$$

where λ_n are the Lagrange multipliers and $Z_{\{\hat{G}\}}(\lambda_1, \ldots \lambda_n)$ is the generalized partition function. By using the derivatives of the partition function we obtain the expectation values G_ν as

$$G_\nu = \mathrm{Tr}(\hat{\sigma}_{\{\hat{G}\}} \hat{G}_\nu) = -\frac{\partial}{\partial \lambda_\nu} \ln Z_{\{\hat{G}\}}(\lambda_1, ..., \lambda_n),$$

where in the case of noncommuting operators the following relation has to be used

$$\frac{\partial}{\partial a} \exp[-\hat{X}(a)] = \exp[-\hat{X}(a)] \int_0^1 \exp[\mu \hat{X}(a)] \frac{\partial \hat{X}(a)}{\partial a} \exp[-\mu \hat{X}(a)]\, d\mu.$$

The Lagrange multipliers can, in principle, be expressed as functions of the expectation values

$$\lambda_\nu = \lambda_\nu(G_1, ..., G_n).$$

Minimal observation level

The total reduction of the complete OL $\mathcal{O}_0$ results in the minimal OL $\mathcal{O}_{\rm th}$ characterized just by one observable, the photon number operator $\hat{n}$

$$\hat{\sigma}_{\rm th} = \frac{1}{Z_{\rm th}} \exp[-\lambda_{\rm th}\hat{n}].$$

To find the Lagrange multiplier $\lambda_{\rm th}$ we have to solve the equation

$$\mathrm{Tr}\,[\sigma_{\rm th}\hat{n}] = \bar{n},$$

from which we find that

$$\lambda_{\rm th} = \ln\left(\frac{\bar{n}+1}{\bar{n}}\right),$$

so that the partition function corresponding to the operator $\hat{\sigma}_{\rm th}$ reads

$$Z_{\rm th} = \{1 - \exp[-\lambda_{\rm th}]\}^{-1} = \bar{n} + 1\,.$$

The generalized canonical density operator $\hat{\sigma}_{\rm th}$ in the Fock basis

$$\hat{\sigma}_{\rm th} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}} |n\rangle\langle n|.$$

The entropy is

$$S_{\rm th} = k_B(\bar{n}+1)\ln(\bar{n}+1) - k_B\bar{n}\ln\bar{n}\,.$$

Eigenenergy measurements I

The most general phase-insensitive observation level corresponds to the case when *all* diagonal elements $P_n = \langle n|\hat{\rho}|n\rangle$ of the density operator $\hat{\rho}$

$$\mathcal{O}_{\rm A} \equiv \{\hat{P}_n = |n\rangle\langle n|;\quad \forall n\}; \qquad \sum \hat{P}_n = \hat{I}$$

The generalized canonical operator $\hat{\sigma}_{\rm A}$ at the observation level $\mathcal{O}_{\rm A}$ reads

$$\hat{\sigma}_{\rm A} = \frac{1}{Z_{\rm A}} \exp\left[-\sum_{n=0}^{\infty} \lambda_n |n\rangle\langle n|\right];$$

with the partition function

$$Z_{\rm A} = \mathrm{Tr}\left\{\exp\left[-\sum_{n=0}^{\infty} \lambda_n |n\rangle\langle n|\right]\right\} = \sum_{n=0}^{\infty} \exp[-\lambda_n].$$

The entropy $S_{\rm A}$ at the observation level $\mathcal{O}_{\rm A}$

$$S_{\rm A} = k_B \ln Z_{\rm A} + k_B \sum_{n=0}^{\infty} \lambda_n P_n.$$

Eigenenergy measurements II

We find Lagrange multipliers λ_n from an infinite set of equations:

$$P_n = \mathrm{Tr}[\hat{\sigma}_{\mathrm{A}} \hat{P}_n] = \frac{\mathrm{e}^{-\lambda_n}}{Z_{\mathrm{A}}}; \quad \forall n,$$

from which we find $\lambda_n = -\ln[Z_{\mathrm{A}} P_n]$. If we insert λ_n into the expression for generalized partition function we obtain for the entropy S_{A} the expression

$$S_{\mathrm{A}} = -k_B \sum_{n=0}^{\infty} P_n \ln P_n,$$

derived by Shannon.

The Lagrange multipliers can be expressed as $\lambda_n = -\ln P_n$ and the generalized canonical density operator reads

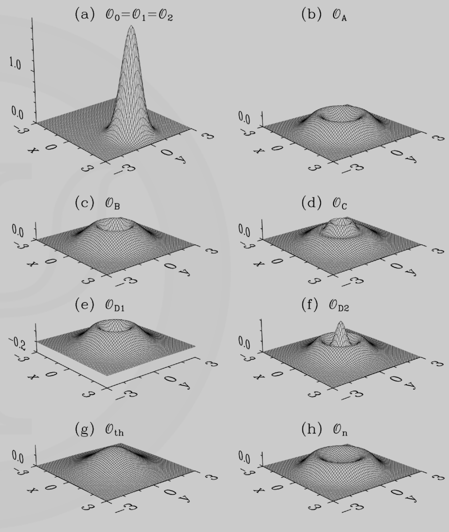
$$\hat{\sigma}_{\mathrm{A}} = \sum_{n=0}^{\infty} P_n |n\rangle\langle n|; \qquad \sum_{n=0}^{\infty} P_n = 1.$$

The probability distribution P_n can be arbitrary as soon as it is normalized!

Example - coherent states

coherent state $|\alpha\rangle$

$\mathcal{O}_0 \equiv \{(\hat{a}^\dagger)^k \hat{a}^l;\ \forall k, l\}$
$\mathcal{O}_{\mathrm{th}} \equiv \{\hat{a}^\dagger \hat{a}\}$
$\mathcal{O}_1 \equiv \{\hat{a}^\dagger \hat{a}, \hat{a}^\dagger, \hat{a}\}$
$\mathcal{O}_2 \equiv \{\hat{a}^\dagger \hat{a}, (\hat{a}^\dagger)^2, \hat{a}^2, \hat{a}^\dagger, \hat{a}\}$
$\mathcal{O}_{\mathrm{A}} \equiv \{\hat{P}_n = |n\rangle\langle n|;\ \ \forall n\}$
$\mathcal{O}_{\mathrm{n}} \equiv \{\hat{n}, \hat{n}^2\}$



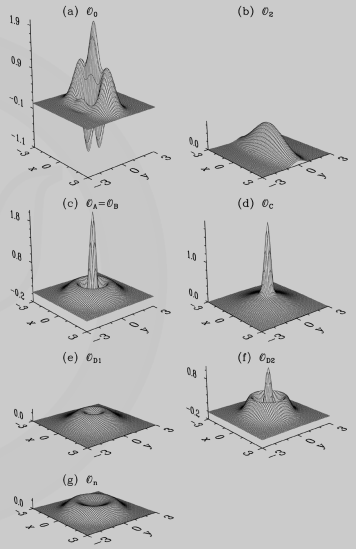
V.Buzek, G.Adam, and G.Drobny, *Ann. Phys. (N.Y.)* 245, 37 (1996);

Example II.

even coherent state

$$|\alpha_{\rm e}\rangle = N_e^{1/2}\left(|\alpha\rangle + |-\alpha\rangle\right)$$

$$N_e^{-1} = 2\left[1+\exp(-2|\alpha|^2)\right]$$



V.Buzek, G.Adam, and G.Drobny, *Ann. Phys. (N.Y.)* 245, 37 (1996);

Q-Tomography & Incomplete Data

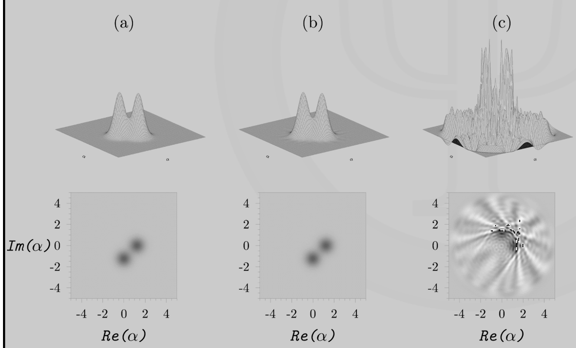
complete data:

$$\hat\rho = \frac{1}{Z_0}\exp\left[-\int_0^\pi d\theta\int_{-\infty}^{\infty}dx_\theta\,|x_\theta\rangle\langle x_\theta|\lambda(x_\theta)\right]$$

$$P_\theta(x_\theta) = \mathrm{Tr}\left[\hat P_\theta(x_\theta)\hat\rho\right]; \qquad \hat P_\theta(x_\theta) = |x_\theta\rangle\langle x_\theta|$$

incomplete data:

$$\boxed{\hat\rho = \frac{1}{Z}\exp\left(\lambda_0\hat n + \sum_{l=1}^{N_x}\sum_{m=1}^{N_\theta}\lambda_{l,m}|x_{\theta_m}^{(l)}\rangle\langle x_{\theta_m}^{(l)}|\right)}$$

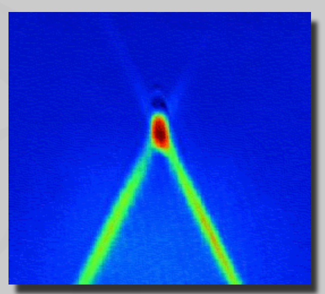


- $N_\theta = 5,\ N_x = 15$
- error $\Delta = \sum_{n_1,n_2}\left[(\rho_1)_{n_1,n_2} - (\tilde\rho_1)_{n_1,n_2}\right]^2$
- MaxEnt $\Delta = 1.0\times 10^{-5}$
- pattern functions D = 0.76
- **number of cuts!**

V.Buzek, G.Drobny, R.Derka, G.Adam, and H.Wiedemann, *Chaos, Solitons & Fractals,* 10, 981 (1999)

Laser Cooling of CS Atoms in Optical Traps

Salomon et al. use cesium atoms pre-cooled in a magneto-optical from a vapor cell to load a dipole trap. This non dissipative trap is realized by crossing two focused Nd:YAG laser beams. The initial density is a few times 10^{12} atoms/cm^3, corresponding to a million atoms at a temperature of 20 micro K, i.e. a velocity dispersion of 10 recoil velocities (v_{rec} = 3.5mm/s for cesium). The lifetime of the crossed dipole trap is of the order of one to two seconds. In this trap, the atoms are further cooled by a subrecoil cooling technique, Raman cooling: the number of atoms is increased with a velocity close to v=0 by controlling the momentum exchanges between the atoms and the laser photons. Using chirped Raman pulses, it is possible to get the final velocity dispersion of 3 v_{rec}. It is also possible to cool the atoms evaporatively just by lowering slowly the trapping Nd:YAG laser power. By combining these two cooling methods, it is possible to cool the atomic sample to 640 nK, which corresponds to a velocity dispersion of 1.8 v_{rec}. The phase space density is then of the order of 5.10^{-4}.



Absorption picture of the crossed dipole trap taken just after the MOT has been switched off. The atoms initially at the crossing of the two Nd:YAG beams remain trapped for one to two seconds, whereas the others fall due to gravity. The trapping volume is of the order of (40 microns)

M.Morinaga, I.Bouchole, J.-C.Karam, and C.Salomon, *Phys. Rev. Lett.* 83, 4037 (1999).

Reconstruction of vibrational states of trapped CS atoms

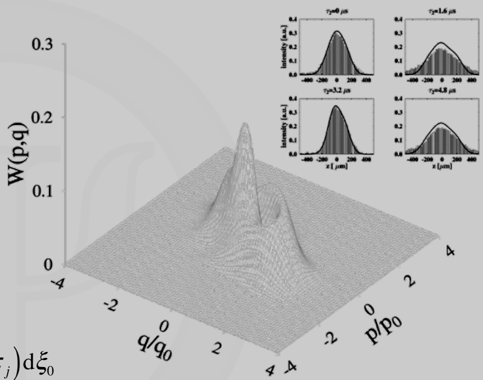
MaxEnt reconstruction of density operator

$$\hat{\rho}_r = \frac{1}{Z}\exp\left(\lambda_n \hat{n} + \sum_{j=1}^{N_x}\sum_{k=1}^{N_\theta}\lambda_{j,k}\hat{F}_{jk}\right)$$

l – Lagrange multipliers

$\hat{F}$ – Observables

$$\hat{F}_{\tau_j}(z_k) = T^{-1}\int F_0(\xi_0)\hat{U}^{+}(\tau_j)\left|\frac{z_k-\xi_0}{T}\right\rangle\left\langle\frac{z_k-\xi_0}{T}\right|\hat{U}(\tau_j)\,\mathrm{d}\xi_0$$



Wigner function reconstructed from experimental data from ENS, Paris.

V.Buzek and R.Derka: "*Quantum observations*" in *Coherence and Statistics of Photons and Atoms* ed. J.Perina (John Wiley & Sons, New York, 2001) pp. 198–261

G.Drobny and V.Buzek: "Reconstruction of motional states of neutral atoms via *MaxEnt* principle." *Phys. Rev. A* 65, 053410 (2002)

WF via the parity operator

$$\boxed{W(\alpha) = 2.Tr\left(\hat{P}.\rho(\alpha)\right)}$$

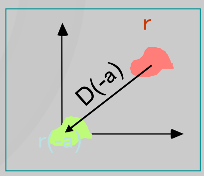
Parity operator

$$P|n\rangle = (-1)^{\hat{N}}|n\rangle = \begin{cases} +|n\rangle & \text{If n even} \\ -|n\rangle & \text{If n odd} \end{cases}$$

$$\rho(\alpha) = D(-\alpha)\rho D(\alpha)$$

$$D(-\alpha) = \exp\left(\alpha^* a - \alpha a^+\right)$$

Displacement operator

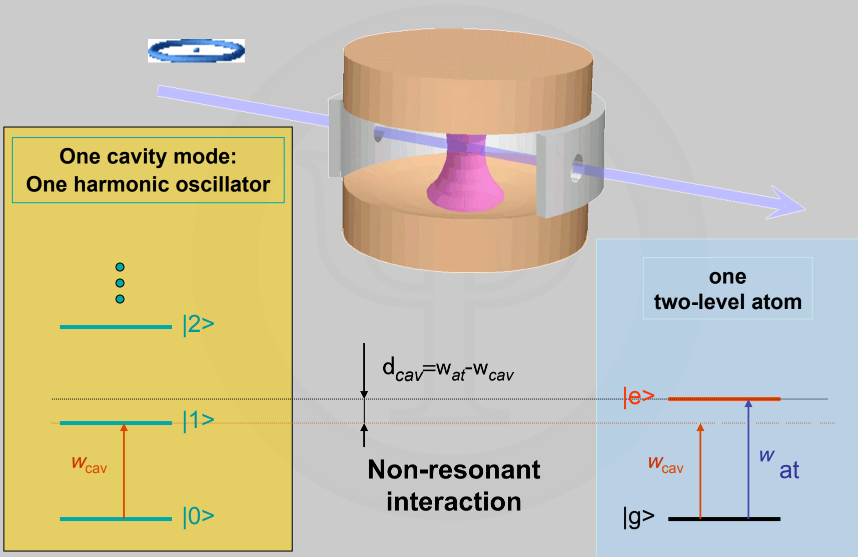


Realized with a classical source

W is the average value of parity operator in the "shifted" state

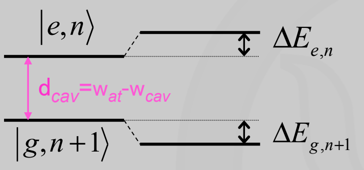
L.G.Lutterbach & L.Davidovich, *PRL 78*, 2547 (1997)

Measuring the field parity?

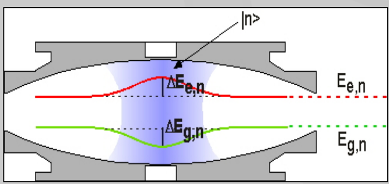


Non-resonant atom-field interaction

- One atom interacting with n photons:



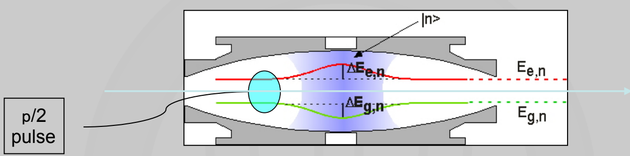
- Copling: $\langle e,n|\hat{V}|g,n+1\rangle = \Omega\sqrt{n+1}$
- "Vacuum Rabi frequency"
- Dispersive regime: $\delta \gg \Omega\sqrt{n+1}$



$$\Delta E_{e,n} \simeq \hbar\frac{\Omega^2}{4\delta}(n+1)$$
$$\Delta E_{g,n} \simeq -\hbar\frac{\Omega^2}{4\delta}n$$

- The cavity frequency is shifted: atom index of refraction
- atomic frequency: light shift and Lamb shift

Measuring the parity: Phase shift of the atomic state



$$\phi_0 = \frac{\Omega^2}{2\delta}.T_{eff}$$

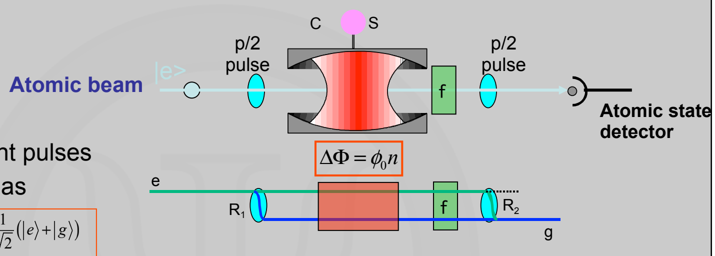
$$\frac{1}{\sqrt{2}}\left(|e\rangle+|g\rangle\right)\otimes|n\rangle \longrightarrow \frac{1}{\sqrt{2}}e^{i\varphi_n}\left(|e\rangle+e^{i\Delta\Phi(n)}|g\rangle\right)\otimes|n\rangle \quad , \quad \Delta\Phi(n)=\phi_0\left(n+1/2\right)$$

Atomic coherence is phase shifted proportionally to n

For $\phi_0=\pi$ $\quad \frac{1}{\sqrt{2}}\left(|e\rangle+|g\rangle\right)\otimes|n\rangle \longrightarrow \frac{1}{\sqrt{2}}e^{i\varphi_n}\left(|e\rangle+i(-1)^n|g\rangle\right)\otimes|n\rangle$

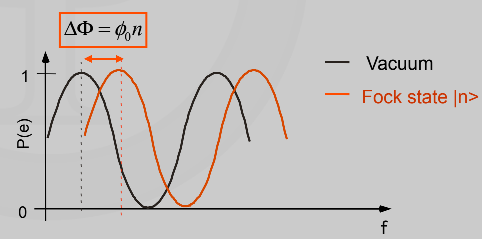
Even and odd number states are correlated to two orthogonal atomic states.

Measuring atomic phase shifts by Ramsey interferometry



apply two $\pi/2$ resonant pulses R1 and R2 which acts as beam splitters

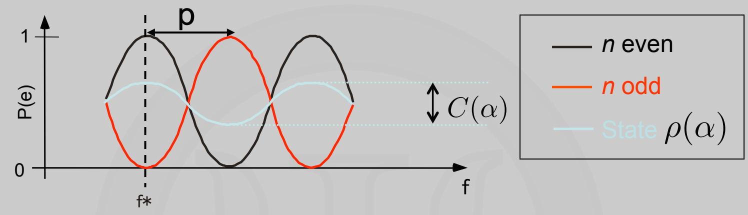
$$|e\rangle \longrightarrow \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$$
$$|g\rangle \longrightarrow \frac{1}{\sqrt{2}}(-|e\rangle + |g\rangle)$$



interferences between two undistinguishable quantum paths: "Ramsey fringes"

Measuring the parity of the photon number

F_0=p



C_{even}=+1 for state |*2n*>

C_{odd}=-1 for state |*2n+1*>

$$C(\alpha) = C_{even}.P_{even} + C_{odd}.P_{odd} = \left\langle (-1)^{\hat{N}} \right\rangle = \frac{W(\alpha)}{2}$$

• Sensitivity to fringe contrast:

Assume:

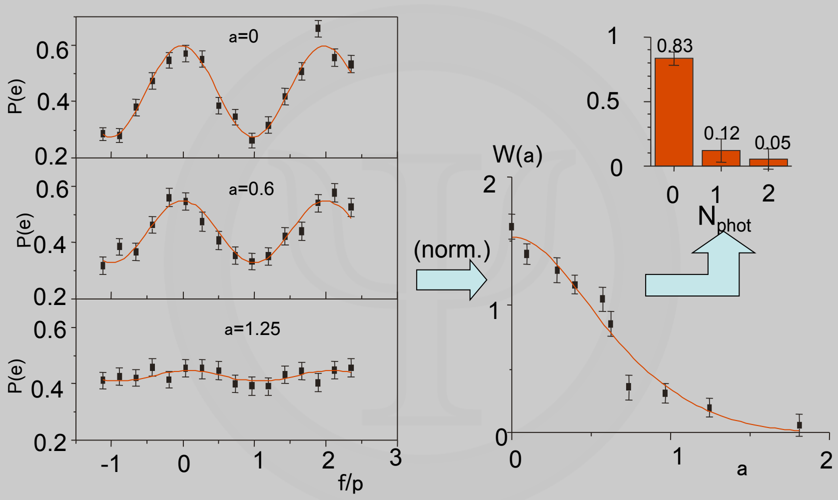
$$C_{even} = -C_{odd} = \eta \leq 1$$

Then:

$$\boxed{\frac{C(\alpha)}{\eta} = \frac{W(\alpha)}{2}}$$

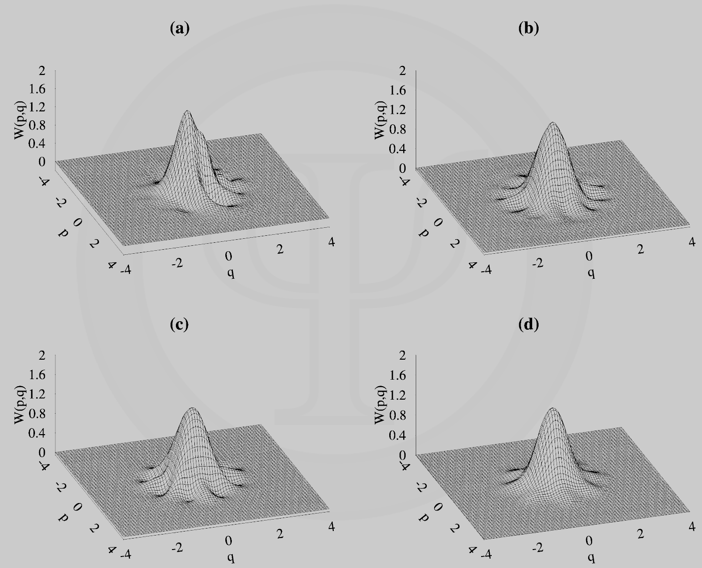
Finite fringe contrast only affects the signal to noise
One still measures W by renormalizing the signal

WF of the vacuum state

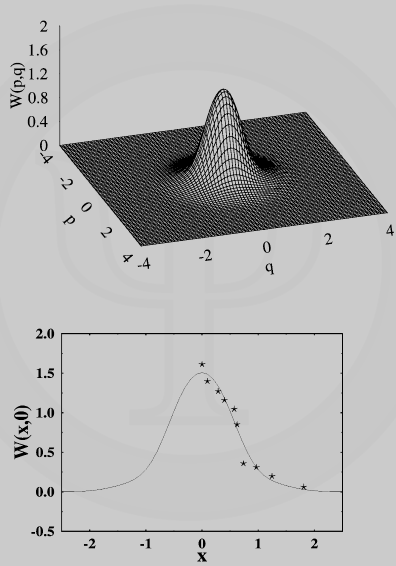


P.Bertet, A.Auffeves, P.Maioli, S.Osnaghi, T.Meunier, M.Brune, J.M.Raimond, & S.Haroche, *PRL.* 89, 200402 (2002)

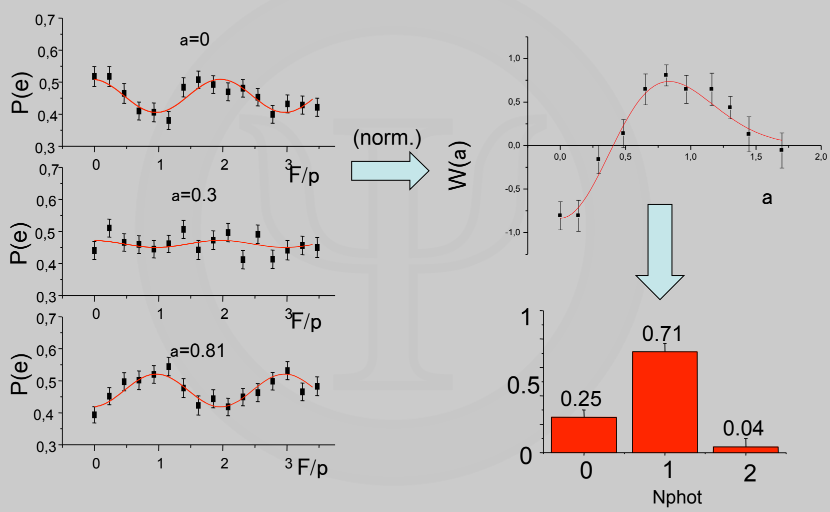
Reconstructed WF of the vacuum state I



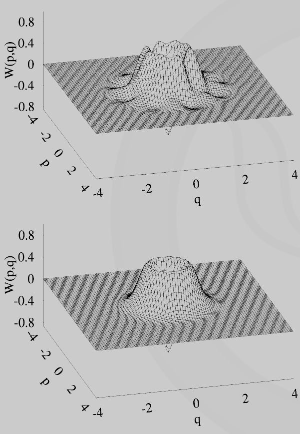
Wigner function of the vacuum state II

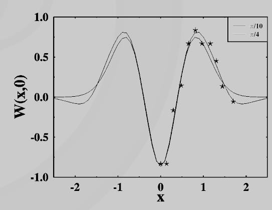


Reconstructed WF of the Fock state



Wigner function of the Fock state II





Qubit – encoding orientation

Pure state of a spin -1/2 particle

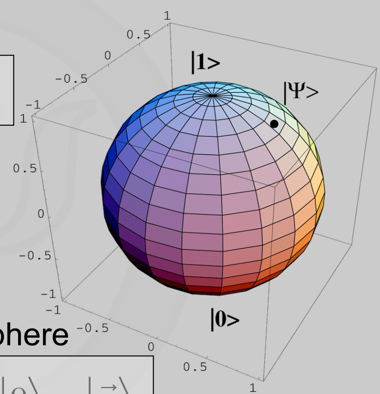
$$\left|\psi\right\rangle = \cos\vartheta/2\left|1\right\rangle + \mathrm{e}^{i\varphi}\sin\vartheta/2\left|0\right\rangle$$

2-d Hilbert space

density operator

$$\hat{\rho} = \frac{1}{2}\left(\hat{I} + \vec{n}.\vec{\sigma}\right) = \frac{1}{2}\left(\hat{I} + n_x\hat{\sigma}_x + n_y\hat{\sigma}_y + n_z\hat{\sigma}_z\right)$$

$$\rho = \frac{1}{2}\left(1 + \vec{n}.\vec{\sigma}\right) = \frac{1}{2}\begin{pmatrix} 1 \\ n_x \\ n_y \\ n_z \end{pmatrix} \leftrightarrow \vec{n} = \left(n_x, n_y, n_z\right)$$

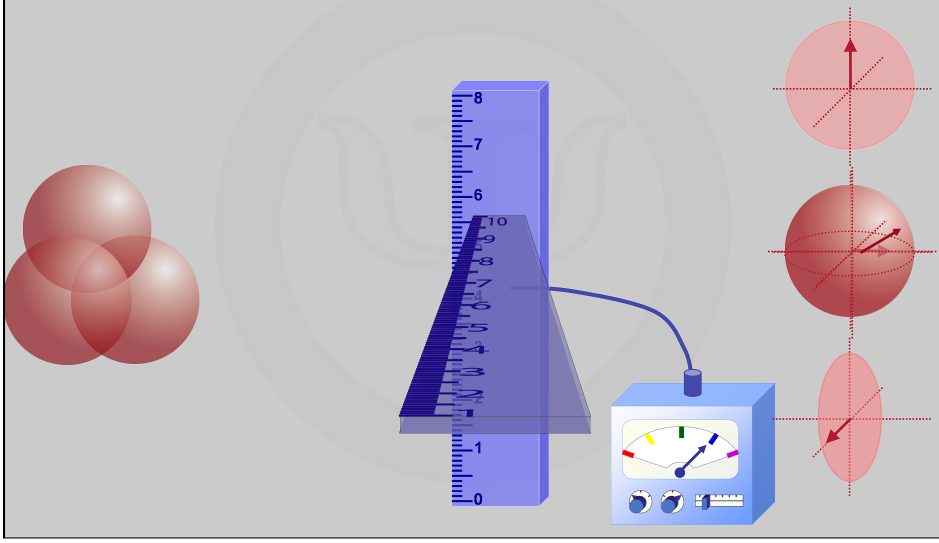


State space – Bloch (Poincare) sphere

$$\left|\psi\right\rangle = \cos\vartheta/2\left|1\right\rangle + \mathrm{e}^{i\varphi}\sin\vartheta/2\left|0\right\rangle = \left|\vec{n}\right\rangle$$

$$\vec{n}.\vec{\sigma}\left|\vec{n}\right\rangle = \vec{n}\left|\vec{n}\right\rangle$$

Complete State Measurement



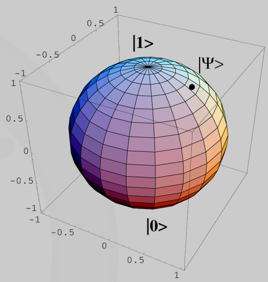
MaxEnt reconstruction of qubit

Pure state of a spin -1/2 particle

$$\left|\psi\right\rangle = \cos\vartheta/2\left|1\right\rangle + e^{i\varphi}\sin\vartheta/2\left|0\right\rangle$$

density operator

$$\hat{\rho} = \frac{1}{2}\left(\hat{I} + \vec{n}.\vec{\sigma}\right) = \frac{1}{2}\left(\hat{I} + n_x\hat{\sigma}_x + n_y\hat{\sigma}_y + n_z\hat{\sigma}_z\right)$$



MaxEnt reconstruction

OL	$\hat{\sigma}_z$	$\hat{\sigma}_x$	$\hat{\sigma}_y$	**reconstructed density operator**
$\mathcal{O}_{\mathrm{A}}^{(1)}$	•			$\hat{\rho}_{\mathrm{A}} = \frac{1}{2}\left(\hat{I} + n_z\hat{\sigma}_z\right)$
$\mathcal{O}_{\mathrm{B}}^{(1)}$	•	•		$\hat{\rho}_{\mathrm{B}} = \frac{1}{2}\left(\hat{I} + n_z\hat{\sigma}_z + n_x\hat{\sigma}_x\right)$
$\mathcal{O}_{\mathrm{comp}}^{(1)}$	•	•	•	$\hat{\rho}_{\mathrm{comp}} = \frac{1}{2}\left(\hat{I} + n_z\hat{\sigma}_z + n_x\hat{\sigma}_x + n_y\hat{\sigma}_y\right)$

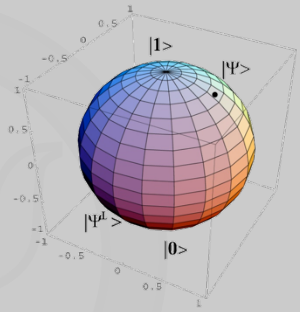
Reconstruction of Qubits

• **Pure state of a qubit**

$$\left|\psi\right\rangle = \cos\vartheta/2\left|1\right\rangle + e^{i\varphi}\sin\vartheta/2\left|0\right\rangle = \left|\vec{n}\right\rangle$$

density operator

$$\hat{\rho} = \frac{1}{2}\left(\hat{I} + \vec{n}.\vec{\sigma}\right) = \frac{1}{2}\left(\hat{I} + n_x\hat{\sigma}_x + n_y\hat{\sigma}_y + n_z\hat{\sigma}_z\right)$$



1) exact meanvalues – infinite ensembles

2) "What is the best a posteriori estimation of a quantum state when a measurement is performed on a finite (arbitrary small) number of elements of the ensemble?"

S.Massar & S.Popescu: ***Phys. Rev. Lett.*** **74, 1259 (1995)**
C.W.Helstrom: ***Quantum Detection and Estimation Theory*** **(Academic, NY, 1976)**
A.S.Holevo: ***Probabilistic and Statistical Aspects of Quantum Theory*** **(North-Holland, Amsterdam, 1982)**

Optimal state reconstructions

- Quantum states of light and the Wigner function
- Quantum tomography
- Quantum state reconstructions on various observation levels
- Maximum entropy (MaxEnt) principle
- Vibrational motion of neutral atoms
- Reconstruction of a cavity field via measurement of parity operator
- Max-Ent reconstruction of a state of a qubit